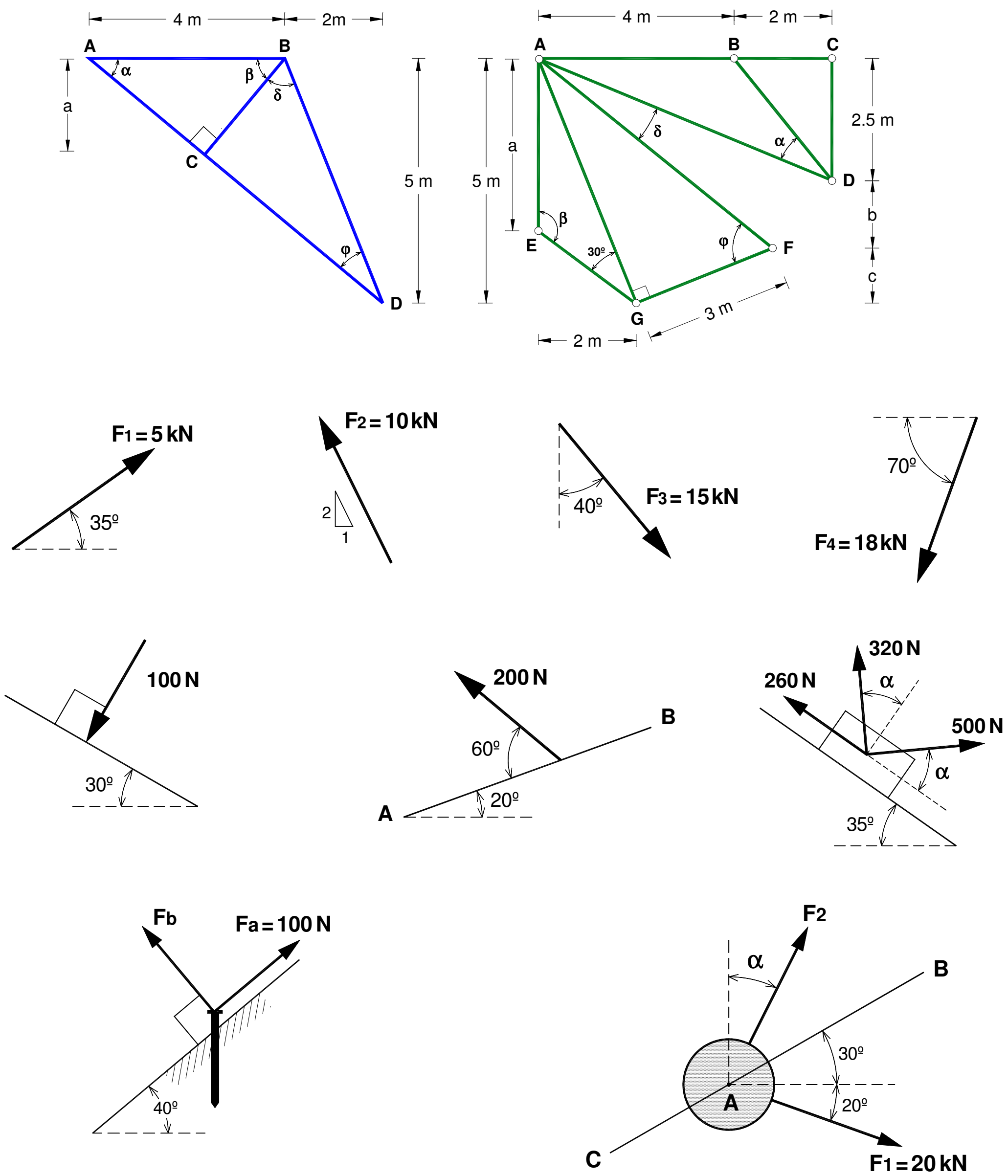
LICENCIATURA EM ENGENHARIA CIVIL

ESTÁTICA



REVISÕES

GEOMETRIA - CÁLCULO VETORIAL

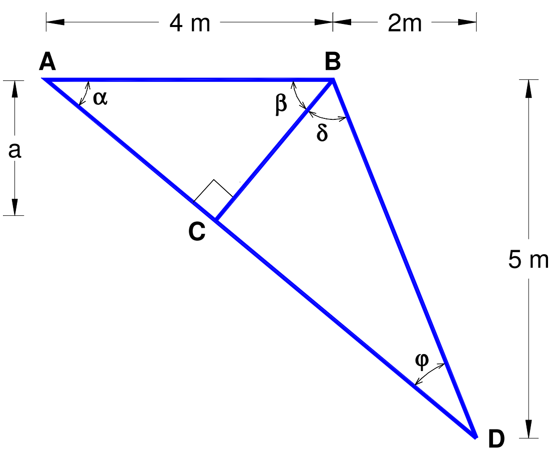
EXERCÍCIOS PROPOSTOS E RESOLVIDOS

ISABEL ALVIM TELES

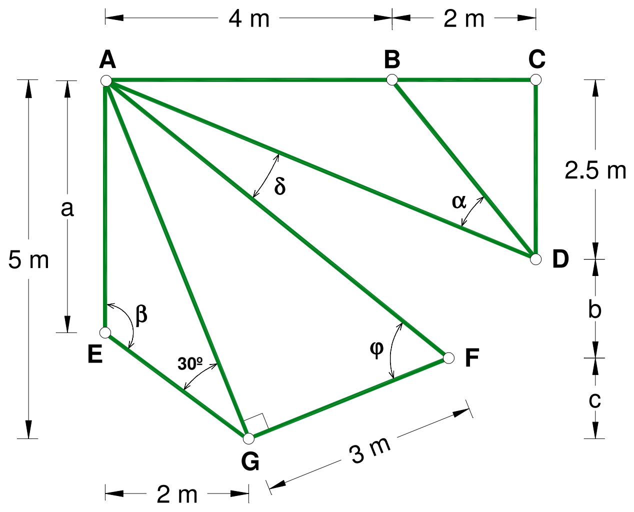
EXERCÍCIO 1

Determine o comprimento de todas as barras, as cotas e os ângulos representados nas figuras seguintes.

