

FICHA 1 – Revisões

Exercício 1

a) Figura 1

Barras	Comprimento
AB	4,00 m
AC	3,073 m
BC	2,561 m
BD	5,385 m
CD	4,737 m

$\alpha = 39,806^\circ$
 $\beta = 50,195^\circ$
 $\varphi = 28,393^\circ$
 cota **a** = 1,967 m

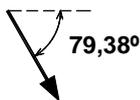
b) Figura 2

Barras	Comprimento
AD	6,50 m
AE	3,426 m
AF	6,164 m
AG	5,385 m
BD	3,202 m
EG	2,545 m
FG	3,00 m

$\alpha = 28,720^\circ$
 $\delta = 16,457^\circ$
 $\varphi = 60,878^\circ$
 $\beta = 128,199^\circ$
 cota **a** = 3,426 m
 cota **b** = 1,386 m
 cota **c** = 1,114 m

Exercício 2

a) $F_a = 16,882 \text{ KN}$



b) $F_b = 24,375 \text{ KN}$



Exercício 3

$F_x = 50 \text{ N} \leftarrow$

$F_y = 86,603 \text{ N} \downarrow$

Exercício 4

a) $F_x = 153,209 \text{ N} \leftarrow$ $F_y = 128,558 \text{ N} \uparrow$

b) $F_{//AB} = 100 \text{ N} \swarrow$ $F_{\perp AB} = 173,205 \text{ N} \nwarrow$

Exercício 5

a) $\alpha = 25^\circ$ $\begin{cases} R_x = 334,992 \text{ N} \rightarrow \\ R_y = 377,444 \text{ N} \uparrow \end{cases} \Rightarrow R = 504,66 \text{ N} \nearrow$ $48,41^\circ$

$\begin{cases} R_{\perp \text{lado menor}} = 57,92 \text{ N} \searrow \\ R_{\perp \text{lado maior}} = 501,33 \text{ N} \nearrow \end{cases}$

b) $\alpha = 50^\circ$ $\begin{cases} R_x = 187,161 \text{ N} \rightarrow \\ R_y = 587,636 \text{ N} \uparrow \end{cases} \Rightarrow R = 616,72 \text{ N} \nearrow$ $72,33^\circ$

$\begin{cases} R_{\perp \text{lado menor}} = 183,74 \text{ N} \nwarrow \\ R_{\perp \text{lado maior}} = 588,72 \text{ N} \nearrow \end{cases}$

Exercício 6

a) $F_b = 119,175 \text{ N}$

b) $R = 155,57 \text{ N}$

Exercício 7

a) $\alpha = 25^\circ$ $\begin{cases} F_2 = 26,71 \text{ kN} \\ R = 34,74 \text{ kN} \nearrow \end{cases}$

b) $|F_2| = 40 \text{ kN}$ $\begin{cases} \alpha = 47,48^\circ \\ R = 24,09 \text{ kN} \swarrow \end{cases}$

FICHA 2 – Estática do ponto material

Exercício 1

a) $\begin{cases} R_x = 39,03 \text{ kN} \nearrow \\ R_y = 114,59 \text{ kN} \nwarrow \end{cases}$

b) $\begin{cases} R_x = 87,36 \text{ kN} \nwarrow \\ R_y = 83,81 \text{ kN} \nearrow \end{cases}$

c) $\begin{cases} R_x = 43,759 \text{ kN} \leftarrow \\ R_y = 112,876 \text{ kN} \uparrow \end{cases} \Rightarrow R = 121,06 \text{ kN} \nwarrow 68,81^\circ$

d) $F = 121,06 \text{ kN} \nwarrow 68,81^\circ$

e) $F = 114,04 \text{ kN} \nwarrow 81,82^\circ$

f) $\begin{cases} F = 29,59 \text{ kN} \nwarrow 40^\circ \\ R = 61,18 \text{ kN} \nwarrow \end{cases}$

Exercício 2

a) $\begin{cases} F_1 = 80,92 \text{ kN} \nearrow \\ F_2 = 45,69 \text{ kN} \searrow \end{cases}$

b) $\begin{cases} F_3 = 11,49 \text{ kN} \nearrow \\ F_4 = 10,59 \text{ kN} \rightarrow \end{cases}$

c) $\begin{cases} F_2 = 26,59 \text{ kN} \searrow \\ F_3 = 21,31 \text{ kN} \swarrow \end{cases}$

d) $F_5 = 42,83 \text{ kN} \nwarrow 26,31^\circ$

FICHA 3 – Estática do corpo rígido

Exercício 1

a) a1) $M_C = 15 \text{ kNm} \curvearrowright$

a2) $M_D = 5 \text{ kNm} \curvearrowright$

b) b1) $M_C = M_B = M_D = 6 \text{ kNm} \curvearrowright$

b2) $M_E = 12 \text{ kNm} \curvearrowright$

c) c1) $M_D = 12,5 \text{ kNm} \curvearrowright$

c2) $M_C = 28,5 \text{ kNm} \curvearrowright$

d) d1) $M_D = 4,302 \text{ kNm} \curvearrowright$

d2) $M_C = 15,706 \text{ kNm} \curvearrowright$

e) e1) e2) e3) $M_A = M_B = M_C = M_D = M_E = 18 \text{ kNm} \curvearrowright$

f) f1) $M_C = 17,321 \text{ kNm} \curvearrowright$

f2) $M_E = 21,071 \text{ kNm} \curvearrowright$

f3) Sistema equivalente em D $\begin{cases} F_x = 25 \text{ kN} \leftarrow \\ F_y = 0 \\ M = 17,321 \text{ kNm} \curvearrowright \end{cases}$

f4) $F_A = 28,87 \text{ kN} \uparrow$

f5) $F_E = 111,90 \text{ kN} \leftarrow$

g) g1) Sistema equivalente em **C** $\left\{ \begin{array}{l} F_x = 33,251 \text{ kN} \leftarrow \\ F_y = 19,069 \text{ kN} \downarrow \\ M = 2,008 \text{ kNm} \curvearrowright \end{array} \right.$

g2) Sistema equivalente a meio da barra **DE** $\left\{ \begin{array}{l} F_x = 33,251 \text{ kN} \leftarrow \\ F_y = 19,069 \text{ kN} \downarrow \\ M = 14,036 \text{ kNm} \curvearrowright \end{array} \right.$

g3) Sistema equivalente a resultante sobre o tramo **BCD** $\left\{ \begin{array}{l} F_x = 33,251 \text{ kN} \leftarrow \\ F_y = 19,069 \text{ kN} \downarrow \end{array} \right.$
 ponto de aplicação: 0,105 m à esquerda de **C**

