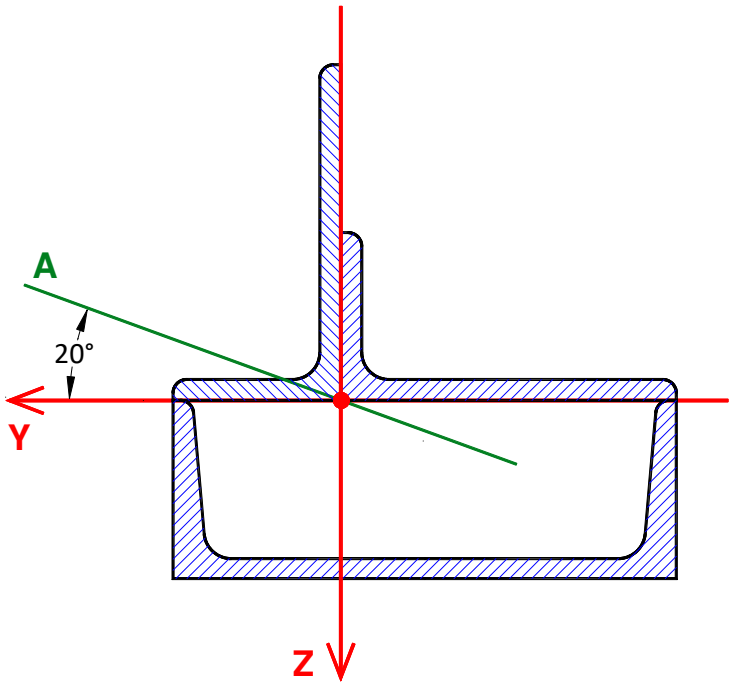
Instituto Superior de Engenharia do Porto

LICENCIATURA EM ENGENHARIA CIVIL

ESTÁTICA



GEOMETRIA DE MASSAS

RESOLUÇÃO DO EXERCÍCIO DO EXAME DE ÉPOCA NORMAL

ANO LETIVO 2011 - 12

ISABEL ALVIM TELES

EXERCÍCIO

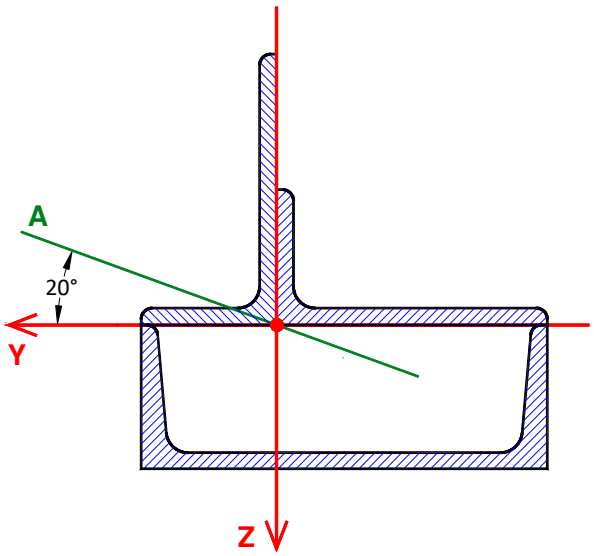
Considere a secção plana representada na figura, composta por três perfis metálicos: duas cantoneiras de abas desiguais **L 160x80x10** e um perfil **UNP 240**.

Relativamente ao sistema de eixos (**Y,Z**) representado, determine:

a) As coordenadas do centro de gravidade, posicionando-o na figura;

b) Os momentos de inércia **Iy** e **Iz** e o produto de inércia **Iyz**;

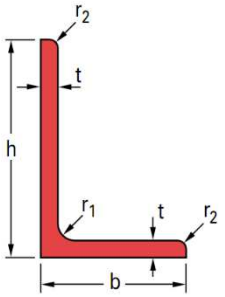
c) O momento de inércia em relação ao eixo **A**: I_A.

