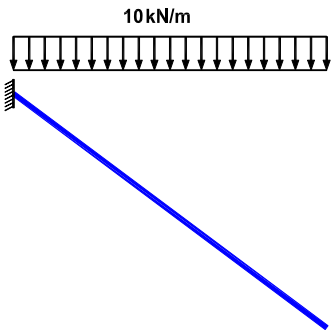
LICENCIATURA EM ENGENHARIA CIVIL

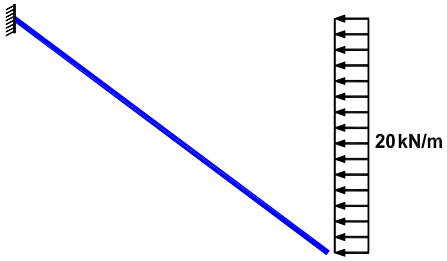
ESTÁTICA

DIAGRAMAS

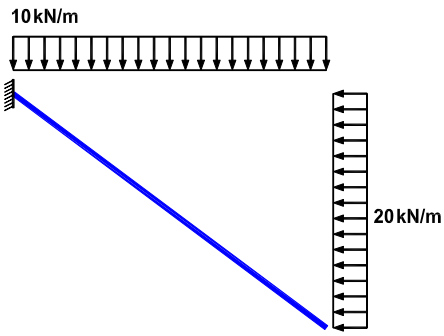
ESFORÇO AXIAL - ESFORÇO TRANSVERSO - MOMENTO FLECTOR



EXERCÍCIO 1



EXERCÍCIO 2



EXERCÍCIO 3

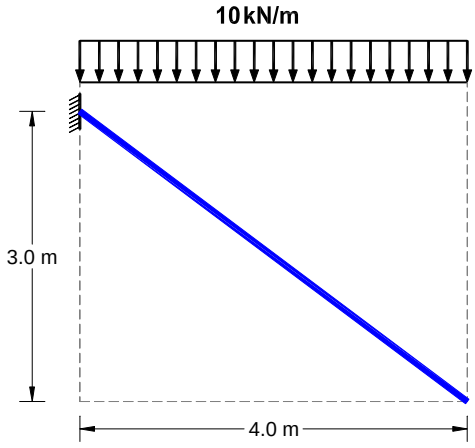
EXERCÍCIOS EXEMPLIFICATIVOS APRESENTADOS NAS AULAS TEÓRICAS

ISABEL ALVIM TELES

EXERCÍCIO 1

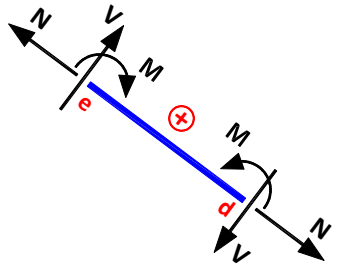
Sem calcular previamente as reacções, determine as expressões analíticas do esforço axial (**N**), esforço transverso (**V**) e momento flector (**M**) da viga encastrada representada na figura.

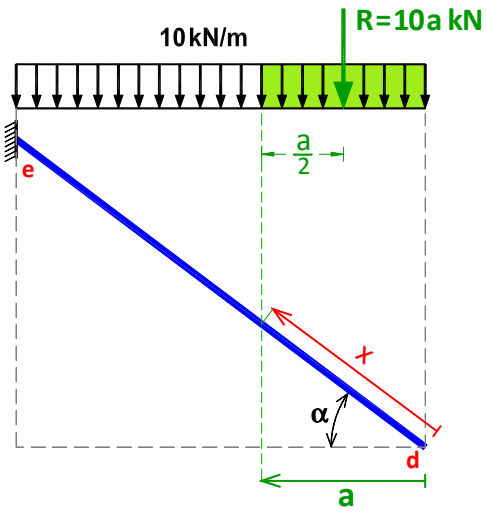
Desenhe os diagramas dos esforços (**N**, **V** e **M**).



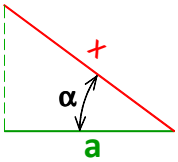
EXERCÍCIO 1 - RESOLUÇÃO

CONVENÇÃO DE SINAIS
DA
RESISTÊNCIA DOS MATERIAIS





$\alpha = \text{arctg}\frac{3}{4} = 36{,}8699^\circ \Rightarrow \begin{cases} \cos\alpha = 0{,}8 \\ \text{sen}\,\alpha = 0{,}6 \end{cases}$



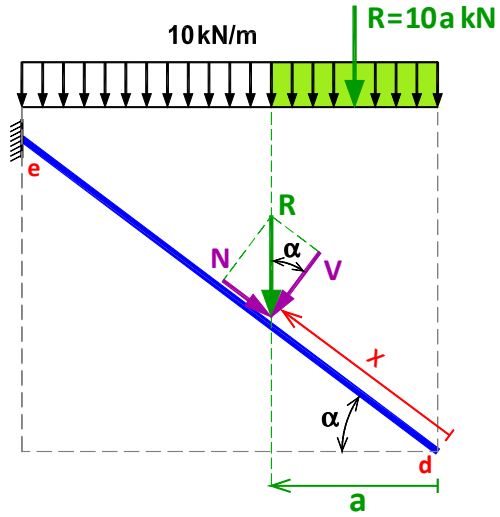
$a = x \cos\alpha$

$a = 0{,}8\,x$

$M(a) = -10\,a\,\frac{a}{2} = -5\,a^2 \quad \text{kNm}$

$M(x) = -5\,(0{,}8\,x)^2 = -3{,}2\,x^2 \quad \text{kNm} \Rightarrow \text{curva (2º grau)}$

Concavidade do diagrama de Momentos: $\frac{d^2M}{dx^2} < 0 \Rightarrow$ concavidade $\cup$



R – resultante de todas as forças à direita do ponto.

$\mathbf{R} = 10\,\mathbf{a}$ kN

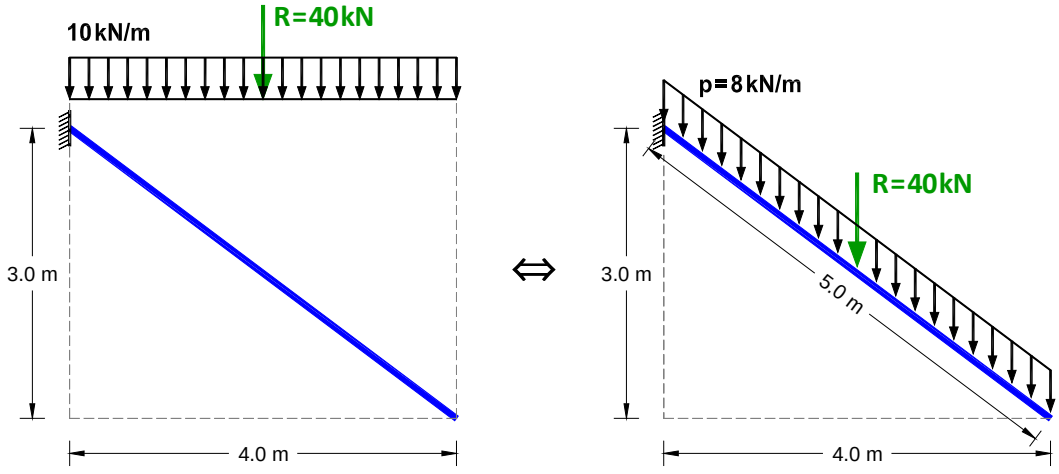
$\mathbf{N} = \mathbf{R}\ \text{sen}\,\boldsymbol{\alpha} = 10\,\mathbf{a}\ \text{sen}\,\boldsymbol{\alpha}$
$\mathbf{V} = \mathbf{R}\ \cos\boldsymbol{\alpha} = 10\,\mathbf{a}\ \cos\boldsymbol{\alpha}$