Exercício 2

a) $\left\{ \begin{array}{l} F_x = 5,51 \text{ kN} \leftarrow \\ F_y = 21,32 \text{ kN} \uparrow \\ M = 38,58 \text{ kNm} \curvearrowright \end{array} \right.$

b) $\left\{ \begin{array}{l} F_{\text{hor.D}} = 12,86 \text{ kN} \rightarrow \\ F_{\text{hor.H}} = 18,37 \text{ kN} \leftarrow \\ F_{\text{vert.G}} = 21,32 \text{ kNm} \uparrow \end{array} \right.$

c) $\left\{ \begin{array}{l} F_x = 5,51 \text{ kN} \rightarrow \\ F_y = 21,32 \text{ kN} \downarrow \\ M = 6,6 \text{ kNm} \curvearrowright \end{array} \right.$

d) $\left\{ \begin{array}{l} F_x = 5,51 \text{ kN} \rightarrow \\ F_y = 21,32 \text{ kN} \downarrow \\ M = 23,43 \text{ kNm} \curvearrowright \end{array} \right.$

e) Sistema equivalente a resultante sobre o tramo **ABC** $\left\{ \begin{array}{l} F_x = 5,51 \text{ kN} \rightarrow \\ F_y = 21,32 \text{ kN} \downarrow \end{array} \right.$
 ponto de aplicação: 0,31 m à esquerda de **A**

Exercício 3

a) Sistema resultante na extremidade inferior do poste $\left\{ \begin{array}{l} F_x = 0 \\ F_y = 180 \text{ kN} \downarrow \\ M = 80 \text{ kNm} \curvearrowright \end{array} \right.$

b) $\left\{ \begin{array}{l} \text{parafuso da esquerda: } F = 110 \text{ kN} \uparrow \\ \text{parafuso da direita: } F = 290 \text{ kN} \downarrow \end{array} \right.$

c) Cota $a \geq 0,31 \text{ m}$

FICHA 4 – Equilíbrio de corpos Rígidos - Reacções

Exercício 2

Figura 1: $H_A = 0$

$V_A = 6 \text{ kN} \uparrow$

$V_B = 9 \text{ kN} \uparrow$

Figura 2: $H_A = 0$

$V_A = 7,5 \text{ kN} \downarrow$

$V_B = 27,5 \text{ kN} \uparrow$

Figura 3: $V_B = 44,167 \text{ kN} \downarrow$

$H_C = 0$

$V_C = 44,167 \text{ kN} \uparrow$

Figura 4: $H_B = 43,32 \text{ kN} \rightarrow$

$V_B = 56,519 \text{ kN} \uparrow$

$V_B = 34,041 \text{ kN} \downarrow$

Figura 5: $H_B = 0$

$V_B = 4,0476 \text{ kN} \uparrow$

$V_B = 4,0476 \text{ kN} \downarrow$

Figura 6: $H_A = 0$

$V_A = 10,75 \text{ kN} \uparrow$

$M_A = 25,48 \text{ kNm} \curvearrowright$

Exercício 3

- a) Ponto de aplicação: 1,20 m à esquerda de **B**
- b) $M = 92,817 \text{ kNm}$ ↻
- c) Carga vertical uniformemente distribuída: $p = 13,09 \text{ kN/m}$ ↓↓↓↓

Exercício 4

Figura 1: $R_H = 12.262 \text{ kN}$ → $R_V = 3,5 \text{ kN}$ ↑ $M = 11,581 \text{ kNm}$ ↻

Figura 2: $H_A = 18.453 \text{ kN}$ ← $V_A = 31,965 \text{ kN}$ ↑ $R_B = 17,695 \text{ kN}$ ↗

Figura 3: apoio duplo: $\begin{cases} R_H = 31,070 \text{ kN} \rightarrow \\ R_V = 27,946 \text{ kN} \uparrow \end{cases}$ Apoio simples: $R = 34,738 \text{ kN}$ ↗

Figura 4: $R_H = 18.058 \text{ kN}$ → $R_V = 65,934 \text{ kN}$ ↑ $M = 117,915 \text{ kNm}$ ↻

FICHA 5 – Equilíbrio de corpos rígidos – Arco de três rótulas

Exercício 1

A $\begin{cases} H_A = 41,88 \text{ kN} \rightarrow \\ V_A = 50,21 \text{ kN} \uparrow \end{cases}$ **B** $\begin{cases} H_B = 110,17 \text{ kN} \leftarrow \\ V_B = 33,37 \text{ kN} \uparrow \end{cases}$ Rótula **C** (forças de ligação) $\begin{cases} H_C = 110,17 \text{ kN} \leftarrow \rightarrow \\ V_C = 2,63 \text{ kN} \downarrow \uparrow \end{cases}$

Exercício 2

Figura 1: **A** $\begin{cases} H_A = 141,79 \text{ kN} \rightarrow \\ V_A = 79,30 \text{ kN} \uparrow \\ M_A = 153,32 \text{ kNm} \curvearrowright \end{cases}$ $R_G = 48,79 \text{ kN}$ ↙

Figura 2: $V_A = 20,78 \text{ kN}$ ↓ **C** $\begin{cases} H_C = 108,18 \text{ kN} \leftarrow \\ V_C = 19,53 \text{ kN} \uparrow \end{cases}$ $R_E = 9,09 \text{ kN}$ ↙ (com a direcção da barra ED)

Figura 3: **A** $\begin{cases} H_A = 126,83 \text{ kN} \leftarrow \\ V_A = 28,465 \text{ kN} \downarrow \end{cases}$ **I** $\begin{cases} H_I = 9,50 \text{ kN} \leftarrow \\ V_I = 23,76 \text{ kN} \downarrow \end{cases} \Rightarrow R_I = 25,59 \text{ kN}$ ↙ (dir. barra IR)

Figura 4: $R_A = 35,48 \text{ kN}$ ↗ **F** $\begin{cases} H_F = 79,03 \text{ kN} \rightarrow \\ V_F = 24,11 \text{ kN} \downarrow \\ M_F = 97,27 \text{ kNm} \curvearrowright \end{cases}$