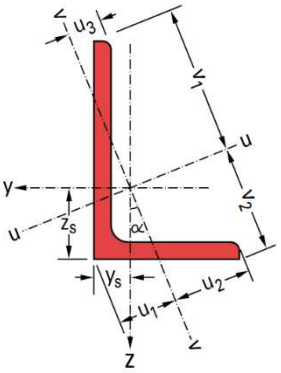


Notas importantes:

- Utilize a unidade de comprimento *"centímetro"* [cm].
- Utilize nos cálculos o mínimo de 2 casas decimais.

Cantoneiras de abas desiguais

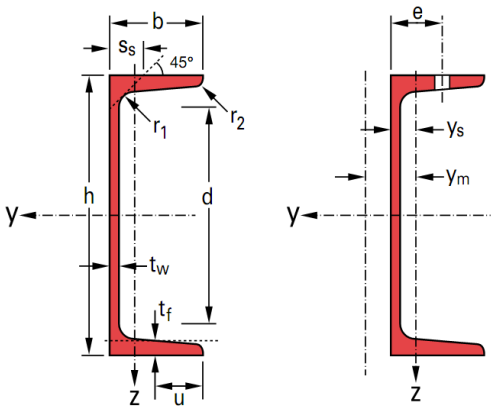




Désignation Designation Bezeichnung		Dimensions Abmessungen						Position des axes Position of axes Lage der Achsen						
	G kg/m	h mm	b mm	t mm	r_1 mm	r_2 mm	A cm^2	z_s cm	y_s cm	v_1 cm	v_2 cm	u_1 cm	u_2 cm	u_3 cm
L 160 x 80 x 10	18.2	160	80	10	13	6.5	23.18	5.63	1.69	10.46	7.04	3.06	4.70	1.73
L 160 x 80 x 12	21.6	160	80	12	13	6.5	27.54	5.72	1.77	10.40	7.10	3.15	4.66	1.80

Désignation Designation Bezeichnung		Valeurs statiques / Section properties / Statische Kennwerte											
		axe y-y axis y-y Achse y-y			axe z-z axis z-z Achse z-z			axe u-u axis u-u Achse u-u		axe v-v axis v-v Achse v-v			
	G kg/m	I_y cm^4	$W_{el,y}$ cm^3	i_y cm	I_z cm^4	$W_{el,z}$ cm^3	i_z cm	I_u cm^4	i_u cm	I_v cm^4	i_v cm	I_{yz} cm^4	α °
L 160 x 80 x 10	18.2	611.3	58.94	5.14	104.4	16.55	2.12	648.7	5.29	67.01	1.70	-142.7	14.69
L 160 x 80 x 12	21.6	719.5	69.98	5.11	122.0	19.59	2.10	762.8	5.26	78.77	1.69	-166.5	14.57

UNP

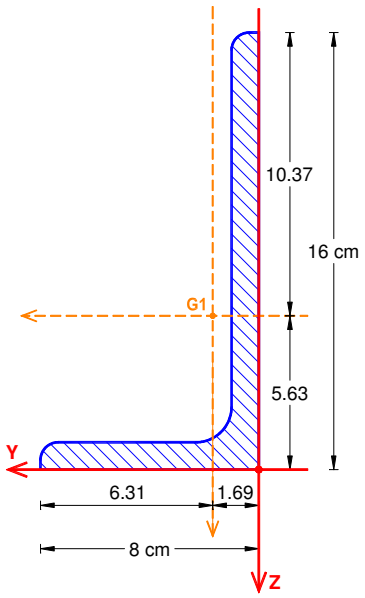


Désignation Designation Bezeichnung		Dimensions Abmessungen							Dimensions de construction Dimensions for detailing Konstruktionsmaße			
	G kg/m	h mm	b mm	t_w mm	t_f mm	r_1 mm	r_2 mm	A cm^2	d mm	∅	e_{min} mm	e_{max} mm
UPN 200	25.3	200	75	8.5	11.5	11.5	6	32.2	151	M16	39	46
UPN 220	29.4	220	80	9	12.5	12.5	6.5	37.4	167	M16	40	51
UPN 240	33.2	240	85	9.5	13	13	6.5	42.3	184	M20	46	50
UPN 260	37.9	260	90	10	14	14	7	48.3	200	M22	50	52
UPN 280	41.8	280	95	10	15	15	7.5	53.3	216	M22	52	57