1. Figura 1

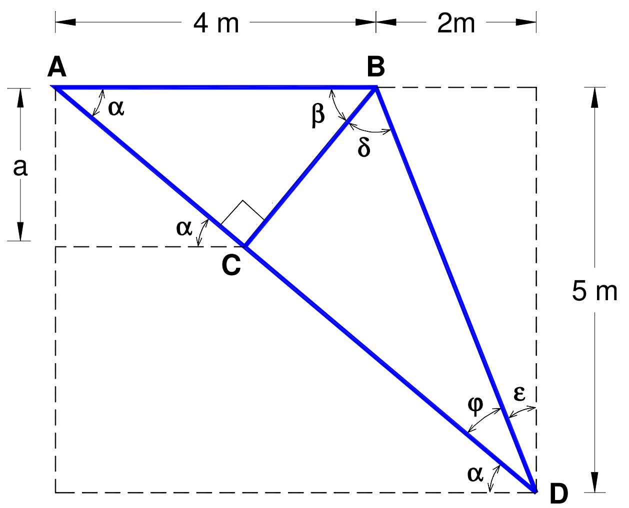


2. Figura 2



RESOLUÇÃO

FIGURA 1



$$\alpha = \text{arctg}\,\frac{5}{6} = 39{,}806^\circ$$

Triângulo retângulo **ABC**:

$$\alpha + \beta + 90^\circ = 180^\circ$$

$$\beta = 180^\circ - 90^\circ - 39{,}806^\circ = 50{,}195^\circ$$

$$\varepsilon = \text{arctg}\,\frac{2}{5} = 21{,}801^\circ$$

$$\alpha + \varphi + \varepsilon = 90^\circ \quad \Rightarrow \quad \varphi = 90^\circ - \alpha - \varepsilon = 90^\circ - 39{,}806^\circ - 21{,}801^\circ$$

$$\varphi = 28{,}393^\circ$$

Triângulo retângulo **BCD**: $\varphi + \delta + 90^\circ = 180^\circ \quad \Rightarrow \quad \delta = 180^\circ - 90^\circ - \varphi \quad \Rightarrow \quad \delta = 61{,}607^\circ$

BARRAS

$$\overline{AB} = 4\,m$$

$$\overline{BD} = \sqrt{2^2 + 5^2} = 5{,}385\,m$$

$$\overline{AC} = 4\ \cos\boldsymbol{\alpha} = 4\ \cos 39{,}803^\circ = 3{,}073\,m$$

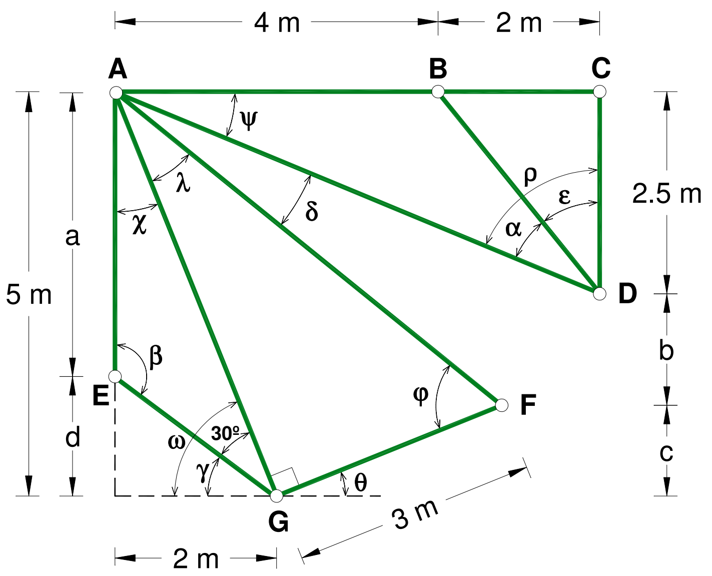
$$\overline{BC} = 4\ \mathrm{sen}\,\boldsymbol{\alpha} = 4\ \mathrm{sen}\,39{,}803^\circ = 2{,}561\,m$$

$$\overline{CD} = \frac{\overline{BC}}{\mathrm{tg}\,\boldsymbol{\varphi}} = \frac{2{,}561}{\mathrm{tg}\,28{,}393^\circ} = 4{,}737\,m$$

COTAS

$$\mathbf{a} = \overline{AC}\ \ \mathrm{sen}\,\boldsymbol{\alpha} = 3{,}073\ \mathrm{sen}\,39{,}803^\circ = 1{,}967\,m$$

FIGURA 2



$$\boldsymbol{\rho} = \mathrm{arctg}\,\frac{6}{2{,}5} = 67{,}380^\circ$$

$$\boldsymbol{\varepsilon} = \mathrm{arctg}\,\frac{2}{2{,}5} = 38{,}660^\circ$$

$$\boldsymbol{\alpha} = \boldsymbol{\rho} - \boldsymbol{\varepsilon} = 28{,}720^\circ$$

$$\boldsymbol{\psi} = 180^\circ - 90^\circ - \boldsymbol{\rho} = 22{,}620^\circ$$

$$\boldsymbol{\omega} = \mathrm{arctg}\,\frac{5}{2} = 68{,}1986^\circ$$

$$\boldsymbol{\chi} = \mathrm{arctg}\,\frac{2}{5} = 21{,}801^\circ$$

$$\boldsymbol{\gamma} = \boldsymbol{\omega} - 30 = 38{,}1986^\circ$$

$$\boldsymbol{\beta} = 180^\circ - 30^\circ - \boldsymbol{\chi} = 128{,}199^\circ$$

Cota $\mathbf{d} = 2 \times \mathrm{tg}\,\boldsymbol{\gamma} = 2 \times \mathrm{tg}\,38{,}1986^\circ = 1{,}574\,m$

Cota $\mathbf{a} = 5 - 1{,}574 = 3{,}426\,m = \overline{\mathbf{AE}}$

Triângulo $\overline{AEG}$ **Lei dos sen:** $\frac{\overline{\mathbf{AG}}}{\text{sen }\boldsymbol{\beta}} = \frac{\overline{\mathbf{EG}}}{\text{sen }\boldsymbol{\chi}} = \frac{\overline{\mathbf{AE}}}{\text{sen }30^\circ} \Rightarrow \frac{\overline{\mathbf{AG}}}{\text{sen }\boldsymbol{\beta}} = \frac{\overline{\mathbf{AE}}}{\text{sen }30^\circ}$

$$\overline{\mathbf{AG}} = \frac{\overline{\mathbf{AE}}}{\text{sen }30^\circ} \times \text{sen }\boldsymbol{\beta} = \frac{3{,}426}{\text{sen }30^\circ} \times \text{sen }128{,}199^\circ = 5{,}385\,\text{m}$$