FICHA 6 – Equilíbrio de corpos rígidos – Vigas Gerber

Figura 1: $V_A = 18,72 \text{ kN } \uparrow$ **C** $\begin{cases} H_C = 22,94 \text{ kN } \rightarrow \\ V_C = 23,40 \text{ kN } \uparrow \end{cases}$ $V_D = 17,64 \text{ kN } \uparrow$

Figura 2: $V_A = 8,44 \text{ kN } \downarrow$ $V_B = 63,36 \text{ kN } \uparrow$ **E** $\begin{cases} H_E = 8,25 \text{ kN } \leftarrow \\ V_E = 24,32 \text{ kN } \uparrow \end{cases}$ $V_F = 14,94 \text{ kN } \downarrow$

Figura 3: **A** $\begin{cases} H_A = 66,14 \text{ kN } \rightarrow \\ V_A = 4,88 \text{ kN } \uparrow \end{cases}$ $V_B = 4,40 \text{ kN } \uparrow$ $V_D = 30,68 \text{ kN } \uparrow$ $V_F = 42,08 \text{ kN } \uparrow$ $V_G = 16,15 \text{ kN } \uparrow$

Figura 4: $V_A = 71,82 \text{ kN } \downarrow$ $V_C = 126,32 \text{ kN } \uparrow$ $V_D = 53,88 \text{ kN } \downarrow$ **G** $\begin{cases} H_G = 51,42 \text{ kN } \leftarrow \\ V_G = 21,72 \text{ kN } \uparrow \\ M_G = 13,725 \text{ kNm } \curvearrowright \end{cases}$

Figura 5: $V_A = 18,97 \text{ kN } \uparrow$ $V_C = 46,315 \text{ kN } \uparrow$ **D** $\begin{cases} H_D = 30 \text{ kN } \leftarrow \\ V_D = 12,72 \text{ kN } \uparrow \end{cases}$ $V_E = 4,0 \text{ kN } \uparrow$

FICHA 7 – Equilíbrio de corpos rígidos – Associação de corpos

Figura 1

A $\begin{cases} H_A = 46,118 \text{ kN } \leftarrow \\ V_A = 75,086 \text{ kN } \downarrow \end{cases}$ $V_B = 163,701 \text{ kN } \uparrow$ $V_G = 6 \text{ kN } \uparrow$ $R_K = 52,393 \text{ kN } \nearrow$

Figura 2

A $\begin{cases} H_A = 121,17 \text{ kN } \rightarrow \\ V_A = 68,49 \text{ kN } \uparrow \\ M_A = 767,85 \text{ kNm } \curvearrowright \end{cases}$ $V_H = 153,37 \text{ kN } \uparrow$ $R_I = 101,28 \text{ kN } \nwarrow$

Figura 3

A $\begin{cases} H_A = 47,57 \text{ kN } \rightarrow \\ V_A = 12,57 \text{ kN } \downarrow \end{cases}$ $V_B = 165,22 \text{ kN } \downarrow$ **E** $\begin{cases} H_E = 17 \text{ kN } \leftarrow \\ V_E = 265,79 \text{ kN } \uparrow \end{cases}$

Figura 4

A $\begin{cases} H_A = 37,75 \text{ kN } \leftarrow \\ V_A = 55 \text{ kN } \uparrow \end{cases}$ **F** $\begin{cases} H_F = 59,75 \text{ kN } \leftarrow \\ V_F = 24 \text{ kN } \uparrow \end{cases}$ $V_H = 5 \text{ kN } \uparrow$

Figura 5

A $\begin{cases} H_A = 94,731 \text{ kN } \leftarrow \\ V_A = 74,731 \text{ kN } \downarrow \end{cases}$ **B** $\begin{cases} H_B = 153,078 \text{ kN } \leftarrow \\ V_B = 25,300 \text{ kN } \uparrow \end{cases}$ $R_J = 16,553 \text{ kN } \nwarrow$

Figura 6

A $\begin{cases} H_A = 5,713 \text{ kN } \rightarrow \\ V_A = 21,610 \text{ kN } \downarrow \\ M_A = 50,330 \text{ kNm } \curvearrowright \end{cases}$ $R_F = 9,144 \text{ kN } \swarrow$ $V_G = 31,25 \text{ kN } \downarrow$

FICHA 8 – Sistemas articulados planos (SAP)

Exercícios 1 e 2

Figura 1

Reacções:

$$\begin{array}{l}
 H_A = 15 \text{ kN} \leftarrow \\
 \left. \begin{array}{l} H_C = 5 \text{ kN} \rightarrow \\ V_C = 20 \text{ kN} \uparrow \end{array} \right\} C
 \end{array}$$

Barras	Compressão (kN)	Tracção (kN)
AB	15	—
AC	—	21,213
BC	10	—
BD	—	11,180
CD	5	—

Figura 2

Reacções:

$$\begin{array}{l}
 H_A = 90 \text{ kN} \rightarrow \\
 \left. \begin{array}{l} H_E = 80 \text{ kN} \leftarrow \\ V_E = 30 \text{ kN} \uparrow \end{array} \right\} E
 \end{array}$$

Barras	Compressão (kN)	Tracção (kN)
AB	0	0
AC	90	—
BC	20	—
BD	22,36	—
CD	10	—
CE	82,46	—
DE	10	—

Figura 3

Reacções:

$$\begin{array}{l}
 \left. \begin{array}{l} H_A = 137,712 \text{ kN} \leftarrow \\ V_A = 113,284 \text{ kN} \uparrow \end{array} \right\} A \\
 HB = 141,977 \text{ kN} \rightarrow
 \end{array}$$

Barras	Compressão (kN)	Tracção (kN)
AC	24,428	—
AD	—	194,754
BC	141,977	—
BE	0	0
CD	115,996	—
CE	9,428	—
DE	—	47,140

Figura 4

Reacções:

$$\begin{array}{l}
 V_A = 1,633 \text{ kN} \uparrow \\
 \left. \begin{array}{l} H_E = 3,384 \text{ kN} \leftarrow \\ V_E = 22,633 \text{ kN} \uparrow \end{array} \right\} E
 \end{array}$$

Barras	Compressão (kN)	Tracção (kN)
AD	—	1,633
BD	—	14,373
BE	21,749	—
CD	—	7,071
CF	5	—
DE	1,250	—
DF	—	15,590
EF	2,5	—