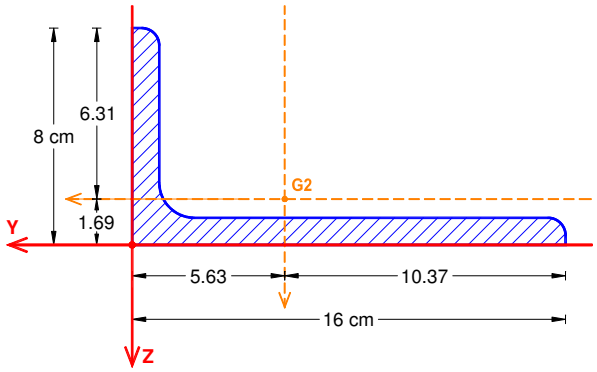
Désignation Designation Bezeichnung		Valeurs statiques / Section properties / Statische Kennwerte													
		axe fort y-y strong axis y-y starke Achse y-y					axe faible z-z weak axis z-z schwache Achse z-z								
	G kg/m	I_y cm^4	$W_{el.y}$ cm^3	$W_{pl.y}$ cm^3	i_y cm	A_{vz} cm^2	I_z cm^4	$W_{el.z}$ cm^3	$W_{pl.z}$ cm^3	i_z cm	s_s mm	I_t cm^4	$I_w \times 10^{-3}$ cm^6	y_s cm	y_m cm
UPN 200	25.3	1910	191	228	7.7	17.71	148	27	51.8	2.14	28.1	11.9	9.07	2.01	3.94
UPN 220	29.4	2690	245	292	8.48	20.62	197	33.6	64.1	2.3	30.3	16	14.6	2.14	4.2
UPN 240	33.2	3600	300	358	9.22	23.71	248	39.6	75.7	2.42	31.7	19.7	22.1	2.23	4.39
UPN 260	37.9	4820	371	442	9.99	27.12	317	47.7	91.6	2.56	33.9	25.5	33.3	2.36	4.66
UPN 280	41.8	6280	448	532	10.9	29.28	399	57.2	109	2.74	35.6	31	48.5	2.53	5.02

RESOLUÇÃO

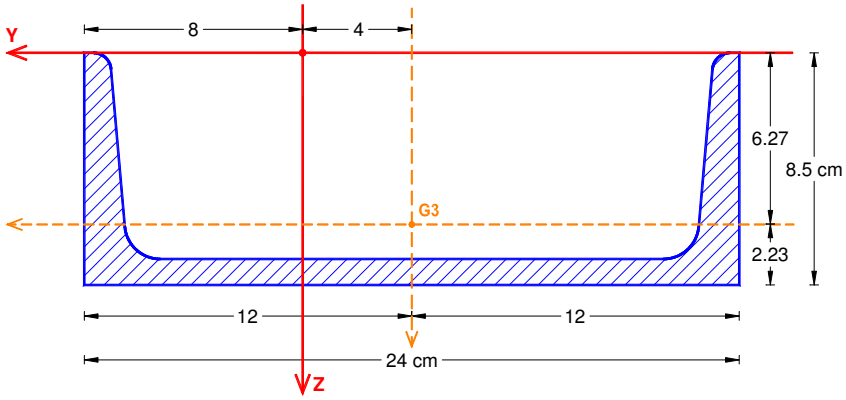
Alínea a)



Secção **1**: $\begin{cases} A_1 = 23{,}18\ cm^2 \\ Y_{G1} = 1{,}69\,cm \\ Z_{G1} = -5{,}63\ cm \end{cases}$



Secção **2**: $\begin{cases} A_2 = 23{,}18\ cm^2 \\ Y_{G2} = -5{,}63\ cm \\ Z_{G2} = -1{,}69\,cm \end{cases}$