Triângulo $\overline{AFG}$ $\varphi = \text{arctg}\,\frac{\overline{\mathbf{AG}}}{\overline{\mathbf{GF}}} = \text{arctg}\,\frac{5{,}385}{3} = 60{,}878^\circ$

$$\boldsymbol{\lambda} = 180^\circ - 90^\circ - \varphi = 29{,}122^\circ$$

$$\boldsymbol{\delta} = 90^\circ - \chi - \lambda - \psi = 90^\circ - 21{,}801^\circ - 29{,}122^\circ - 22{,}620^\circ = 16{,}457^\circ$$

$$\boldsymbol{\theta} = 180^\circ - 90^\circ - \boldsymbol{\omega} = 180^\circ - 90^\circ - 68{,}1986^\circ = 21{,}801^\circ$$

Cota $\mathbf{c} = \overline{\mathbf{GF}}\ \text{sen}\,\boldsymbol{\theta} = 3 \times \text{sen}\,21{,}801^\circ = 1{,}114\,\text{m}$

Cota $\mathbf{b} = 5 - 2{,}5 - 1{,}114 = 1{,}386\,\text{m}$

<u>BARRAS</u>

$\overline{AB} = 4\,\text{m}$

$\overline{BC} = 2\,\text{m}$

$\overline{AD} = \sqrt{6^2 + 2{,}5^2} = 6{,}5\,\text{m}$

$\overline{BD} = \sqrt{2^2 + 2{,}5^2} = 3{,}202\,\text{m}$

$\overline{CD} = 2{,}5\,\text{m}$

$\overline{AF} = \frac{3}{\cos\varphi} = \frac{3}{\cos 60{,}878^\circ} = 6{,}164\,\text{m}$

$\overline{AG} = 5{,}385\,\text{m}$

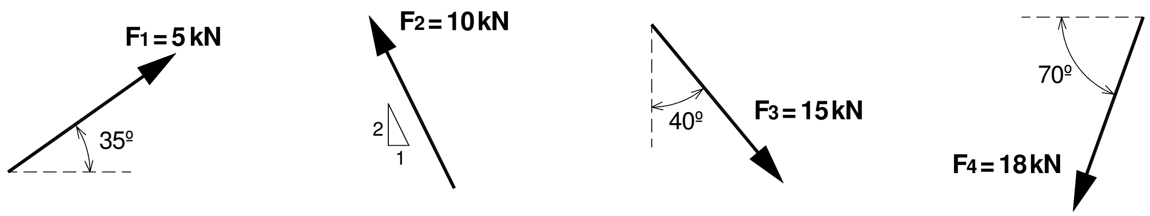
$\overline{AE} = 3{,}426\,\text{m}$

$\overline{EG} = \frac{\overline{AE}}{\text{sen }30^\circ} \times \text{sen }\chi = \frac{3{,}426}{\text{sen }30^\circ} \times \text{sen }21{,}801^\circ = 2{,}545\,\text{m}$

$\overline{GF} = 3\,\text{m}$

EXERCÍCIO 2

Considere as forças **F1**, **F2**, **F3** e **F4** representadas na figura.



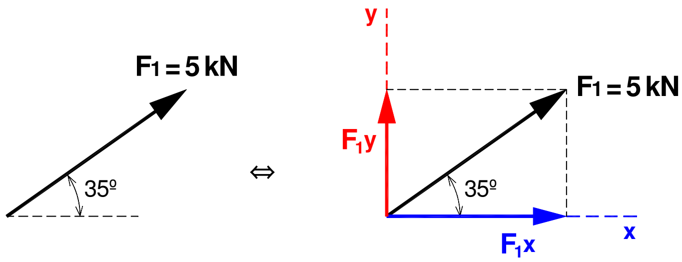
a) Determine as componentes segundo os eixos coordenados ***x*** e ***y***, de cada uma das forças.

b) Considerando que todas as forças estão a actuar num ponto **P**, determine a força **Fa = F1 + F2 + F3 + F4**, caracterizando a sua grandeza, ângulo com o eixo ***x*** e sentido.

c) Considerando que todas as forças estão a actuar num ponto **P**, determine a força **Fb = F1 - F2 + F3 - F4**, caracterizando a sua grandeza, ângulo com o eixo ***y*** e sentido.

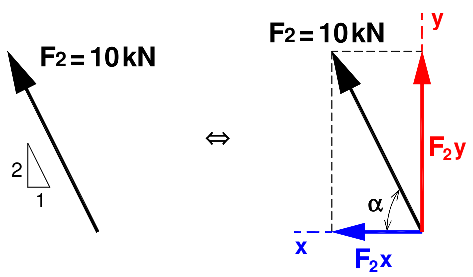
RESOLUÇÃO

Alínea a)



$F_1x = F_1 \cos 35° = 5 \cos 35° = 4{,}096\ \text{kN}\ \rightarrow$

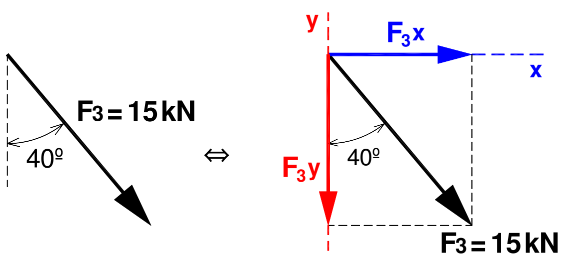
$F_1y = F_1 \text{ sen } 35° = 5 \text{ sen } 35° = 2{,}868\ \text{kN}\ \uparrow$



$$\alpha = \text{arctg}\ \frac{2}{1} = 63{,}435^{\circ}$$

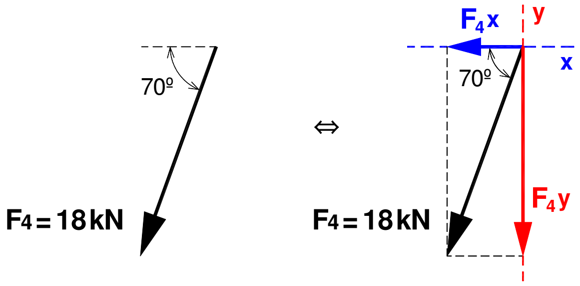
$F_2x = F_2 \cos \alpha = 10 \cos 63{,}435° = 4{,}472\ \text{kN}\ \leftarrow$

$F_2y = F_2 \text{ sen } 35° = 10 \text{ sen } 63{,}435° = 8{,}944\ \text{kN}\ \uparrow$



$F_3x = F_3 \text{ sen } 40° = 15 \text{ sen } 40° = 9{,}642\ \text{kN}\ \rightarrow$