Figura 5 Reacções:

$$A \begin{cases} H_A = 5,314 \text{ kN} \rightarrow \\ V_A = 41,265 \text{ kN} \uparrow \\ R_F = 67,587 \text{ kN} \uparrow \\ R_H = 33,380 \text{ kN} \leftarrow \end{cases}$$

Barras	Compressão (kN)	Tracção (kN)
AB	—	44,20
AC	64,46	—
BC	—	41,27
BD	—	7,54
CD	49,52	—
DE	33,11	—
DG	11,67	—
EF	0	0
EG	—	25
EH	32,19	—
FH	—	67,59
GH	11,67	—

Exercício 3

- a) $N_{AD} = 59,27 \text{ kN}$ (comp.) b) $N_{EG} = 37,33 \text{ kN}$ (tracção)
 $N_{AE} = 10,73 \text{ kN}$ (tracção)



Exercício 4

- a) $N_{DI} = 10,00 \text{ kN}$ (compressão) b) $N_{GJ} = 9,12 \text{ kN}$ (compressão) c) $\alpha = 59,87^\circ$
 $N_{DJ} = 27,04 \text{ kN}$ (tracção)
 $N_{IJ} = 19,12 \text{ kN}$ (compressão)

Exercício 5

- a) $N_{BF} = 6,66 \text{ kN}$ (tracção) b) $N_{AC} = 3,78 \text{ kN}$ (tracção) c) $F = 6,01 \text{ kN} \uparrow$
 $N_{BG} = 9,615 \text{ kN}$ (compressão)

Exercício 6

- a) $N_{BE} = 4,30 \text{ kN}$ (compressão) b) $N_{DE} = 62,91 \text{ kN}$ (compressão)
 $N_{CE} = 33,39 \text{ kN}$ (compressão)
 Reacções: $A \begin{cases} H_A = 14,835 \text{ kN} \leftarrow \\ V_A = 45,761 \text{ kN} \uparrow \end{cases}$ $R_G = 65,4286 \text{ kN} \nearrow$

Exercício 7

- a) $N_{BE} = 45,753 \text{ kN}$ (compressão)
 b) $N_{AD} = 0$ (barra descarregada) Reacção no apoio em C: $\sum M_A = 0 \Rightarrow V_C = 49,3235 \text{ kN} \uparrow$
 c) $N_{DF} = 23,048 \text{ kN}$ (tracção)

FICHA 11 – Geometria de Massas – Centro de Gravidade

EXERCÍCIO 1

Figura 1 $\begin{cases} X_G = 19,42 \text{ cm} \\ Y_G = 44,55 \text{ cm} \end{cases}$

Figura 2 $\begin{cases} X_G = 13,02 \text{ cm} \\ Y_G = -2,69 \text{ cm} \end{cases}$

Figura 3 $\begin{cases} X_G = -2,039 \text{ cm} \\ Y_G = -0,733 \text{ cm} \end{cases}$

Figura 4 $\begin{cases} Y_G = -4,17 \text{ mm} \\ Z_G = -7,37 \text{ mm} \end{cases}$

Figura 5 $\begin{cases} Y_G = 4,36 \text{ cm} \\ Z_G = 14,37 \text{ cm} \end{cases}$

Figura 6 $\begin{cases} Y_G = 0,731 \text{ cm} \\ Z_G = 1,041 \text{ cm} \end{cases}$

EXERCÍCIO 2

Figura 1 $\begin{cases} Y_G = 22,48 \text{ mm} \\ Z_G = -21,36 \text{ mm} \end{cases}$

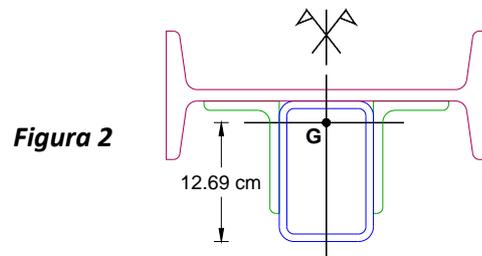


Figura 3

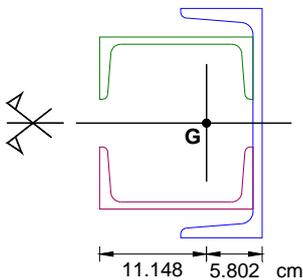


Figura 4 $\begin{cases} X_G = 2,95 \text{ cm} \\ Y_G = 2,95 \text{ cm} \end{cases}$

EXERCÍCIO 3

Figura 1

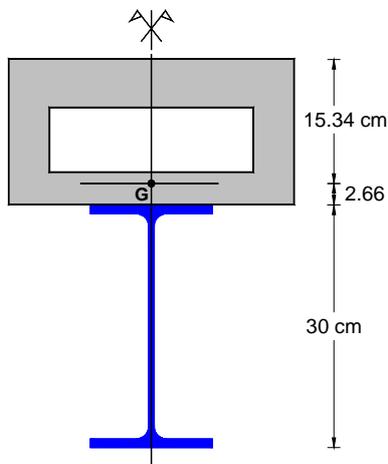
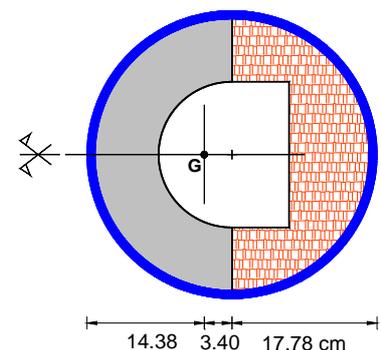


Figura 2 $\begin{cases} Y_G = -0,1116 \text{ m} \\ Z_G = -0,0319 \text{ m} \end{cases}$

Figura 3

$\begin{cases} X_G = 0,32 \text{ cm} \\ Y_G = -8,69 \text{ cm} \end{cases}$

Figura 4



FICHA 12 – Geometria de Massas – Momentos e Produto de Inércia. Raio de Giração.