$$\begin{cases} N(a) = 10\,a\,\text{sen}\,\alpha \\ V(a) = 10\,a\cos\alpha \end{cases} \xrightarrow{a = x\cos\alpha} \begin{cases} N(x) = 10\,\mathbf{x}\cos\alpha\,\text{sen}\,\alpha = 10\,\mathbf{x}\times 0{,}8\times 0{,}6 = 4{,}8\,\mathbf{x}\ \text{kN} \Rightarrow \text{recta} \\ V(x) = 10\,\mathbf{x}\cos\alpha\cos\alpha = 10\,\mathbf{x}\times 0{,}8\times 0{,}8 = 6{,}4\,\mathbf{x}\ \text{kN} \Rightarrow \text{recta} \end{cases}$$

Extremidade da consola: (x=0) $\begin{cases} N = 0 \\ V = 0 \\ M = 0 \end{cases}$

Encastramento: (x=5 m) $\begin{cases} N = 4{,}8\times 5 = 24\ \text{kN} \\ V = 6{,}4\times 5 = 32\ \text{kN} \\ M = -3{,}2\times 5^2 = -80\ \text{kNm} \end{cases}$

EXERCÍCIO 1 - OUTRA RESOLUÇÃO

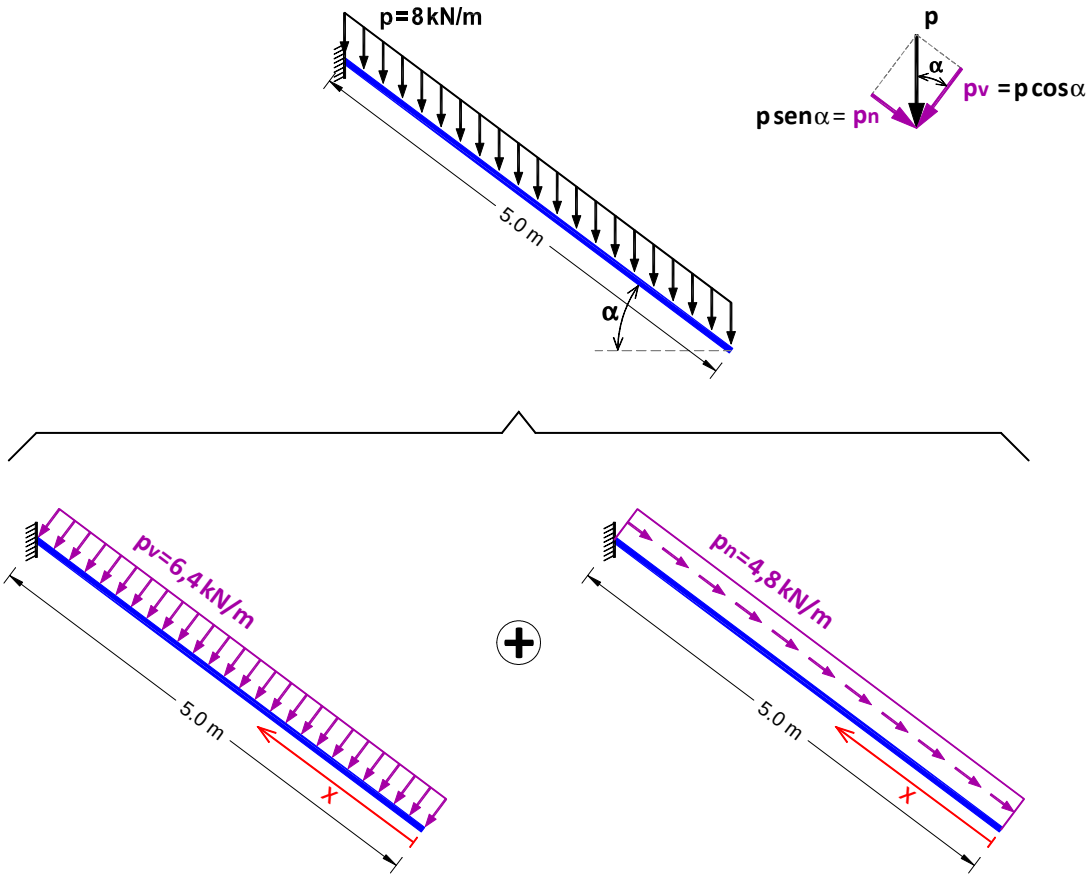


Para os dois sistemas serem equivalentes, a Resultante tem que ser igual, ou seja:

R = 10 x 4 = 40 kN ⇒ p x 5 = R = 40 kN ⇒ p = 8 kN/m

$p_v = p \cos\alpha = 8 \times 0{,}8 = 6{,}4$ kN/m

$p_n = p \operatorname{sen}\alpha = 8 \times 0{,}6 = 4{,}8$ kN/m



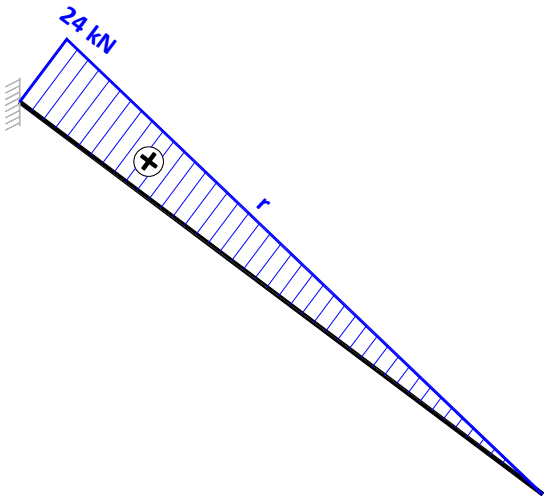
$$\begin{cases} \mathbf{N}(x) = pn\,.\,\mathbf{x} = 4{,}8\ \mathbf{x}\ \text{kN} & \Rightarrow \text{recta} \\ \mathbf{V}(x) = pv\,.\,\mathbf{x} = 6{,}4\ \mathbf{x}\ \text{kN} & \Rightarrow \text{recta} \\ \mathbf{M}(x) = -6{,}4\mathbf{x}\,\frac{\mathbf{x}}{2} = -3{,}2\,\mathbf{x}^2\ \text{kNm} & \Rightarrow \text{curva (2º grau)} \end{cases}$$