$F_3y = F_3 \cos 40° = 15 \cos 40° = 11{,}491\ \text{kN}\ \downarrow$

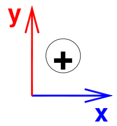


$F_{4x} = F_4 \cos 70° = 18 \cos 70° = 6{,}156 \text{ kN} \leftarrow$

$F_{4y} = F_4 \text{ sen } 70° = 5 \text{ sen } 70° = 16{,}914 \text{ kN} \downarrow$

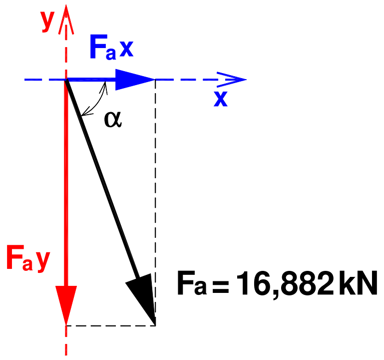
Alínea b)

$$F_a = F_1 + F_2 + F_3 + F_4 \quad \Rightarrow \quad \begin{cases} F_{ax} = F_{1x} + F_{2x} + F_{3x} + F_{4x} \\ F_{ay} = F_{1y} + F_{2y} + F_{3y} + F_{4y} \end{cases}$$



$$\begin{cases} F_{ax} = 4{,}096 - 4{,}472 + 9{,}642 - 6{,}156 = 3{,}11 \text{kN} \rightarrow \\ F_{ay} = 2{,}868 + 8{,}944 - 11{,}491 - 16{,}914 = -16{,}593 \text{kN} \downarrow \end{cases}$$

$$|F_a| = \sqrt{F_{ax}^2 + F_{ay}^2} = \sqrt{3{,}11^2 + (-16{,}593)^2} = 16{,}882 \text{kN}$$

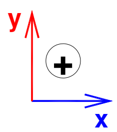


$F_a = 16{,}882 \text{ kN} \searrow$

$$\alpha = \text{arctg} \frac{F_{ay}}{F_{ax}} = \text{arctg} \frac{16{,}593}{3{,}11} = 79{,}38°$$

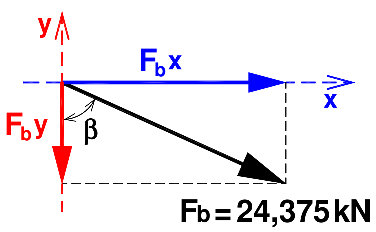
Alínea c)

$$F_b = F_1 - F_2 + F_3 - F_4 \quad \Rightarrow \quad \begin{cases} F_{bx} = F_{1x} - F_{2x} + F_{3x} - F_{4x} \\ F_{by} = F_{1y} - F_{2y} + F_{3y} - F_{4y} \end{cases}$$



$$\begin{cases} F_{bx} = 4{,}096 - (-4{,}472) + 9{,}642 - (-6{,}156) = 24{,}366 \text{kN} \rightarrow \\ F_{by} = 2{,}868 - 8{,}944 - 11{,}491 - (-16{,}914) = -0{,}653 \text{kN} \downarrow \end{cases}$$

$$|F_b| = \sqrt{F_{bx}^2 + F_{by}^2} = \sqrt{(-0{,}653)^2 + 24{,}366^2} = 24{,}375 \text{kN}$$

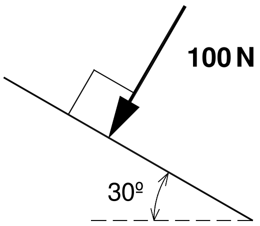


$F_b = 24{,}375 \text{ kN} \searrow$

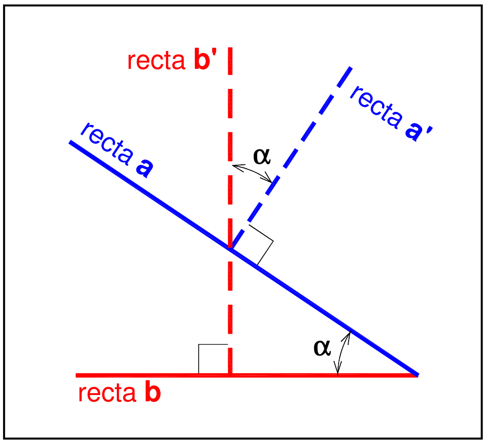
$$\beta = \text{arctg} \frac{F_{bx}}{F_{by}} = \text{arctg} \frac{24{,}366}{0{,}653} = 88{,}465°$$

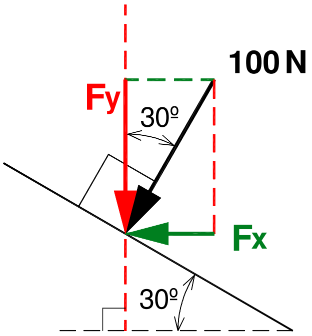
EXERCÍCIO 3

Considere a força de **100 N** representada na figura. Determine as componentes da força segundo os eixos coordenados ***x*** e ***y***.



RESOLUÇÃO





Fx = 100 sen 30° = 50 kN ←

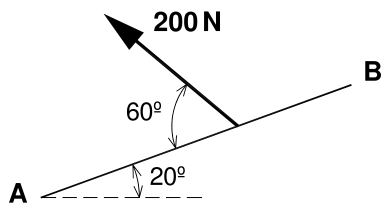
Fy = 100 cos 30° = 86,603 kN ↓

EXERCÍCIO 4

Considere a força de **200 N** representada na figura.

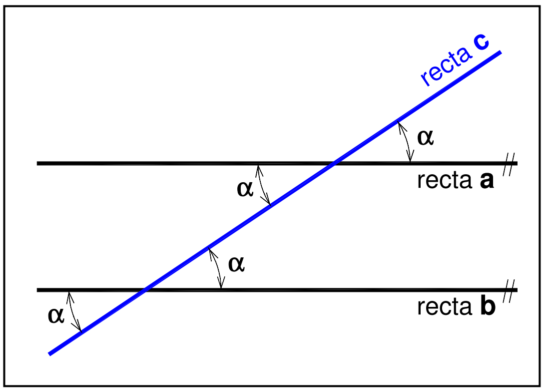
a) Determine as componentes da força segundo os eixos coordenados ***x*** e ***y***.