EXERCÍCIO 1

Figura 1

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_X = 4370751,5 \text{ cm}^4 \\ I_Y = 1150946,8 \text{ cm}^4 \end{cases} & \text{b)} \begin{cases} I_{X_G} = 714504,3 \text{ cm}^4 \\ I_{Y_G} = 456237,7 \text{ cm}^4 \end{cases} & \text{c)} I_O = 5521398,3 \text{ cm}^4 & \text{d)} I_{CG} = 1170742,0 \text{ cm}^4 \\
 \text{e)} \begin{cases} i_X = 48,7 \text{ cm} \\ i_Y = 25,0 \text{ cm} \end{cases} & \text{f)} \begin{cases} i_{X_G} = 19,7 \text{ cm} \\ i_{Y_G} = 15,7 \text{ cm} \end{cases} & \text{g)} I_{XY} = 1843314,9 \text{ cm}^4 & \text{h)} I_{X_G Y_G} = 249633,7 \text{ cm}^4 \\
 \text{i)} \begin{cases} I_{X'} = 5161932,8 \text{ cm}^4 \\ I_{Y'} = 359465,5 \text{ cm}^4 \\ I_{X'Y'} = -472429,0 \text{ cm}^4 \end{cases} & \text{j)} \begin{cases} I_{X'_G} = 477145,8 \text{ cm}^4 \\ I_{Y'_G} = 693596,3 \text{ cm}^4 \\ I_{X'_G Y'_G} = 259383,3 \text{ cm}^4 \end{cases} & &
 \end{array}$$

Figura 2

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_X = 364380,1 \text{ cm}^4 \\ I_Y = 818362,6 \text{ cm}^4 \end{cases} & \text{b)} \begin{cases} I_{X_G} = 350474,4 \text{ cm}^4 \\ I_{Y_G} = 491799,0 \text{ cm}^4 \end{cases} & \text{c)} I_O = 1182742,7 \text{ cm}^4 & \text{d)} I_{CG} = 842273,4 \text{ cm}^4 \\
 \text{e)} \begin{cases} i_X = 13,75 \text{ cm} \\ i_Y = 20,61 \text{ cm} \end{cases} & \text{f)} \begin{cases} i_{X_G} = 13,48 \text{ cm} \\ i_{Y_G} = 15,97 \text{ cm} \end{cases} & \text{g)} I_{XY} = 212654,975 \text{ cm}^4 & \text{h)} I_{X_G Y_G} = 280042,5 \text{ cm}^4 \\
 \text{i)} \begin{cases} I_{X'} = 662040,3 \text{ cm}^4 \\ I_{Y'} = 520702,3 \text{ cm}^4 \\ I_{X'Y'} = 302907,7 \text{ cm}^4 \end{cases} & \text{j)} \begin{cases} I_{X'_G} = 161190,8 \text{ cm}^4 \\ I_{Y'_G} = 681082,6 \text{ cm}^4 \\ I_{X'_G Y'_G} = 125877,4 \text{ cm}^4 \end{cases} & &
 \end{array}$$

Figura 3

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_X = 319,62 \text{ cm}^4 \\ I_Y = 531,24 \text{ cm}^4 \end{cases} & \text{b)} \begin{cases} I_{X_G} = 294,06 \text{ cm}^4 \\ I_{Y_G} = 333,49 \text{ cm}^4 \end{cases} & \text{c)} I_O = 850,86 \text{ cm}^4 & \text{d)} I_{CG} = 627,55 \text{ cm}^4 \\
 \text{e)} \begin{cases} i_X = 2,59 \text{ cm} \\ i_Y = 3,34 \text{ cm} \end{cases} & \text{f)} \begin{cases} i_{X_G} = 2,49 \text{ cm} \\ i_{Y_G} = 2,65 \text{ cm} \end{cases} & \text{g)} I_{XY} = 185,52 \text{ cm}^4 & \text{h)} I_{X_G Y_G} = 114,43 \text{ cm}^4 \\
 \text{i)} \begin{cases} I_{X'} = 211,86 \text{ cm}^4 \\ I_{Y'} = 639,00 \text{ cm}^4 \\ I_{X'Y'} = 1,13 \text{ cm}^4 \end{cases} & \text{j)} \begin{cases} I_{X'_G} = 388,76 \text{ cm}^4 \\ I_{Y'_G} = 238,79 \text{ cm}^4 \\ I_{X'_G Y'_G} = 88,66 \text{ cm}^4 \end{cases} & &
 \end{array}$$

Figura 4

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_Y = 119\,234,04 \text{ mm}^4 \\ I_Z = 125\,081,58 \text{ mm}^4 \end{cases} & \text{b)} \begin{cases} I_{Y_G} = 79\,529,9 \text{ mm}^4 \\ I_{Z_G} = 112\,383,6 \text{ mm}^4 \end{cases} & \text{c)} I_O = 244\,315,6 \text{ mm}^4 & \text{d)} I_{CG} = 191\,913,5 \text{ mm}^4 \\
 \text{e)} \begin{cases} i_Y = 12,78 \text{ mm} \\ i_Z = 13,09 \text{ mm} \end{cases} & \text{f)} \begin{cases} i_{Y_G} = 10,44 \text{ mm} \\ i_{Z_G} = 12,41 \text{ mm} \end{cases} & \text{g)} I_{YZ} = -39\,205,53 \text{ mm}^4 & \text{h)} I_{Y_G Z_G} = -61\,659,1 \text{ mm}^4 \\
 \text{i)} \begin{cases} I_{Y'} = 86\,742,94 \text{ mm}^4 \\ I_{Z'} = 157\,572,67 \text{ mm}^4 \\ I_{Y'Z'} = -17\,070,71 \text{ mm}^4 \end{cases} & \text{j)} \begin{cases} I_{Y'_G} = 132\,631,4 \text{ mm}^4 \\ I_{Z'_G} = 59\,282,1 \text{ mm}^4 \\ I_{Y'_G Z'_G} = -52\,217,4 \text{ mm}^4 \end{cases} & &
 \end{array}$$

Figura 5

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_Y = 419\,754,71 \text{ cm}^4 \\ I_Z = 110\,902,71 \text{ cm}^4 \end{cases} & \text{b)} \begin{cases} I_{Y_G} = 201\,786,28 \text{ cm}^4 \\ I_{Z_G} = 90\,837,07 \text{ cm}^4 \end{cases} & \text{c)} I_O = 530\,657,42 \text{ cm}^4 & \text{d)} I_{CG} = 292\,623,35 \text{ cm}^4 \\
 \text{e)} \begin{cases} i_Y = 19,94 \text{ cm} \\ i_Z = 10,25 \text{ cm} \end{cases} & \text{f)} \begin{cases} i_{Y_G} = 13,83 \text{ cm} \\ i_{Z_G} = 9,28 \text{ cm} \end{cases} & \text{g)} I_{YZ} = 100\,223,64 \text{ cm}^4 & \text{h)} I_{Y_G Z_G} = 34\,089,9 \text{ cm}^4 \\
 \text{i)} \begin{cases} I_{Y'} = 429\,337,9 \text{ cm}^4 \\ I_{Z'} = 101\,319,5 \text{ cm}^4 \\ I_{Y'Z'} = -83\,625,0 \text{ cm}^4 \end{cases} & \text{j)} \begin{cases} I_{Y'_G} = 155\,855,7 \text{ cm}^4 \\ I_{Z'_G} = 136\,767,7 \text{ cm}^4 \\ I_{Y'_G Z'_G} = 64\,408,6 \text{ cm}^4 \end{cases} & &
 \end{array}$$