Concavidade do diagrama de Momentos: $\dfrac{d^2M}{dx^2} < 0 \Rightarrow$ concavidade U

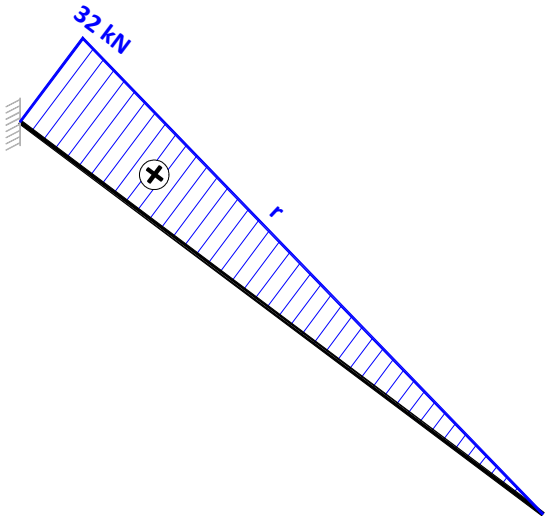
Extremidade da consola: (x=0) $\begin{cases} N = 0 \\ V = 0 \\ M = 0 \end{cases}$

Encastramento: (x=5 m) $\begin{cases} N = 4{,}8 \times 5 = 24\ \text{kN} \\ V = 6{,}4 \times 5 = 32\ \text{kN} \\ M = -3{,}2 \times 5^2 = -80\ \text{kNm} \end{cases}$

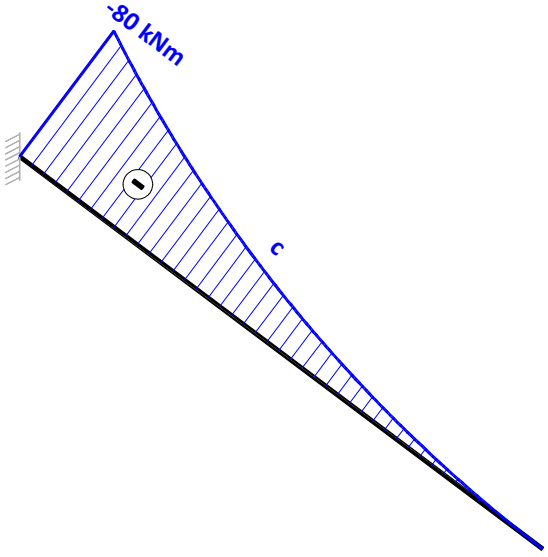
DIAGRAMAS



N
ESFORÇO AXIAL



V
ESFORÇO TRANSVERSO

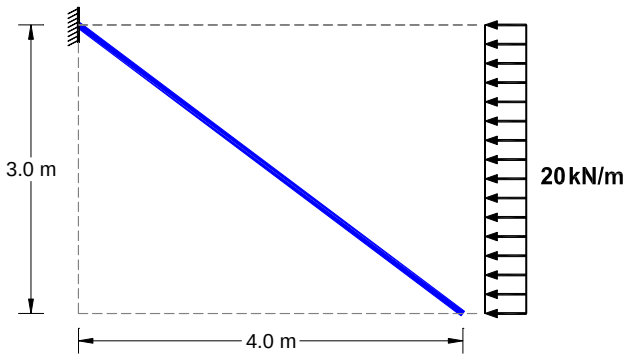


M
MOMENTO FLECTOR

EXERCÍCIO 2

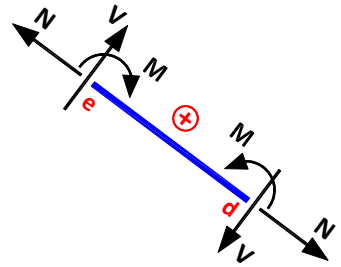
Sem calcular previamente as reacções, determine as expressões analíticas do esforço axial (**N**), esforço transverso (**V**) e momento flector (**M**) da viga encastrada representada na figura.

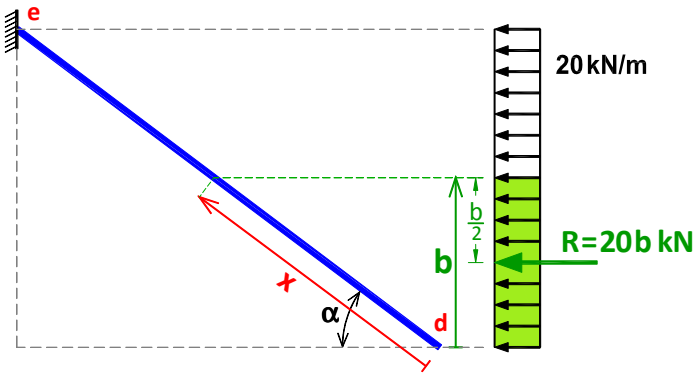
Desenhe os diagramas dos esforços (**N**, **V** e **M**).



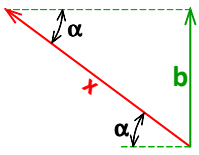
EXERCÍCIO 2 - RESOLUÇÃO

CONVENÇÃO DE SINAIS
DA
RESISTÊNCIA DOS MATERIAIS





$\alpha = \text{arctg}\frac{3}{4} = 36{,}8699^\circ \Rightarrow \begin{cases} \cos\alpha = 0{,}8 \\ \text{sen}\,\alpha = 0{,}6 \end{cases}$

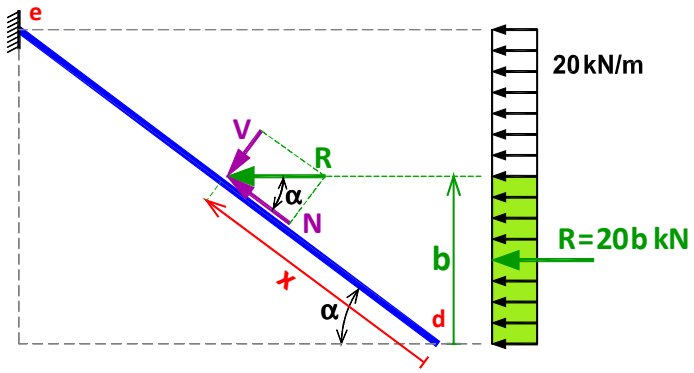


$b = x \text{ sen } \alpha$
$b = 0{,}6\,x$

$M(b) = -20\,b\,\frac{b}{2} = -10\,b^2 \quad \text{kNm}$

$M(x) = -10\,(0{,}6\,x)^2 = -3{,}6\,x^2 \quad \text{kNm} \Rightarrow \text{curva (2º grau)}$

Concavidade do diagrama de Momentos: $\frac{d^2M}{dx^2} < 0 \Rightarrow$ concavidade U



R – resultante de todas as forças à direita do ponto.