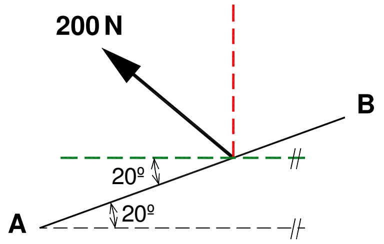
b) Decomponha a força em duas componentes: uma na direcção **AB** e outra na perpendicular à direcção **AB**.

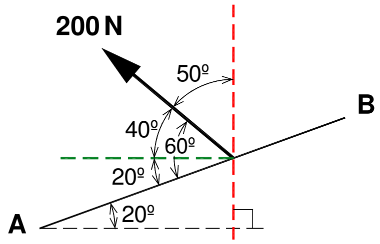


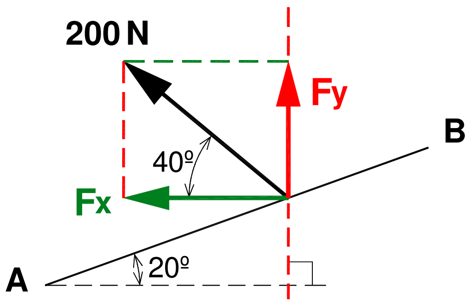
RESOLUÇÃO

Alínea a)





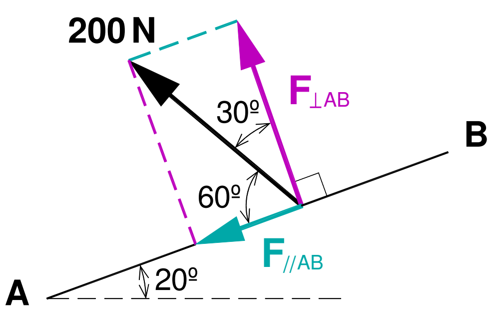




$Fx = 200 \cos 40° = 153{,}209$ kN ←

$Fy = 200 \text{ sen } 40° = 200 \cos 50° = 128{,}558$ kN ↑

Alínea b)



$F_{//AB} = 200 \cos 60° = 100$ kN ↙

$F_{\perp AB} = 200 \cos 30° = 173{,}205$ kN ↖

EXERCÍCIO 5

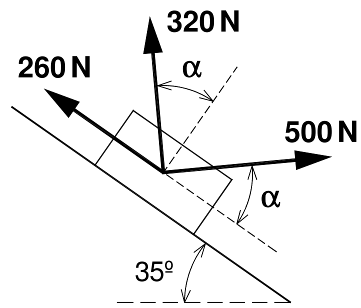
Considere o corpo rectangular representado na figura onde actuam três forças.

a) Considere **α=25º**.

- **a1)** Determine a resultante das três forças caracterizando a sua grandeza, ângulo com a horizontal e sentido.
- **a2)** Determine as componentes da resultante nas direcções perpendiculares aos lados maior e menor do corpo rectangular.

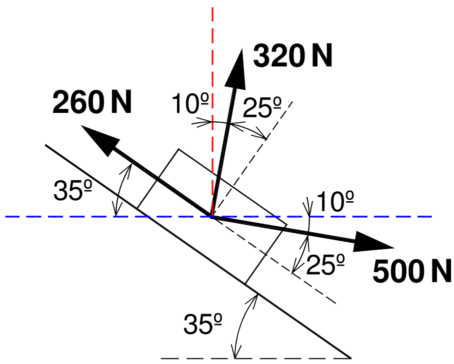
b) Considere **α=50º**.

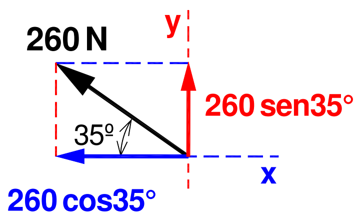
- **b1)** Determine a resultante das três forças caracterizando a sua grandeza, ângulo com a horizontal e sentido.
- **b2)** Determine as componentes da resultante nas direcções perpendiculares aos lados maior e menor do corpo rectangular.

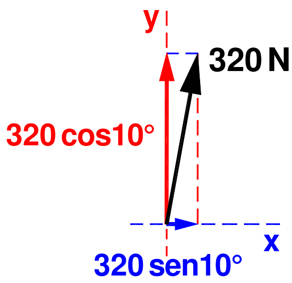


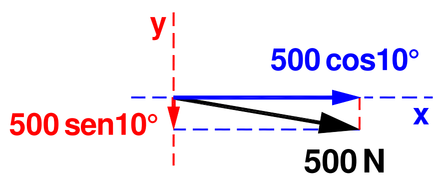
RESOLUÇÃO

Alínea a1)









$F_{1}x = -260 \cos 35° \text{ N}$ ←

$F_{1}y = 260 \text{ sen } 35° \text{ N}$ ↑

$F_{2}x = 320 \text{ sen } 10° \text{ N}$ →

$F_{2}y = 320 \cos 10° \text{ N}$ ↑

$F_{3}x = 500 \cos 10° \text{ N}$ →

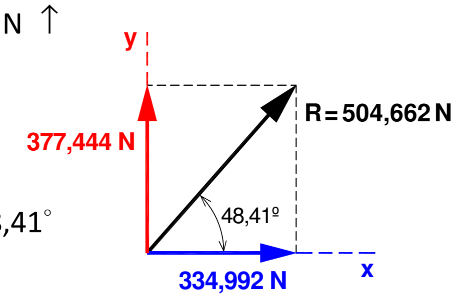
$F_{3}y = -500 \text{ sen } 10° \text{ N}$ ↓

Resultante **R** $\Rightarrow$ R = F1 + F2 + F3

$$\begin{cases} R_x = F_{1x} + F_{2x} + F_{3x} \\ R_y = F_{1y} + F_{2y} + F_{3y} \end{cases}$$

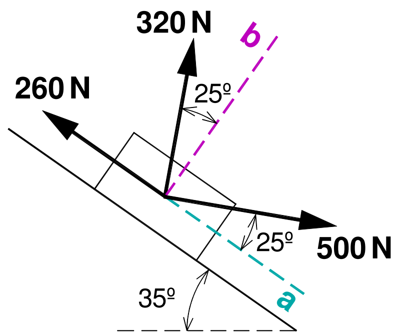
$$\begin{cases} R_x = -260\cos 35^\circ + 320\,\text{sen}\,10^\circ + 500\cos 10^\circ = 334{,}992\ \text{N}\ \rightarrow \\ R_y = 260\,\text{sen}\,35^\circ + 320\cos 10^\circ - 500\,\text{sen}\,10^\circ = 377{,}444\ \text{N}\ \uparrow \end{cases}$$