Figura 6

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_Y = 337,612 \text{ cm}^4 \\ I_Z = 158,572 \text{ cm}^4 \end{cases} & \text{b)} \begin{cases} I_{Y_G} = 294,77 \text{ cm}^4 \\ I_{Z_G} = 137,475 \text{ cm}^4 \end{cases} & \text{c)} I_O = 496,18 \text{ cm}^4 & \text{d)} I_{CG} = 432,245 \text{ cm}^4 \\
 \text{e)} \begin{cases} i_Y = 2,92 \text{ cm} \\ i_Z = 2,00 \text{ cm} \end{cases} & \text{f)} \begin{cases} i_{Y_G} = 2,73 \text{ cm} \\ i_{Z_G} = 1,87 \text{ cm} \end{cases} & \text{g)} I_{YZ} = 3,375 \text{ cm}^4 & \text{h)} I_{Y_G Z_G} = -26,7 \text{ cm}^4 \\
 \text{i)} \begin{cases} I_{Y'} = 295,77 \text{ cm}^4 \\ I_{Z'} = 200,41 \text{ cm}^4 \\ I_{Y'Z'} = -75,84 \text{ cm}^4 \end{cases} & \text{j)} \begin{cases} I_{Y'_G} = 287,13 \text{ cm}^4 \\ I_{Z'_G} = 145,11 \text{ cm}^4 \\ I_{Y'_G Z'_G} = 43,09 \text{ cm}^4 \end{cases} & &
 \end{array}$$

Figura 7

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_Y = 9\,971,87 \text{ cm}^4 \\ I_Z = 15\,125,23 \text{ cm}^4 \end{cases} & \text{b)} \begin{cases} I_{Y_G} = 9\,267,28 \text{ cm}^4 \\ I_{Z_G} = 14\,344,82 \text{ cm}^4 \end{cases} & \text{c)} I_O = 25\,097,1 \text{ cm}^4 & \text{d)} I_{CG} = 23\,612,1 \text{ cm}^4 \\
 \text{e)} \begin{cases} i_Y = 8,04 \text{ cm} \\ i_Z = 9,90 \text{ cm} \end{cases} & \text{f)} \begin{cases} i_{Y_G} = 7,75 \text{ cm} \\ i_{Z_G} = 9,64 \text{ cm} \end{cases} & \text{g)} I_{YZ} = 3\,554,45 \text{ cm}^4 & \text{h)} I_{Y_G Z_G} = 4\,295,98 \text{ cm}^4 \\
 \text{i)} \begin{cases} I_{Y'} = 14\,338,45 \text{ cm}^4 \\ I_{Z'} = 10\,758,65 \text{ cm}^4 \\ I_{Y'Z'} = 4\,008,70 \text{ cm}^4 \end{cases} & \text{j)} \begin{cases} I_{Y'_G} = 6\,883,25 \text{ cm}^4 \\ I_{Z'_G} = 16\,728,85 \text{ cm}^4 \\ I_{Y'_G Z'_G} = 816,59 \text{ cm}^4 \end{cases} & &
 \end{array}$$

Figura 8

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_X = 5002,18 \text{ cm}^4 \\ I_Y = 22357,27 \text{ cm}^4 \end{cases} & \text{b)} \begin{cases} I_{X_G} = 4139,01 \text{ cm}^4 \\ I_{Y_G} = 18313,27 \text{ cm}^4 \end{cases} & \text{c)} } I_O = 27359,45 \text{ cm}^4 & \text{d)} } I_{CG} = 22452,28 \text{ cm}^4 \\
 \text{e)} \begin{cases} i_X = 5,56 \text{ cm} \\ i_Y = 11,76 \text{ cm} \end{cases} & \text{f)} \begin{cases} i_{X_G} = 5,06 \text{ cm} \\ i_{Y_G} = 10,64 \text{ cm} \end{cases} & \text{g)} } I_{XY} = 1865,46 \text{ cm}^4 & \text{h)} } I_{X_G Y_G} = 0 \\
 \text{i)} \begin{cases} I_{X'} = 7725,4 \text{ cm}^4 \\ I_{Y'} = 19634,0 \text{ cm}^4 \\ I_{X'Y'} = -6582,2 \text{ cm}^4 \end{cases} & \text{j)} \begin{cases} I_{X'_G} = 6671 \text{ cm}^4 \\ I_{Y'_G} = 15782 \text{ cm}^4 \\ I_{X'_G Y'_G} = 5429 \text{ cm}^4 \end{cases} & &
 \end{array}$$

Figura 9

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_Y = 6231,35 \text{ cm}^4 \\ I_Z = 4008,20 \text{ cm}^4 \end{cases} & \text{b)} \begin{cases} I_{Y_G} = 6231,35 \text{ cm}^4 \\ I_{Z_G} = 3113,34 \text{ cm}^4 \end{cases} & \text{c)} } I_O = 10239,55 \text{ cm}^4 & \text{d)} } I_{CG} = 9344,7 \text{ cm}^4 \\
 \text{e)} \begin{cases} i_Y = 8,31 \text{ cm} \\ i_Z = 6,66 \text{ cm} \end{cases} & \text{f)} \begin{cases} i_{Y_G} = 8,31 \text{ cm} \\ i_{Z_G} = 5,87 \text{ cm} \end{cases} & \text{g)} } I_{YZ} = 0 & \text{h)} } I_{Y_G Z_G} = 0 \\
 \text{i)} \begin{cases} I_{Y'} = 5675,56 \text{ cm}^4 \\ I_{Z'} = 4563,99 \text{ cm}^4 \\ I_{Y'Z'} = -962,65 \text{ cm}^4 \end{cases} & \text{j)} \begin{cases} I_{Y'_G} = 5674,4 \text{ cm}^4 \\ I_{Z'_G} = 3670,2 \text{ cm}^4 \\ I_{Y'_G Z'_G} = 0194,3 \text{ cm}^4 \end{cases} & &
 \end{array}$$