$\mathbf{R} = 20\ \mathbf{b}$ kN

$\mathbf{N} = \mathbf{R}\cos\boldsymbol{\alpha} = 20\ \mathbf{b}\cos\boldsymbol{\alpha}$

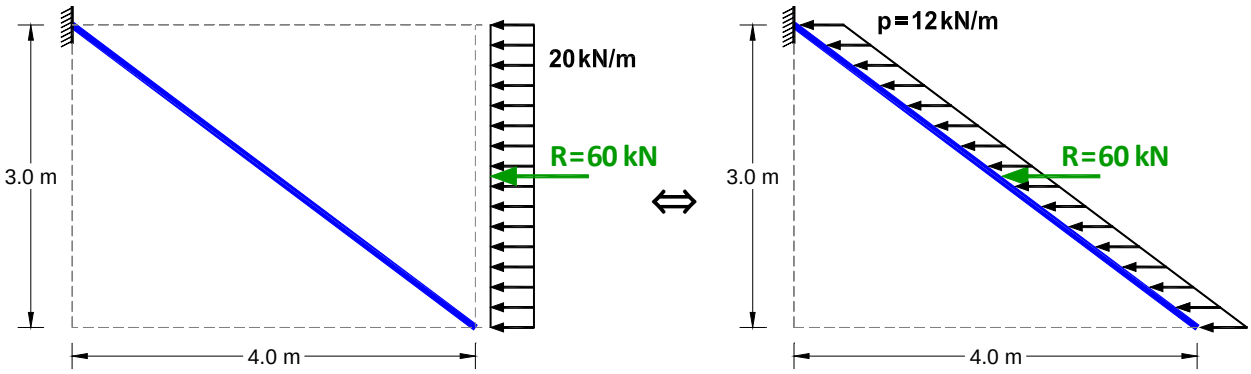
$\mathbf{V} = \mathbf{R}\,\text{sen}\,\boldsymbol{\alpha} = 20\ \mathbf{b}\,\text{sen}\,\boldsymbol{\alpha}$

$$\begin{cases} N(b) = -20\,b\cos\alpha \\ V(b) = 20\,b\,\text{sen}\,\alpha \end{cases} \xrightarrow{b = x\,\text{sen}\,\alpha} \begin{cases} N(x) = -20\,\mathbf{x}\,\text{sen}\,\alpha\cos\alpha = -20\,\mathbf{x}\times 0{,}6\times 0{,}8 = -9{,}6\,\mathbf{x}\ \text{kN} \Rightarrow \text{recta} \\ V(x) = 20\,\mathbf{x}\,\text{sen}\,\alpha\,\text{sen}\,\alpha = 20\,\mathbf{x}\times 0{,}6\times 0{,}6 = 7{,}2\,\mathbf{x}\ \text{kN} \Rightarrow \text{recta} \end{cases}$$

Extremidade da consola: (x=0) $\begin{cases} N = 0 \\ V = 0 \\ M = 0 \end{cases}$

Encastramento: (x=5 m) $\begin{cases} N = -9{,}6\times 5 = -48\ \text{kN} \\ V = 7{,}2\times 5 = 36\ \text{kN} \\ M = -3{,}6\times 5^2 = -90\ \text{kNm} \end{cases}$

EXERCÍCIO 2 - OUTRA RESOLUÇÃO

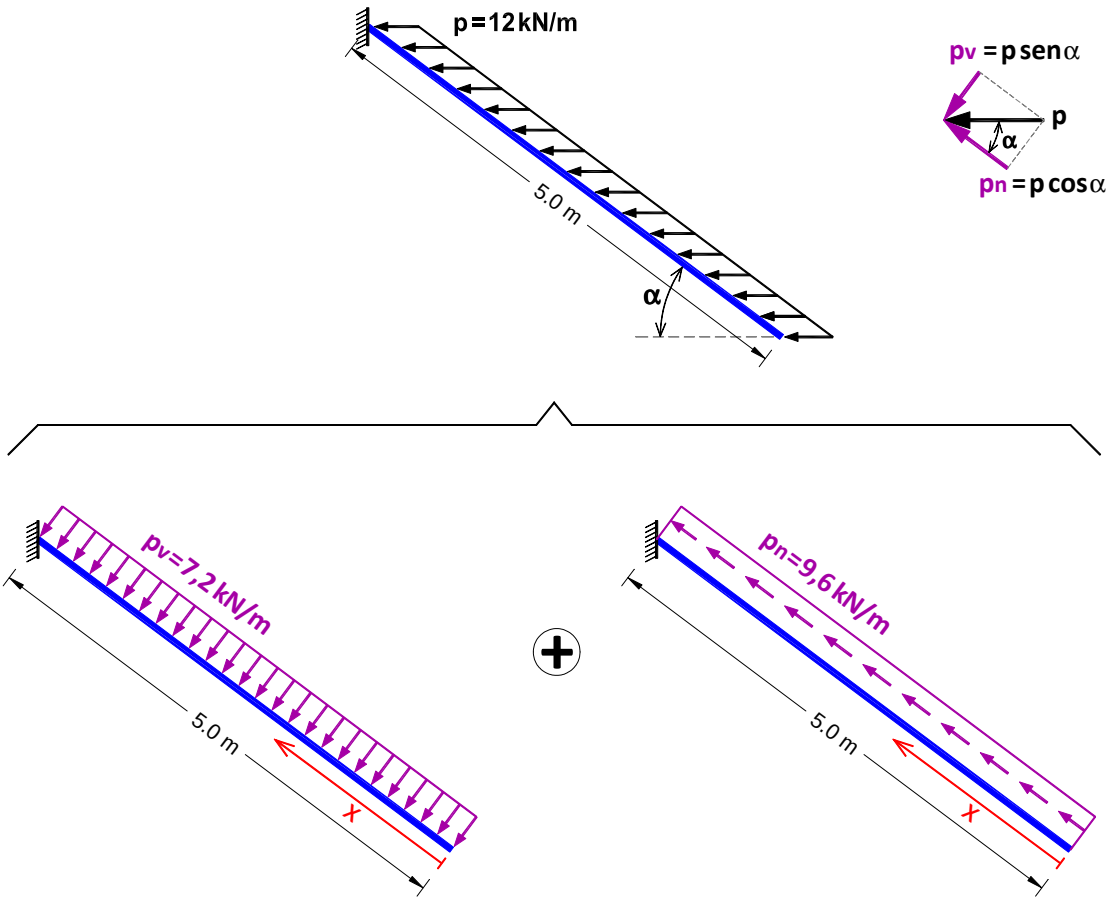


Para os dois sistemas serem equivalentes, a Resultante tem que ser igual, ou seja:

R = 20 x 3 = 60 kN ⇒ p x 5 = R = 60 kN ⇒ p = 12 kN/m

$p_v = p \operatorname{sen} \alpha = 12 \times 0{,}6 = 7{,}2$ kN/m

$p_n = p \cos \alpha = 12 \times 0{,}8 = 9{,}6$ kN/m



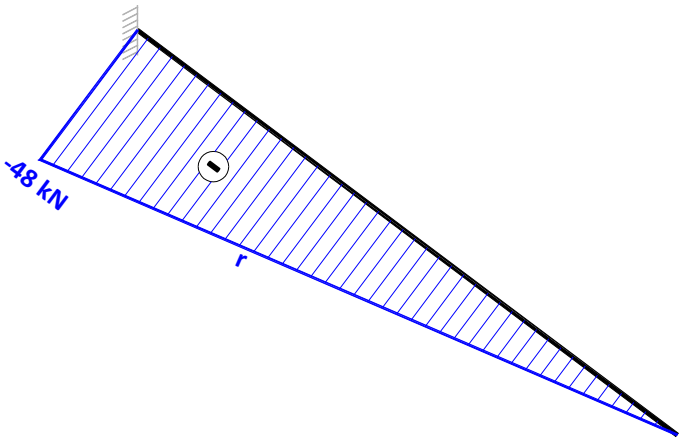
$$\begin{cases} \mathbf{N}(x) = -pn \,.\, \mathbf{x} = -9{,}6\ \mathbf{x} \ \text{kN} & \Rightarrow \text{recta} \\ \mathbf{V}(x) = pv \,.\, \mathbf{x} = 7{,}2\ \mathbf{x} \ \text{kN} & \Rightarrow \text{recta} \\ \mathbf{M}(x) = -7{,}2\,\mathbf{x}\,\frac{\mathbf{x}}{2} = -3{,}6\,\mathbf{x}^2 \ \text{kNm} & \Rightarrow \text{curva (2º grau)} \end{cases}$$

Concavidade do diagrama de Momentos: $\frac{d^2M}{dx^2} < 0 \Rightarrow$ concavidade U

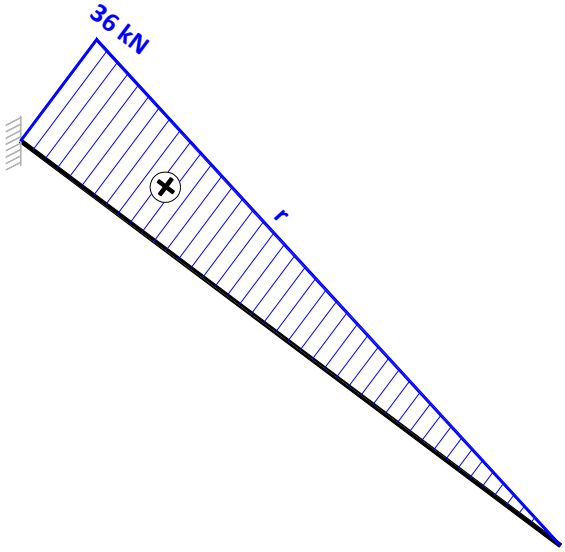
Extremidade da consola: (x=0) $\begin{cases} N = 0 \\ V = 0 \\ M = 0 \end{cases}$

Encastramento: (x=5 m) $\begin{cases} N = -9{,}6 \times 5 = -48 \ \text{kN} \\ V = 7{,}2 \times 5 = 36 \ \text{kN} \\ M = -3{,}6 \times 5^2 = -90 \ \text{kNm} \end{cases}$

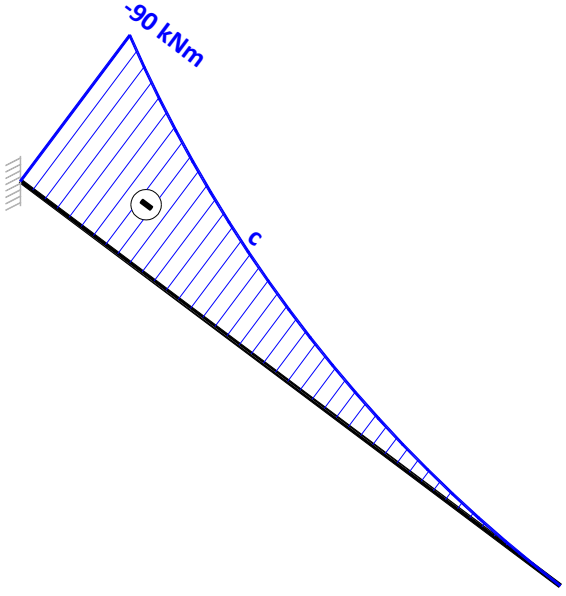
DIAGRAMAS



N
ESFORÇO AXIAL



V
ESFORÇO TRANSVERSO

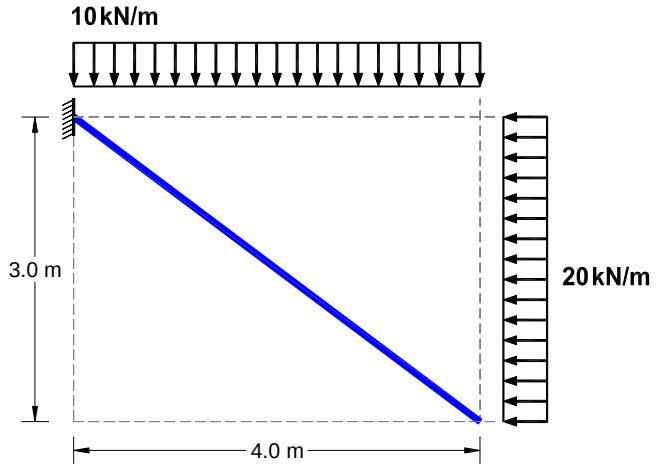


M
MOMENTO FLECTOR

EXERCÍCIO 3

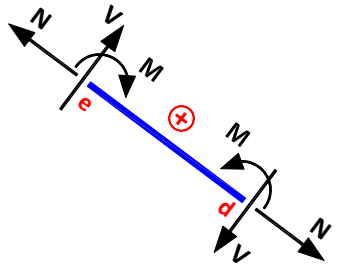
Sem calcular previamente as reacções, determine as expressões analíticas do esforço axial (**N**), esforço transverso (**V**) e momento flector (**M**) da viga encastrada representada na figura.

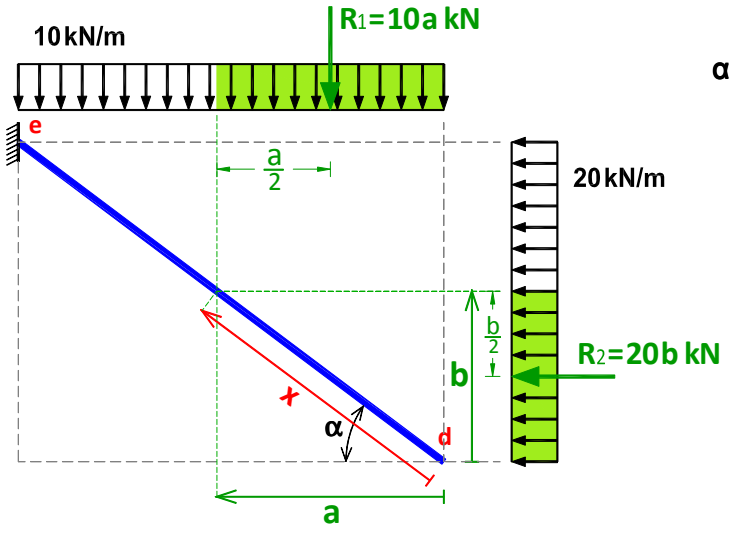
Desenhe os diagramas dos esforços (**N**, **V** e **M**).