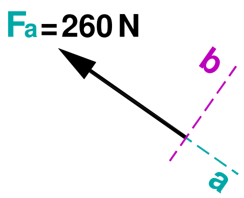
$$\mathbf{R} = \sqrt{R_x^2 + R_y^2} = \sqrt{334{,}992^2 + 377{,}444^2} = 504{,}662\ \text{N}\ \nearrow$$

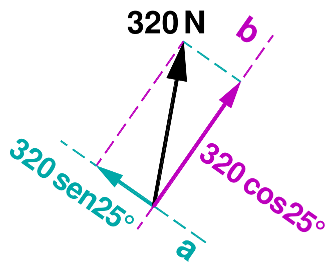


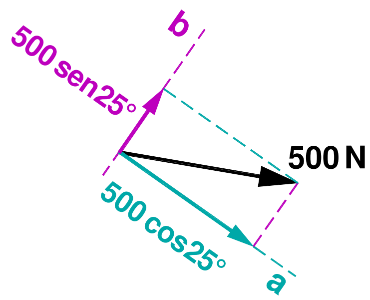
<u>Ângulo com a horizontal</u>: $\beta = \text{arctg}\dfrac{R_y}{R_x} = \text{arctg}\dfrac{377{,}444}{334{,}992} = 48{,}41^\circ$

<u>Alínea a2)</u>



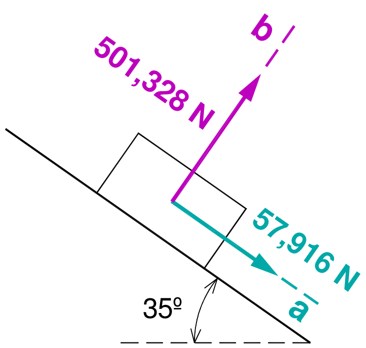






F_1a = - 260 N ↖ $\quad$ F_2a = - 320 sen 25° N ↖ $\quad$ F_3a = 500 cos 25° N ↘

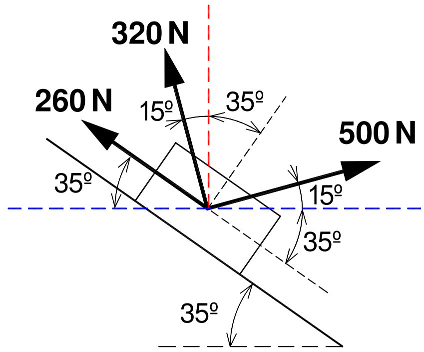
F_1b = 0 $\quad$ F_2b = 320 cos 25° N ↗ $\quad$ F_3b = 500 sen 25° N ↗

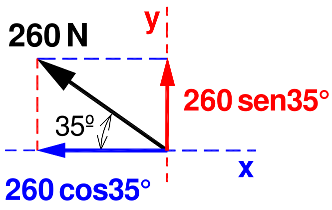


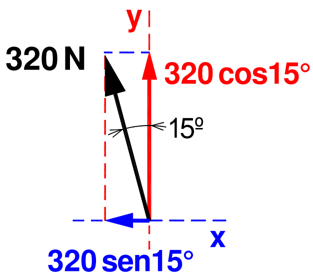
$$\begin{cases} R_a = F_{1a} + F_{2a} + F_{3a} \\ R_b = F_{1b} + F_{2b} + F_{3b} \end{cases}$$

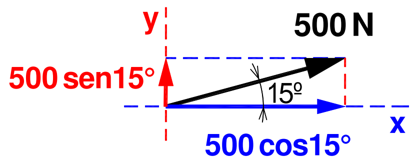
$$\begin{cases} R_a = -260 - 320\,\text{sen}\,25^\circ + 500\cos 25^\circ = 57{,}916\ \text{N}\ \searrow \\ R_b = 320\cos 25^\circ + 500\,\text{sen}\,25^\circ = 501{,}328\ \text{N}\ \nearrow \end{cases}$$

Alínea b1)









F_1x = - 260 cos 35° N ←

F_1y = 260 sen 35° N ↑

F_2x = - 320 sen 15° N ←

F_2y = 320 cos 15° N ↑

F_3x = 500 cos 15° N →

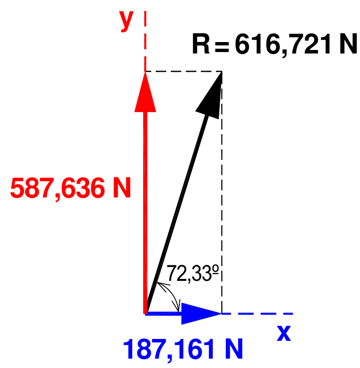
F_3y = 500 sen 15° N ↑

Resultante **R** ⇒ R = F1 + F2 + F3

$$\begin{cases} R_x = F_{1x} + F_{2x} + F_{3x} \\ R_y = F_{1y} + F_{2y} + F_{3y} \end{cases}$$

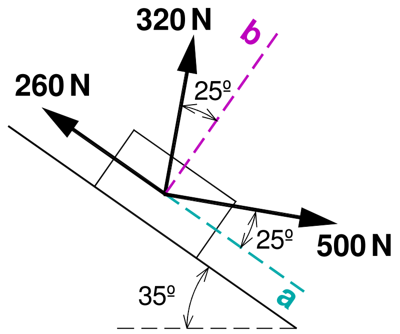
$$\begin{cases} R_x = -260\cos 35^\circ - 320\,\text{sen}\,15^\circ + 500\cos 15^\circ = 187{,}161\ \text{N} \rightarrow \\ R_y = 260\,\text{sen}\,35^\circ + 320\cos 15^\circ + 500\,\text{sen}\,15^\circ = 587{,}636\ \text{N} \uparrow \end{cases}$$

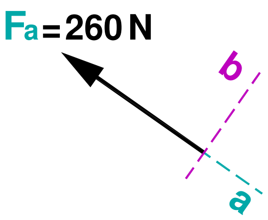
$$\mathbf{R} = \sqrt{R_x^2 + R_y^2} = \sqrt{187{,}161^2 + 587{,}636^2} = 616{,}721\ \text{N} \nearrow$$

