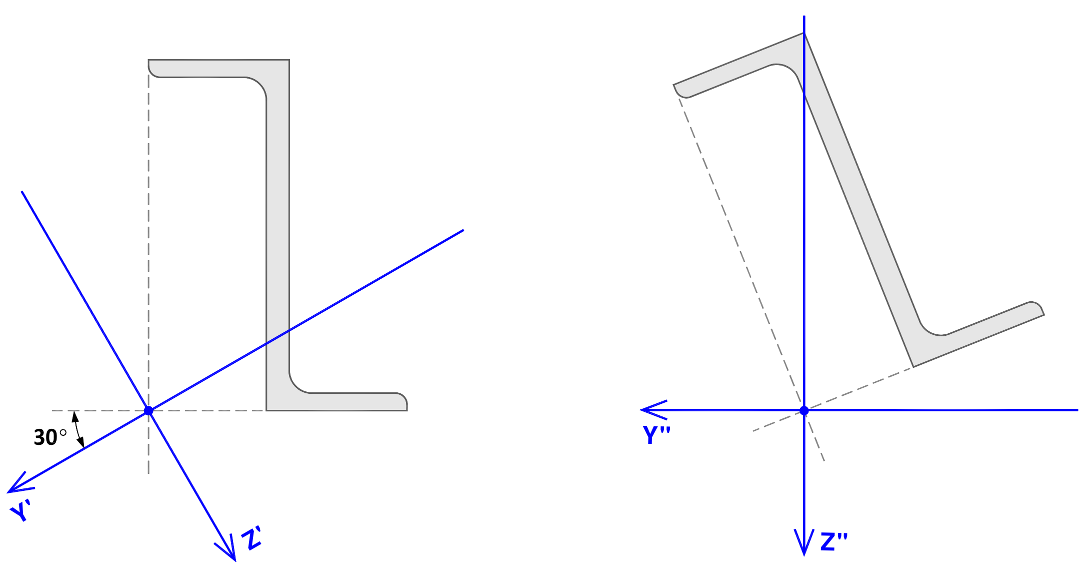
DEPARTAMENTO DE ENGENHARIA CIVIL

LICENCIATURA EM ENGENHARIA CIVIL

ESTÁTICA

GEOMETRIA DE MASSAS

MOMENTOS DE INÉRCIA E PRODUTO DE INÉRCIA EM RELAÇÃO A UM SISTEMA DE EIXOS RODADO



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EXERCÍCIO

Considere o perfil metálico **Z** cujas características estão indicadas na tabela anexa.

1. Considere o sistema de eixos **(Y', Z')** da *Figura 1*.
Determine os momentos de inércia $I_{Y'}$, $I_{Z'}$ e o produto de inércia $I_{Y'Z'}$ do perfil metálico **Z**.

2. Considere o sistema de eixos **(Y'', Z'')** da *Figura 2*.
Determine os momentos de inércia $I_{Y''}$, $I_{Z''}$ e o produto de inércia $I_{Y''Z''}$ do perfil metálico **Z**.