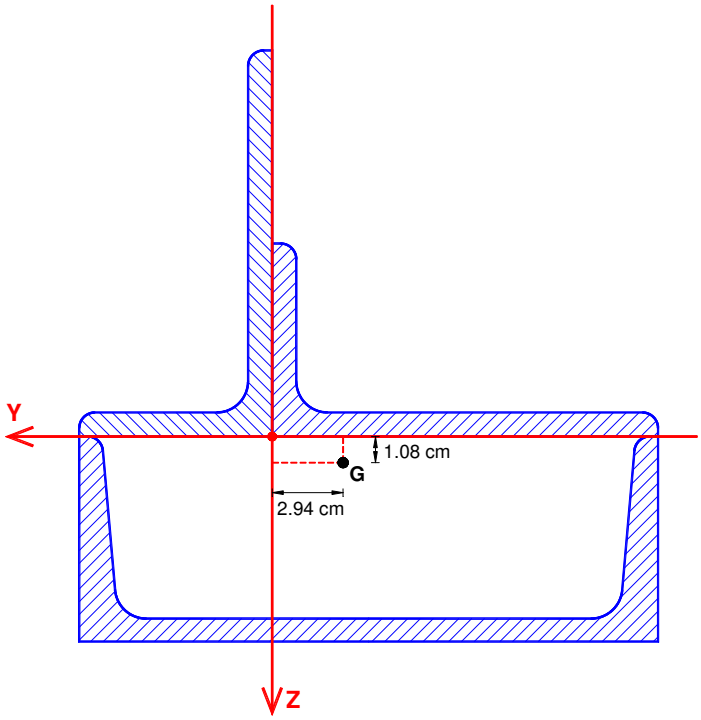


Secção **3**: $\begin{cases} A_3 = 42{,}3\ cm^2 \\ Y_{G3} = -4\ cm \\ Z_{G3} = 6{,}27\,cm \end{cases}$

Secção	A_i (cm^2)	Y_{Gi} (cm)	Z_{Gi} (cm)	$A_i \times Y_{Gi}$	$A_i \times Z_{Gi}$
1	23,18	1,69	- 5,63	39,1742	- 130,5034
2	23,18	- 5,63	- 1,69	- 130,5034	- 39,1742
3	42,30	- 4	6,27	- 169,2000	265,2210
Σ	88,66			- 260,5292	95,5434

$$\mathbf{Y_G} = \frac{\sum A_i \,.\, Y_{Gi}}{\sum A_i} = \frac{-260{,}5292}{88{,}66} = -2{,}94 \text{ cm}$$

$$\mathbf{Z_G} = \frac{\sum A_i \,.\, Z_{Gi}}{\sum A_i} = \frac{95{,}5434}{88{,}66} = 1{,}08 \text{ cm}$$



Alínea b)

Determinação dos momentos de inércia I_Y e I_Z

Teorema da transferência: $\mathbf{I_\Delta} = \mathbf{I_{\Delta_G}} + A \,.\, d^2$

$$\begin{cases} \mathbf{I_Y} = \mathbf{I_{Y_G}} + A \,.\, Z_G{}^2 \\ \mathbf{I_Z} = \mathbf{I_{Z_G}} + A \,.\, Y_G{}^2 \end{cases}$$

Secção **1**:
$$\begin{cases} \mathbf{I}_Y^{①} = 611{,}3 + 23{,}18 \times (-5{,}63)^2 = 1346{,}034 \text{ cm}^4 \\ \mathbf{I}_Z^{①} = 104{,}4 + 23{,}18 \times 1{,}69^2 = 170{,}604 \text{ cm}^4 \end{cases}$$

Secção **2**:
$$\begin{cases} \mathbf{I}_Y^{②} = 104{,}4 + 23{,}18 \times (-1{,}69)^2 = 170{,}604 \text{ cm}^4 \\ \mathbf{I}_Z^{②} = 611{,}4 + 23{,}18 \times (-5{,}63)^2 = 1346{,}034 \text{ cm}^4 \end{cases}$$

Secção **3**:
$$\begin{cases} \mathbf{I}_Y^{③} = 248 + 42{,}3 \times 6{,}27^2 = 1910{,}936 \text{ cm}^4 \\ \mathbf{I}_Z^{③} = 3600 + 42{,}3 \times (-4)^2 = 4276{,}8 \text{ cm}^4 \end{cases}$$