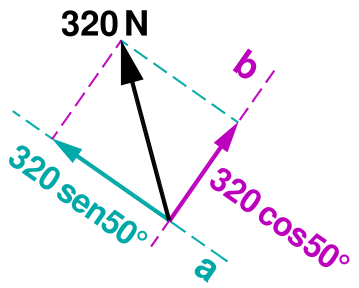


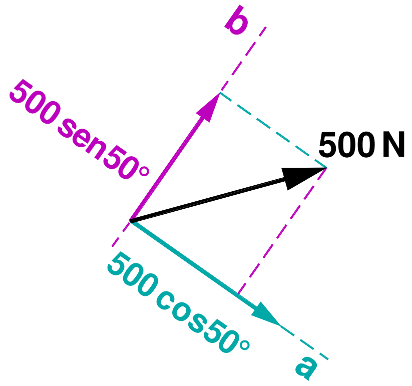
Ângulo com a horizontal: $\beta = \text{arctg}\dfrac{R_y}{R_x} = \text{arctg}\dfrac{587{,}636}{187{,}161} = 72{,}33^\circ$

Alínea b2)







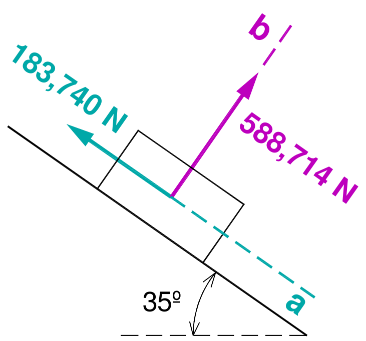


F_1a = - 260 N ↖ F_2a = - 320 sen 50° N ↖ F_3a = 500 cos 50° N ↘

F_1b = 0 F_2b = 320 cos 50° N ↗ F_3b = 500 sen 50° N ↗

$$\begin{cases} R_a = F_{1a} + F_{2a} + F_{3a} \\ R_b = F_{1b} + F_{2b} + F_{3b} \end{cases}$$

$$\begin{cases} R_a = -260 - 320 \operatorname{sen} 50^\circ + 500 \cos 50^\circ = -183{,}740 \text{ N } \nwarrow \\ R_b = 320 \cos 50^\circ + 500 \operatorname{sen} 50^\circ = 588{,}714 \text{ N } \nearrow \end{cases}$$

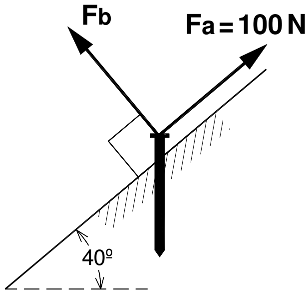


EXERCÍCIO 6

Para que não danifique a tábua de madeira onde está cravado, o prego representado na figura deverá ser arrancado na vertical. As ferramentas existentes permitem aplicar forças paralelas e perpendiculares à superfície da madeira.

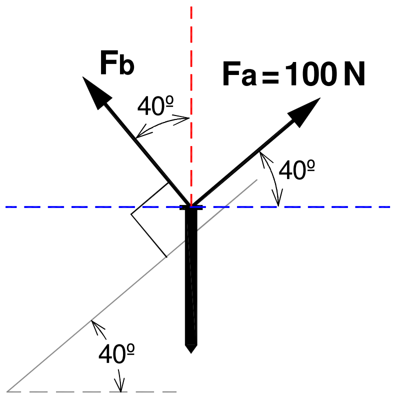
a) Se for aplicada uma força paralela à superfície da madeira de **100 N** (**Fa**), determine a grandeza da força perpendicular (**Fb**) que deverá ser aplicada simultaneamente para que o prego seja arrancado na vertical.

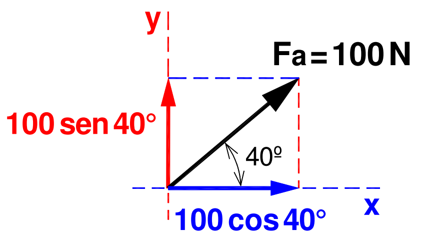
b) Determine a grandeza da força de arranque.

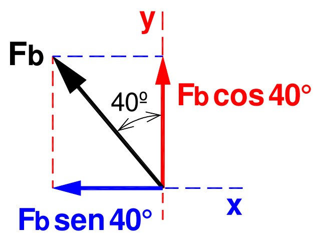


RESOLUÇÃO

Alínea a)







$F_{ax} = 100 \cos 40°$ N →

$F_{ay} = 100 \text{ sen } 40°$ N ↑

$F_{bx} = - F_b \text{ sen } 40°$ N ←

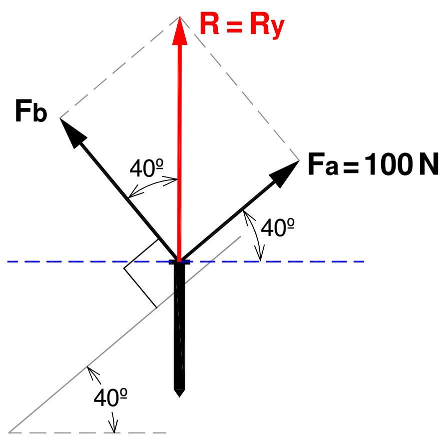
$F_{by} = F_b \cos 40°$ N ↑

Resultante **R** $\Rightarrow$ $R = Fa + Fb$ $\Rightarrow$ $\begin{cases} R_x = F_{ax} + F_{bx} \\ R_y = F_{ay} + F_{by} \end{cases}$

Pretende-se que a resultante seja vertical, ou seja, só tenha componente segundo **y**, pelo que a componente segundo **x** deverá ser nula $\Rightarrow R_x = 0$.

$$R_x = F_{ax} + F_{bx} = 0 \quad \Rightarrow \quad 100 \cos 40^\circ - Fb \text{ sen } 40^\circ = 0$$

$$Fb = \frac{100 \cos 40^\circ}{\text{sen } 40^\circ} = 119{,}175 \text{ N}$$



Alínea b)

Grandeza da força de arranque = Resultante **R** = Ry

$$R = \frac{Fb}{\cos 40^\circ} = \frac{119{,}175}{\cos 40^\circ} = 155{,}57 \text{ N} \uparrow$$

ou

$$R = \frac{Fa}{\text{sen } 40^\circ} = \frac{100}{\text{sen } 40^\circ} = 155{,}57 \text{ N} \uparrow$$

EXERCÍCIO 7

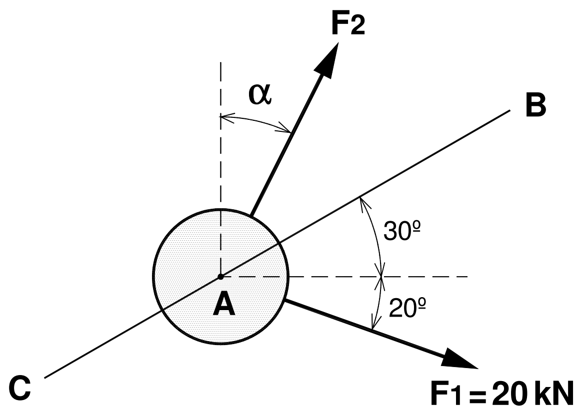
Considere a esfera representada na figura, sob a acção de duas forças: **F1** e **F2**.

a) Considere **α=25°**.