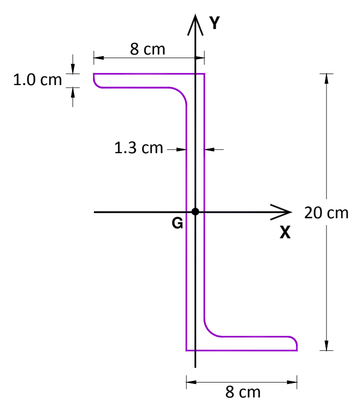
PERFIL **Z**

Área = 38,7 cm^2

I_X = 2300 cm^4

I_Y = 357 cm^4

I_{XY} = -674 cm^4



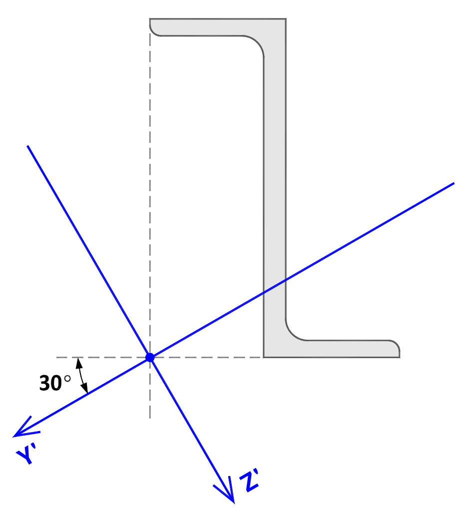


Figura 1

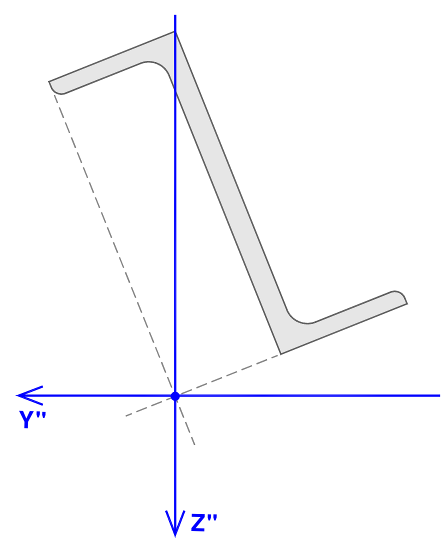


Figura 2

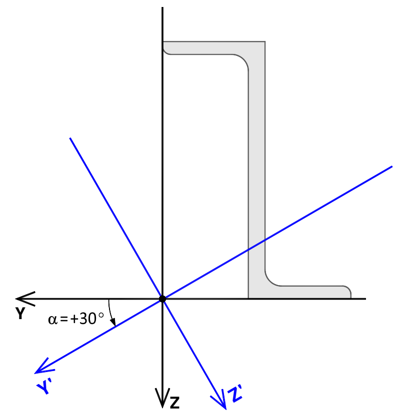
RESOLUÇÃO

1.

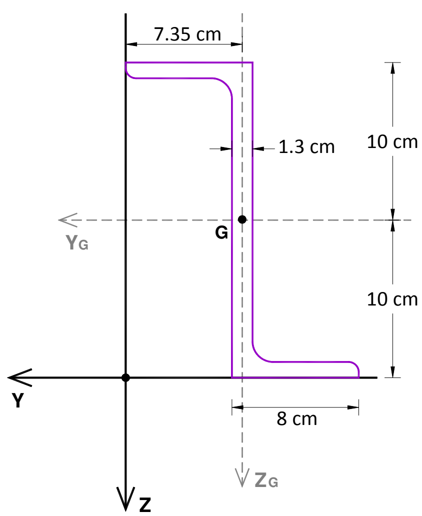
$$\mathbf{I}_{Y'} = \frac{\mathbf{I}_Y + \mathbf{I}_Z}{2} + \frac{\mathbf{I}_Y - \mathbf{I}_Z}{2}\cos 2\alpha - \mathbf{I}_{YZ}\,\mathrm{sen}\,2\alpha$$

$$\mathbf{I}_{Z'} = \frac{\mathbf{I}_Y + \mathbf{I}_Z}{2} - \frac{\mathbf{I}_Y - \mathbf{I}_Z}{2}\cos 2\alpha + \mathbf{I}_{YZ}\,\mathrm{sen}\,2\alpha$$

$$\mathbf{I}_{Y'Z'} = \frac{\mathbf{I}_Y - \mathbf{I}_Z}{2}\,\mathrm{sen}\,2\alpha + \mathbf{I}_{YZ}\cos 2\alpha$$



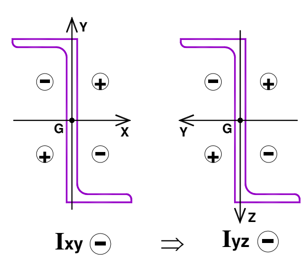
Determinação de $\mathbf{I}_Y$, $\mathbf{I}_Z$ e $\mathbf{I}_{YZ}$



$$\mathbf{I}_Y = \mathbf{I}_{Y_G} + d^2.A = 2300 + 10^2 \times 38{,}7 = 6170\ \mathrm{cm}^4$$

$$\mathbf{I}_Z = \mathbf{I}_{Z_G} + d^2.A = 357 + 7{,}35^2 \times 38{,}7 = 2447{,}671\ \mathrm{cm}^4$$

$$\mathbf{I}_{YZ} = \mathbf{I}_{Y_G Z_G} + a.b.A = -674 + (-7{,}35)\times(-10)\times 38{,}7 = 2170{,}45\ \mathrm{cm}^4$$