Figura 10

$$\begin{array}{llll}
 \text{a)} \begin{cases} I_X = 6261,21 \text{ cm}^4 \\ I_Y = 6261,21 \text{ cm}^4 \end{cases} & \text{b)} \begin{cases} I_{X_G} = 5209,95 \text{ cm}^4 \\ I_{Y_G} = 5209,95 \text{ cm}^4 \end{cases} & \text{c)} } I_O = 12522,4 \text{ cm}^4 & \text{d)} } I_{CG} = 10419,9 \text{ cm}^4 \\
 \text{e)} \begin{cases} i_X = 8,54 \text{ cm} \\ i_Y = 8,54 \text{ cm} \end{cases} & \text{f)} \begin{cases} i_{X_G} = 7,79 \text{ cm} \\ i_{Y_G} = 7,79 \text{ cm} \end{cases} & \text{g)} } I_{XY} = 2173,19 \text{ cm}^4 & \text{h)} } I_{X_G Y_G} = 1121,86 \text{ cm}^4 \\
 \text{i)} \begin{cases} I_{X'} = 8143,25 \text{ cm}^4 \\ I_{Y'} = 4379,17 \text{ cm}^4 \\ I_{X'Y'} = 1086,59 \text{ cm}^4 \end{cases} & \text{j)} \begin{cases} I_{X'_G} = 4350,6 \text{ cm}^4 \\ I_{Y'_G} = 6069,3 \text{ cm}^4 \\ I_{X'_G Y'_G} = 721,1 \text{ cm}^4 \end{cases} & &
 \end{array}$$

EXERCÍCIO 2 a) $d = 7,73 \text{ cm}$

EXERCÍCIO 3 Espessura das chapas: $e = 13 \text{ mm}$

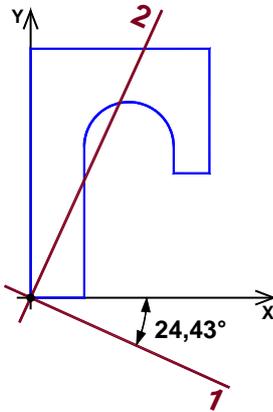
FICHA 13 – Geometria de Massas – Eixos e Momentos Principais de Inércia.

EXERCÍCIO 1

Figura 1

a) $\alpha_p = -24,43^\circ$

$$\begin{cases} I_1 = 5207965,5 \text{ cm}^4 \\ I_2 = 313432,9 \text{ cm}^4 \end{cases}$$



b) $\alpha_p = -31,32^\circ$

$$\begin{cases} I_1 = 835004,8 \text{ cm}^4 \\ I_2 = 335737,2 \text{ cm}^4 \end{cases}$$

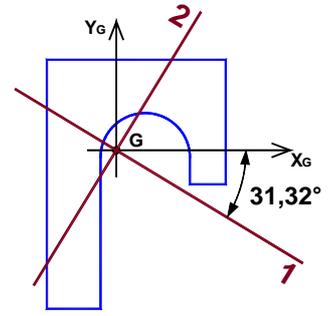
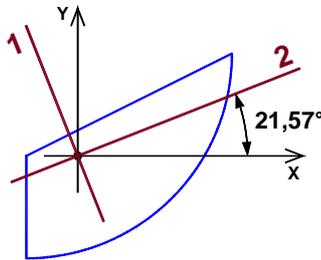


Figura 2

a) $\alpha_p = 21,57^\circ$

$$\begin{cases} I_1 = 902413,4 \text{ cm}^4 \\ I_2 = 280329,3 \text{ cm}^4 \end{cases}$$



b) $\alpha_p = 37,92^\circ$

$$\begin{cases} I_1 = 709956,6 \text{ cm}^4 \\ I_2 = 132316,8 \text{ cm}^4 \end{cases}$$

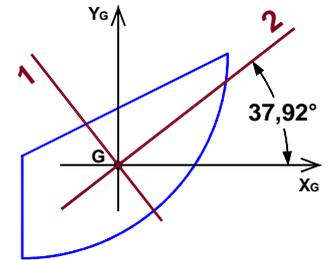
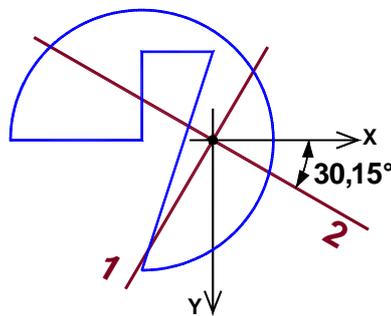


Figura 3

a) $\alpha_p = 30,15^\circ$

$$\begin{cases} I_1 = 639,0 \text{ cm}^4 \\ I_2 = 211,9 \text{ cm}^4 \end{cases}$$



b) $\alpha_p = 40,11^\circ$

$$\begin{cases} I_1 = 429,9 \text{ cm}^4 \\ I_2 = 197,7 \text{ cm}^4 \end{cases}$$

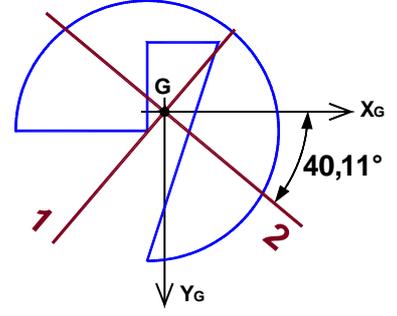
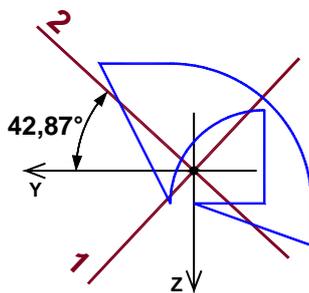


Figura 4

a) $\alpha_p = -42,87^\circ$

$$\begin{cases} I_1 = 161472,2 \text{ mm}^4 \\ I_2 = 82842,4 \text{ mm}^4 \end{cases}$$



b) $\alpha_p = -37,54^\circ$

$$\begin{cases} I_1 = 159766,5 \text{ mm}^4 \\ I_2 = 32147,0 \text{ mm}^4 \end{cases}$$