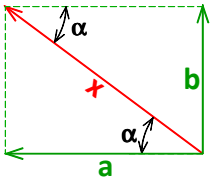
EXERCÍCIO 3 - RESOLUÇÃO

CONVENÇÃO DE SINAIS
DA
RESISTÊNCIA DOS MATERIAIS





$\alpha = \operatorname{arctg}\frac{3}{4} = 36{,}8699^\circ \Rightarrow \begin{cases} \cos\alpha = 0{,}8 \\ \operatorname{sen}\alpha = 0{,}6 \end{cases}$



$a = x \cos\alpha$
$a = 0{,}8\,x$

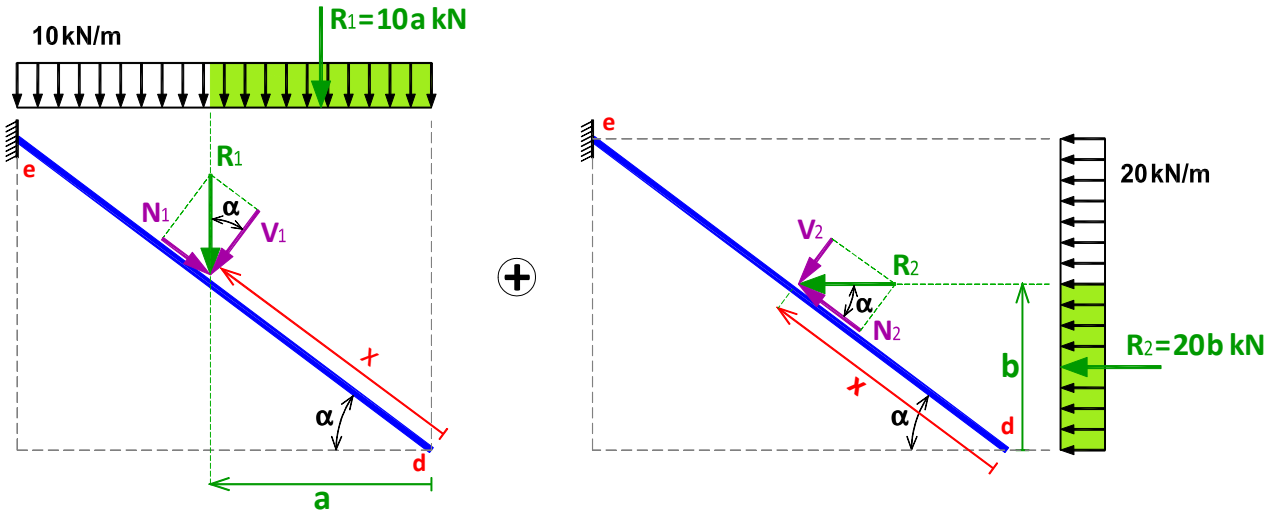
$b = x \operatorname{sen}\alpha$
$b = 0{,}6\,x$

$$M(a,b) = -10\,a\,\frac{a}{2} - 20\,b\,\frac{b}{2} = -5\,a^2 - 10\,b^2 \quad \text{kNm}$$

Substituindo $\boxed{\begin{array}{l}\mathbf{a} = \mathbf{x}\cos\alpha = 0{,}8\,\mathbf{x}\\ \mathbf{b} = \mathbf{x}\,\text{sen}\,\alpha = 0{,}6\,\mathbf{x}\end{array}}$

$$M(x) = -5\,(0{,}8\,\mathbf{x})^2 - 10\,(0{,}6\,\mathbf{x})^2 = -3{,}2\,\mathbf{x}^2 - 3{,}6\,\mathbf{x}^2 = -6{,}8\,\mathbf{x}^2 \text{ kNm} \quad \Rightarrow \text{ curva (2º grau)}$$

Concavidade do diagrama de Momentos: $\frac{d^2M}{dx^2} < 0 \quad \Rightarrow$ concavidade U



R1 – resultante da carga distribuida de 10 kN/m à direita do ponto

R1 = 10 **a** kN

N1 = **R1** sen **α** = 10 **a** sen **α**
V1 = **R1** cos **α** = 10 **a** cos **α**

R2 – resultante da carga distribuida de 20 kN/m à direita do ponto

R2 = 20 **b** kN

N2 = **R2** cos **α** = 20 **b** cos **α**
V2 = **R2** sen **α** = 20 **b** sen **α**

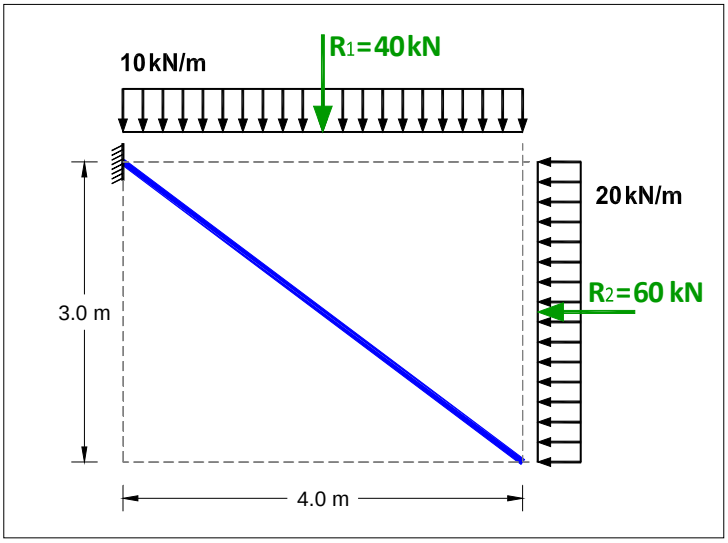
$$\begin{cases} N(a,b) = N_1 - N_2 = 10\,\mathbf{a}\,\text{sen}\,\alpha - 20\,\mathbf{b}\cos\alpha = 10\times 0{,}6\,\mathbf{a} - 20\times 0{,}8\,\mathbf{b} = 6\,\mathbf{a} - 16\,\mathbf{b} \text{ kN} \\ V(a,b) = V_1 + V_2 = 10\,a\cos\alpha + 20\,b\,\text{sen}\,\alpha = 10\times 0{,}8\,\mathbf{a} + 20\times 0{,}6\,\mathbf{b} = 8\,\mathbf{a} + 12\,\mathbf{b} \text{ kN} \end{cases}$$