- Determine a grandeza da força **F2**, que actuando conjuntamente com **F1**, faz a esfera deslocar-se na trajetória **AB**.
- Determine a grandeza da resultante **R = F1 + F2**.

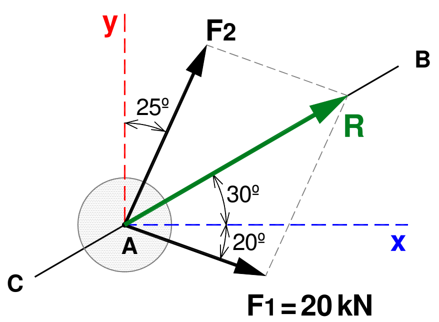
b) Considere a força **|F2| = 40 kN**.

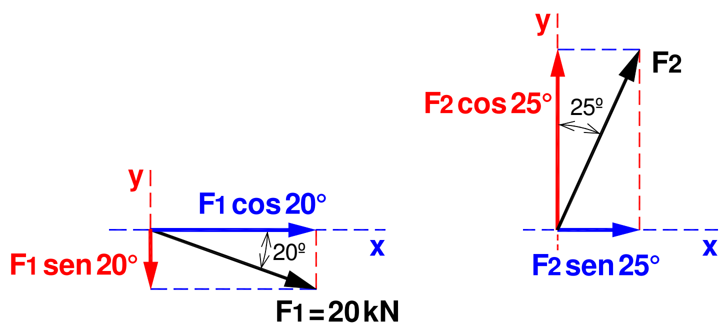
- Determine o ângulo α da direcção da força **F2** que, actuando conjuntamente com **F1**, faz a esfera deslocar-se na trajetória **AC**.
- Determine a grandeza da resultante **R = F1 + F2**.



RESOLUÇÃO

Alínea a)





F_1x = 20 cos 20° kN → F_2x = F2 sen 25° kN →

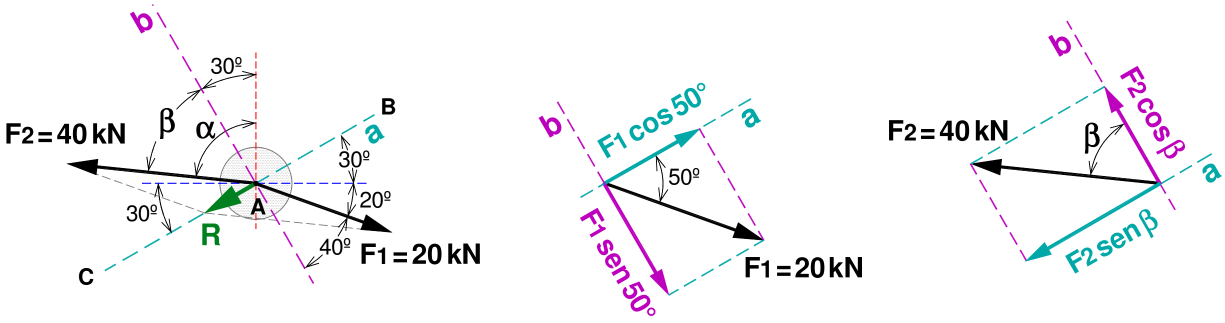
F_1y = -20 sen 20° kN ↓ F_2y = F2 cos 25° kN ↑

Para que a esfera se desloque na trajetória **AB**, a resultante do sistema de forças (F1 + F2) terá que ter a direcção **AB**, ou seja:

Resultante **R** $\Rightarrow \begin{cases} R_x = R\cos 30^\circ \\ R_y = R\,\text{sen}\,30^\circ \end{cases}$ e $R = F1 + F2 \Rightarrow \begin{cases} R_x = F_{1x} + F_{2x} \\ R_y = F_{1y} + F_{2y} \end{cases}$

$$\begin{cases} R\cos 30^\circ = F_{1x} + F_{2x} \\ R\,\text{sen}\,30^\circ = F_{1y} + F_{2y} \end{cases} \Rightarrow \begin{cases} R\cos 30^\circ = 20\cos 20^\circ + F_2\,\text{sen}\,25^\circ \\ R\,\text{sen}\,30^\circ = -20\,\text{sen}\,20^\circ + F_2\cos 25^\circ \end{cases} \Rightarrow \begin{cases} F_2 = 26{,}711 \text{ kN} \nearrow \\ R = 34{,}736 \text{ kN} \nearrow \end{cases}$$

Alínea b)



$F_{1}a = 20 \cos 50° \text{ kN}$ $\quad$ $F_{2}a = -40 \cos \beta \text{ kN}$

$F_{1}b = -20 \text{ sen } 50° \text{ kN}$ $\quad$ $F_{2}b = 40 \text{ sen } \beta \text{ kN}$

Para facilitar a resolução será utilizado o referencial (**a**,**b**) com a direção **a** coincidente com **AB**.

Para que a esfera se desloque na trajetória **AC**, a resultante do sistema de forças (F1 + F2) terá que ter a direcção **AC**. Para que isto aconteça, a força F2 deverá estar no 2º ou 3º quadrantes.

$$\text{Resultante } \mathbf{R} \Rightarrow \begin{cases} R_a = -R \\ R_b = 0 \end{cases} \quad \text{e} \quad R = F1 + F2 \Rightarrow \begin{cases} R_x = F_{1a} + F_{2a} \\ R_y = F_{1a} + F_{2a} \end{cases}$$

$$\begin{cases} -R = F_{1a} + F_{2a} \\ 0 = F_{1b} + F_{2b} \end{cases} \Rightarrow \begin{cases} -R = 20\cos 50^\circ - 40 \text{ sen}\beta \\ 0 = -20 \text{ sen } 50^\circ + 40 \cos\beta \end{cases} \Rightarrow \begin{cases} R = 24{,}094 \text{ kN} \\ \beta = 67{,}48^\circ \end{cases}$$

$$\alpha = \beta + 30^\circ \Rightarrow \alpha = 67{,}48^\circ + 30^\circ = 97{,}488^\circ$$

A força F2 está no 3º quadrante.