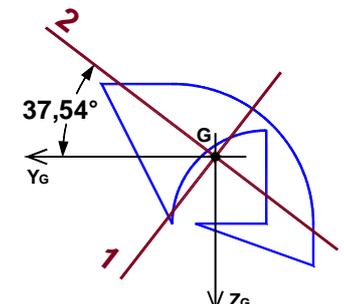
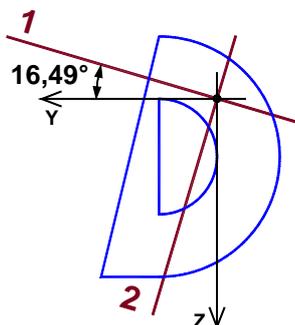


Figura 5

a) $\alpha_p = -16,49^\circ$

$$\begin{cases} I_1 = 449426,96 \text{ cm}^4 \\ I_2 = 81230,45 \text{ cm}^4 \end{cases}$$



b) $\alpha_p = -15,78^\circ$

$$\begin{cases} I_1 = 211468 \text{ cm}^4 \\ I_2 = 81205 \text{ cm}^4 \end{cases}$$

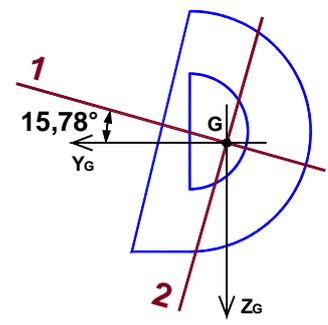
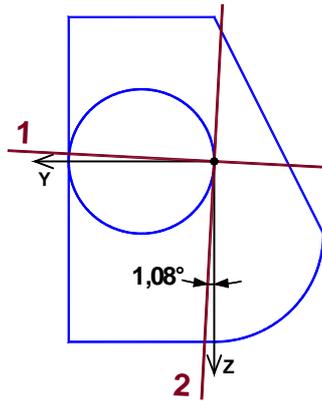


Figura 6

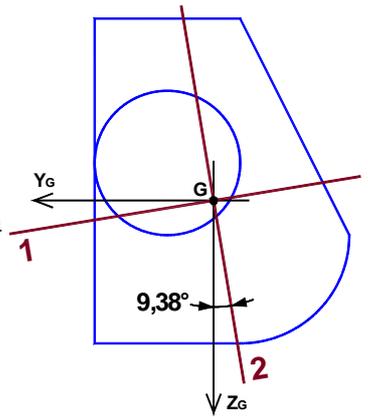
a) $\alpha_p = -1,08^\circ$

$$\begin{cases} I_1 = 337,7 \text{ cm}^4 \\ I_2 = 158,5 \text{ cm}^4 \end{cases}$$



b) $\alpha_p = 9,38^\circ$

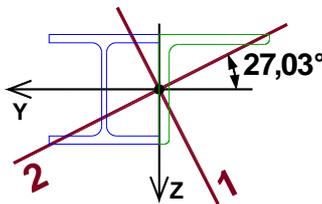
$$\begin{cases} I_1 = 299,2 \text{ cm}^4 \\ I_2 = 133,07 \text{ cm}^4 \end{cases}$$



EXERCÍCIO 2

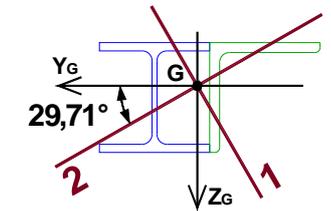
a) $\alpha_p = 27,03^\circ$

$$\begin{cases} I_1 = 16938,7 \text{ cm}^4 \\ I_2 = 8158,4 \text{ cm}^4 \end{cases}$$



b) $\alpha_p = 29,71^\circ$

$$I_{\text{máx}} = I_1 = 16796,1 \text{ cm}^4$$

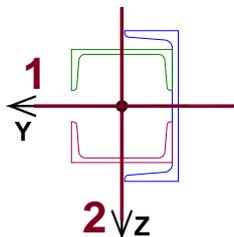


EXERCÍCIO 3

Figura 1

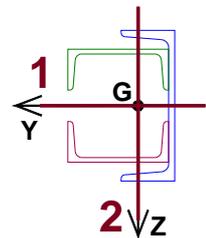
a) $\alpha_p = 0^\circ \pm 90^\circ$

$$\begin{cases} I_1 = 6231,25 \text{ cm}^4 \\ I_2 = 4008,20 \text{ cm}^4 \end{cases}$$



b) $\alpha_p = 0^\circ \pm 90^\circ$

$$\begin{cases} I_1 = 6231,35 \text{ cm}^4 \\ I_2 = 3113,34 \text{ cm}^4 \end{cases}$$

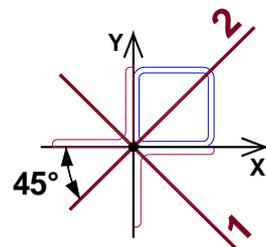


c) $I_\Delta = 4989,45 \text{ cm}^4$

Figura 2

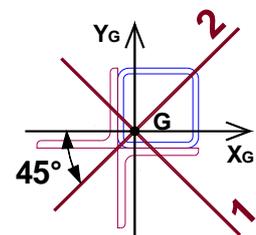
a) $\alpha_p = 45^\circ$

$$\begin{cases} I_1 = 8434,40 \text{ cm}^4 \\ I_2 = 4088,02 \text{ cm}^4 \end{cases}$$



b) $\alpha_p = 45^\circ$

$$\begin{cases} I_1 = 6331,81 \text{ cm}^4 \\ I_2 = 4088,02 \text{ cm}^4 \end{cases}$$

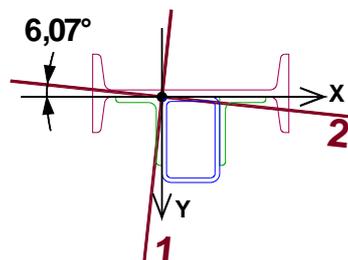


c) $I_\Delta = 4864,31 \text{ cm}^4$

EXERCÍCIO 4

a) $\alpha_p = 6,07^\circ$

$$\begin{cases} I_1 = 22555,5 \text{ cm}^4 \\ I_2 = 4803,9 \text{ cm}^4 \end{cases}$$



b) eixo 2 = eixo X_G

$$I_{\text{mín}} = I_2 = 4139 \text{ cm}^4$$