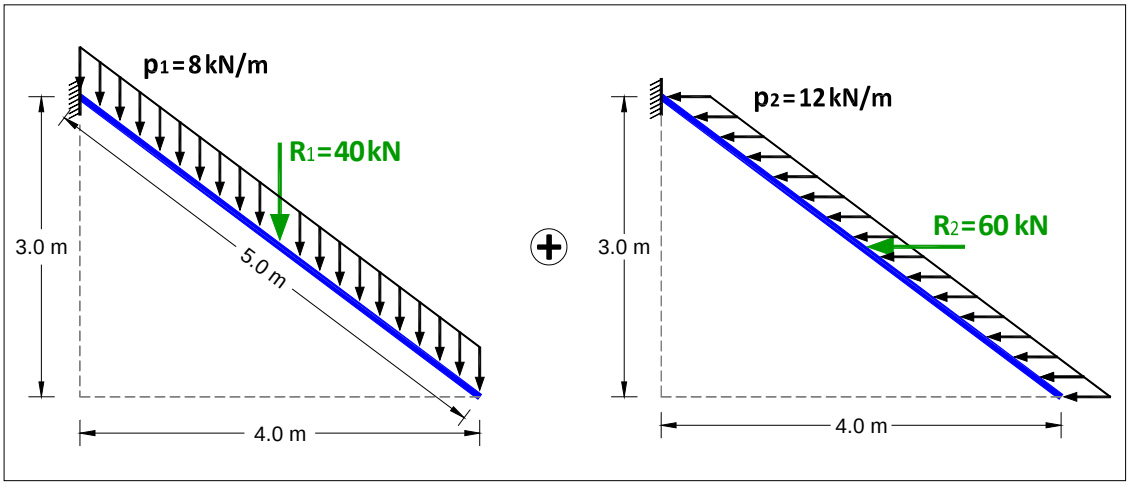
Substituindo: $\boxed{\begin{array}{l}\mathbf{a} = \mathbf{x}\cos\alpha = 0{,}8\,\mathbf{x}\\ \mathbf{b} = \mathbf{x}\,\text{sen}\,\alpha = 0{,}6\,\mathbf{x}\end{array}} \Rightarrow \begin{cases} N(x) = 6\times 0{,}8\,\mathbf{x} - 16\times 0{,}6\,\mathbf{x} = -4{,}8\,\mathbf{x} \text{ kN} & \Rightarrow \text{recta} \\ V(x) = 8\times 0{,}8\,\mathbf{x} + 12\times 0{,}6\,\mathbf{x} = 13{,}6\,\mathbf{x} \text{ kN} & \Rightarrow \text{recta} \end{cases}$

Expressões analíticas dos esforços:
$$\begin{cases} N(x) = -4{,}8\,\mathbf{x} \text{ kN} & \Rightarrow \text{recta} \\ V(x) = 13{,}6\,\mathbf{x} \text{ kN} & \Rightarrow \text{recta} \\ M(x) = -6{,}8\,\mathbf{x}^2 \text{ kNm} & \Rightarrow \text{curva (2º grau)} \end{cases}$$

Extremidade da consola: (x=0) $\begin{cases} N = 0 \\ V = 0 \\ M = 0 \end{cases}$ **Encastramento**: (x=5 m) $\begin{cases} N = -4{,}8 \times 5 = -24 \text{ kN} \\ V = 13{,}6 \times 5 = 68 \text{ kN} \\ M = -6{,}8 \times 5^2 = -170 \text{ kNm} \end{cases}$

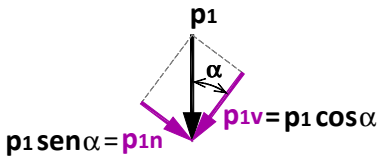
EXERCÍCIO 3 - OUTRA RESOLUÇÃO

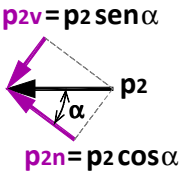




Para os dois sistemas serem equivalentes, as Resultantes R1 e R2 têm que ser igual, ou seja:

$$\begin{cases} p_1 \times 5 = R_1 = 40 \text{ kN} \\ p_2 \times 5 = R_2 = 60 \text{ kN} \end{cases} \Rightarrow \begin{cases} p_1 = 8 \text{ kN/m} \\ p_2 = 12 \text{ kN/m} \end{cases}$$



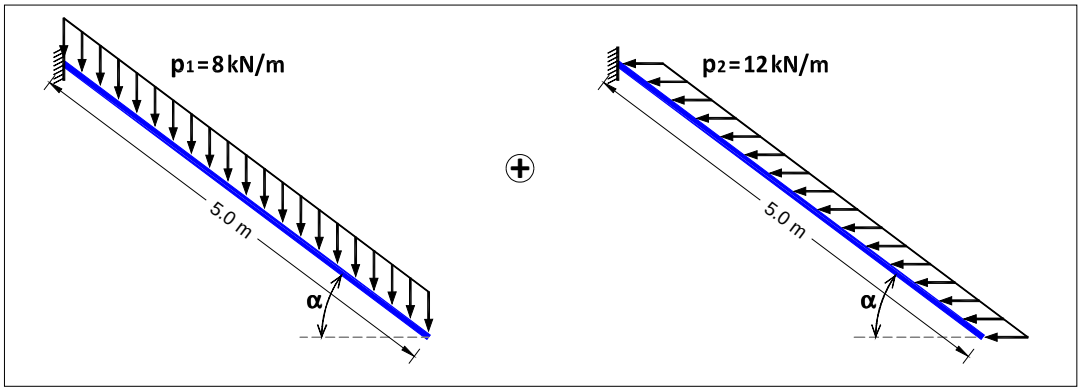


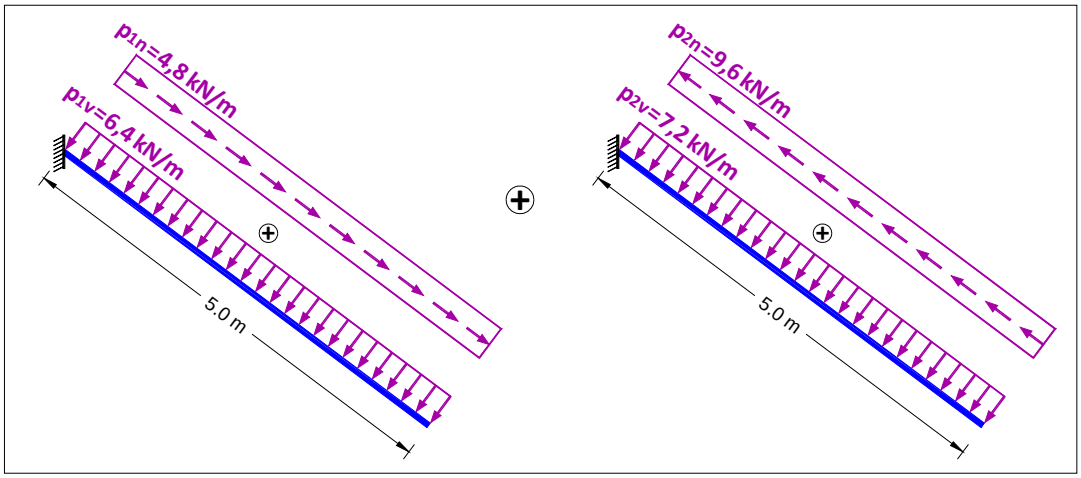
$p_{1n} = p_1 \text{ sen } \alpha = 8 \times 0{,}6 = 4{,}8$ kN/m