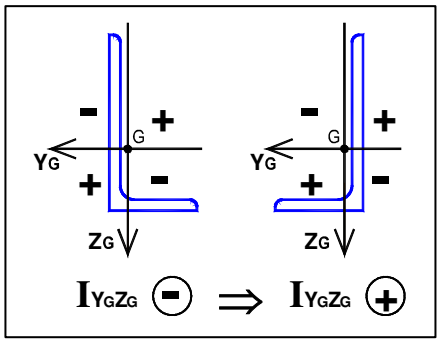
- **Momentos de Inércia**

$$I_Y = I_Y^{①} + I_Y^{②} + I_Y^{③} = 1346{,}034 + 170{,}604 + 1910{,}936 = 3427{,}57 \text{ cm}^4$$

$$I_Z = I_Z^{①} + I_Z^{②} + I_Z^{③} = 170{,}604 + 1346{,}034 + 4276{,}8 = 5793{,}44 \text{ cm}^4$$

Determinação do produto de inérciao I_{YZ}

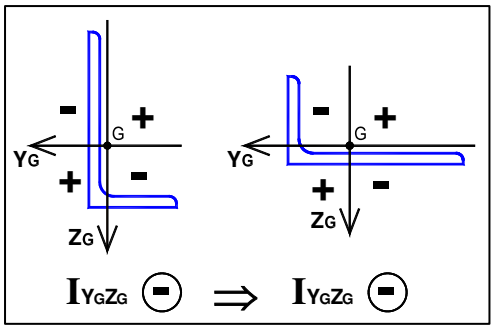
Teorema da transferência: $I_{YZ} = I_{Y_GZ_G} + A \cdot Y_G \cdot Z_G$



- **Secção 1**

$$I_{YZ}^{①} = I_{Y_{G1}Z_{G1}}^{①} + A_1 \cdot Y_{G1} \cdot Z_{G1}$$

$$I_{YZ}^{①} = 142{,}7 + 23{,}18 \times 1{,}69 \times (-5{,}63) = -77{,}8507 \text{ cm}^4$$



- **Secção 2**

$$I_{YZ}^{②} = I_{Y_{G2}Z_{G2}}^{②} + A_2 \cdot Y_{G2} \cdot Z_{G2}$$

$$I_{YZ}^{②} = -142{,}70 + 23{,}18 \times (-5{,}63) \times (-1{,}69) = 77{,}8507 \text{ cm}^4$$

- **Secção 3**

$$I_{YZ}^{③} = I_{Y_{G3}Z_{G3}}^{③} + A_3 \cdot Y_{G3} \cdot Z_{G3}$$

$$I_{XY}^{③} = 0 + 42{,}3 \times (-4) \times 6{,}27 = -1060{,}884 \text{ cm}^4$$

- **Produto de Inércia**

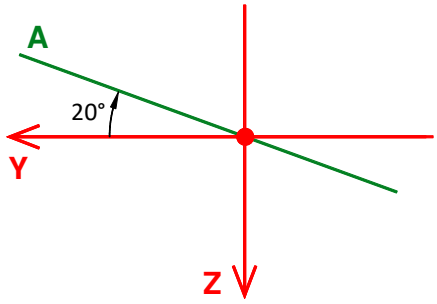
$$I_{YZ} = I_{YZ}^{①} + I_{YZ}^{②} + I_{YZ}^{③} = -77{,}8507 + 77{,}8507 - 1060{,}884 = -1060{,}88 \text{ cm}^4$$

Alínea c)

Determinação do momento de inércia I_A

$$I_A = \frac{I_Y + I_Z}{2} + \frac{I_Y - I_Z}{2}\cos 2\alpha - I_{YZ}\,\text{sen}\,2\alpha$$

$\alpha = -20° \quad \Rightarrow \quad 2\alpha = -40°$



$$I_A = \frac{3427{,}57 + 5793{,}44}{2} + \frac{3427{,}57 - 5793{,}44}{2}\cos(-40^\circ) - (-1060{,}88).\,\text{sen}(-40^\circ) = 3022{,}40\ \text{cm}^4$$