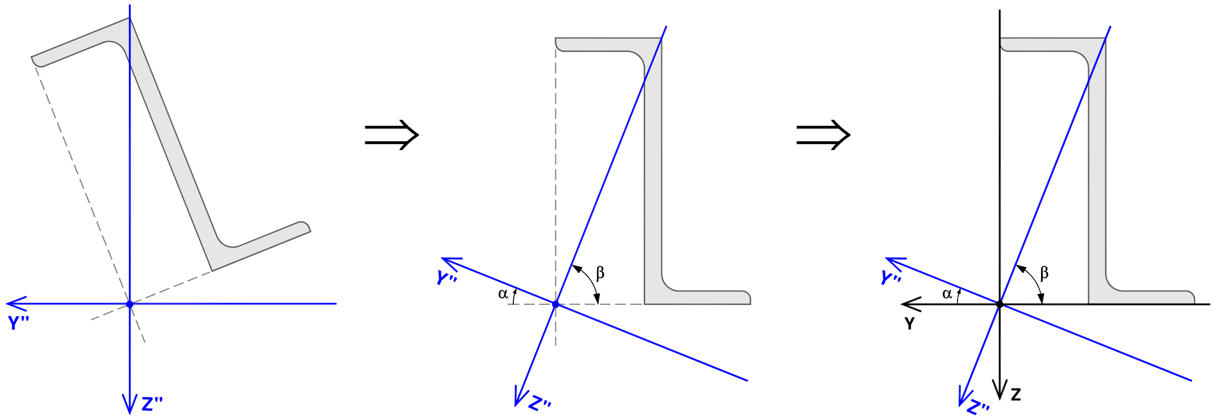
Determinação de $\mathbf{I}_{Y'}$, $\mathbf{I}_{Z'}$ e $\mathbf{I}_{Y'Z'}$

$$\mathbf{I}_{Y'} = \frac{\mathbf{I}_Y + \mathbf{I}_Z}{2} + \frac{\mathbf{I}_Y - \mathbf{I}_Z}{2}\cos 2\alpha - \mathbf{I}_{YZ}\,\mathrm{sen}\,2\alpha =$$
$$= \frac{6170 + 2447{,}671}{2} + \frac{6170 - 2447{,}671}{2}\cos 60^0 - 2170{,}45\ \mathrm{sen}\ 60^0 = 3359{,}75\ \ \mathrm{cm}^4$$

$$\mathbf{I}_{Z'} = \frac{\mathbf{I}_Y + \mathbf{I}_Z}{2} - \frac{\mathbf{I}_Y - \mathbf{I}_Z}{2}\cos 2\alpha + \mathbf{I}_{YZ}\,\mathrm{sen}\,2\alpha =$$
$$= \frac{6170 + 2447{,}671}{2} - \frac{6170 - 2447{,}671}{2}\cos 60^0 + 2170{,}45\,\mathrm{sen}\,60^0 = 5257{,}92\ \ \mathrm{cm}^4$$

$$\mathbf{I}_{Y'Z'} = \frac{\mathbf{I}_Y - \mathbf{I}_Z}{2}\,\mathrm{sen}\,2\alpha + \mathbf{I}_{YZ}\cos 2\alpha =$$
$$= \frac{6170 - 2447{,}671}{2}\,\mathrm{sen}\,60^0 + 2170{,}45\cos 60^0 = 2698{,}04\ \ \mathrm{cm}^4$$

2.



$$\beta = \text{arctg}\,\frac{20}{8} = 68{,}1986^{o}$$

$$|\alpha| = 90^{o} - \beta = 90^{o} - 68{,}1986^{o} = 21{,}8014^{o} \quad \Rightarrow \quad \alpha = -21{,}8014^{o}$$

Determinação de $\mathbf{I}_{Y''}$, $\mathbf{I}_{Z''}$ e $\mathbf{I}_{Y''Z''}$

$$\mathbf{I}_{Y''} = \frac{\mathbf{I}_Y + \mathbf{I}_Z}{2} + \frac{\mathbf{I}_Y - \mathbf{I}_Z}{2}\cos 2\alpha - \mathbf{I}_{YZ}\,\text{sen}\,2\alpha =$$

$$= \frac{6170 + 2447{,}671}{2} + \frac{6170 - 2447{,}671}{2}\cos(-43{,}603^{o}) - 2170{,}45\,\text{sen}\,(-43{,}603^{o}) = 7153{,}44\ \text{cm}^4$$

$$\mathbf{I}_{Z''} = \frac{\mathbf{I}_Y + \mathbf{I}_Z}{2} - \frac{\mathbf{I}_Y - \mathbf{I}_Z}{2}\cos 2\alpha + \mathbf{I}_{YZ}\,\text{sen}\,2\alpha =$$

$$= \frac{6170 + 2447{,}671}{2} - \frac{6170 - 2447{,}671}{2}\cos(-43{,}603^{o}) + 2170{,}45\,\text{sen}\,(-43{,}603^{o}) = 1464{,}23\ \text{cm}^4$$

$$\mathbf{I}_{Y''Z''} = \frac{\mathbf{I}_Y - \mathbf{I}_Z}{2}\,\text{sen}\,2\alpha + \mathbf{I}_{YZ}\cos 2\alpha =$$

$$= \frac{6170 - 2447{,}671}{2}\,\text{sen}\,(-43{,}603^{o}) + 2170{,}45\cos(-43{,}603^{o}) = 288{,}14\ \text{cm}^4$$