$p_{1v} = p_1 \cos \alpha = 8 \times 0{,}8 = 6{,}4$ kN/m

$p_{2n} = p_2 \text{ sen } \alpha = 12 \times 0{,}8 = 9{,}6$ kN/m

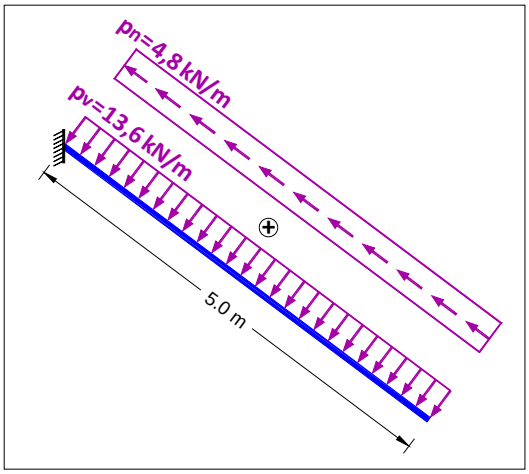
$p_{2v} = p_2 \text{ sen } \alpha = 12 \times 0{,}6 = 7{,}2$ kN/m





$p_n = p_{1n} + p_{2n}$

$p_v = p_{1v} + p_{2v}$



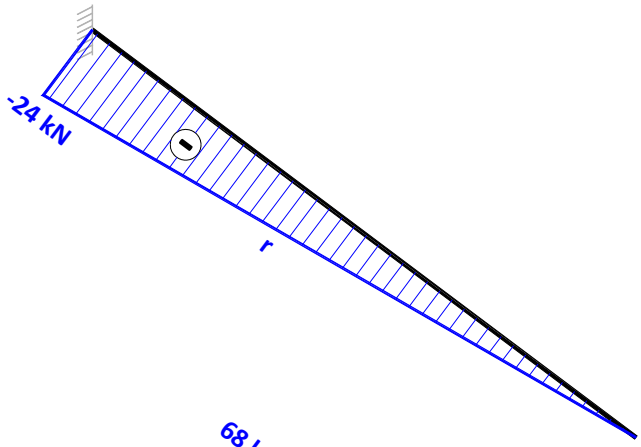
$$\begin{cases} \mathbf{N}(x) = -pn \,.\, \mathbf{x} \;=\; -4{,}8\;\mathbf{x} \;\; \text{kN} & \Rightarrow \text{recta} \\ \mathbf{V}(x) = \; pv \,.\, \mathbf{x} \;=\; 13{,}6\;\mathbf{x} \;\; \text{kN} & \Rightarrow \text{recta} \\ \mathbf{M}(x) = -pv \,.\, \mathbf{x} \,.\, \dfrac{\mathbf{x}}{2} \;=\; -13{,}6\;\mathbf{x}\,\dfrac{\mathbf{x}}{2} \;=\; -6{,}8\,\mathbf{x}^2 \;\; \text{kNm} & \Rightarrow \text{curva (2º grau)} \end{cases}$$

Concavidade do diagrama de Momentos: $\dfrac{d^2M}{d\,x^2} < 0 \quad \Rightarrow \quad$ concavidade $\cup$

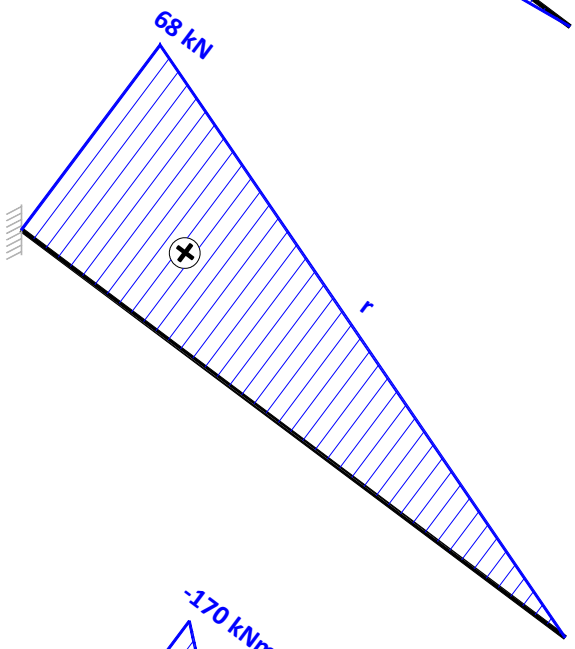
Extremidade da consola: (x=0) $\begin{cases} N = 0 \\ V = 0 \\ M = 0 \end{cases}$

Encastramento: (x=5 m) $\begin{cases} N = -4{,}8 \times 5 = -24 \;\; \text{kN} \\ V = 13{,}6 \times 5 = 68 \;\; \text{kN} \\ M = -6{,}8 \times 5^2 = -170 \;\; \text{kNm} \end{cases}$

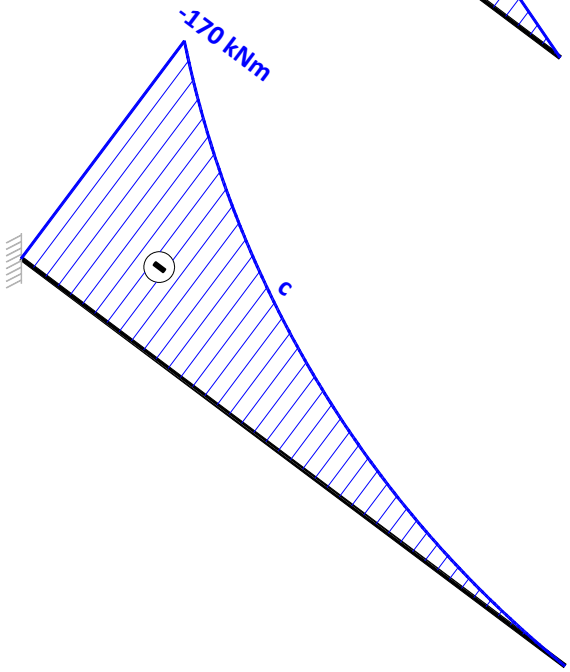
DIAGRAMAS



N
ESFORÇO AXIAL



V
ESFORÇO TRANSVERSO



M
MOMENTO FLECTOR