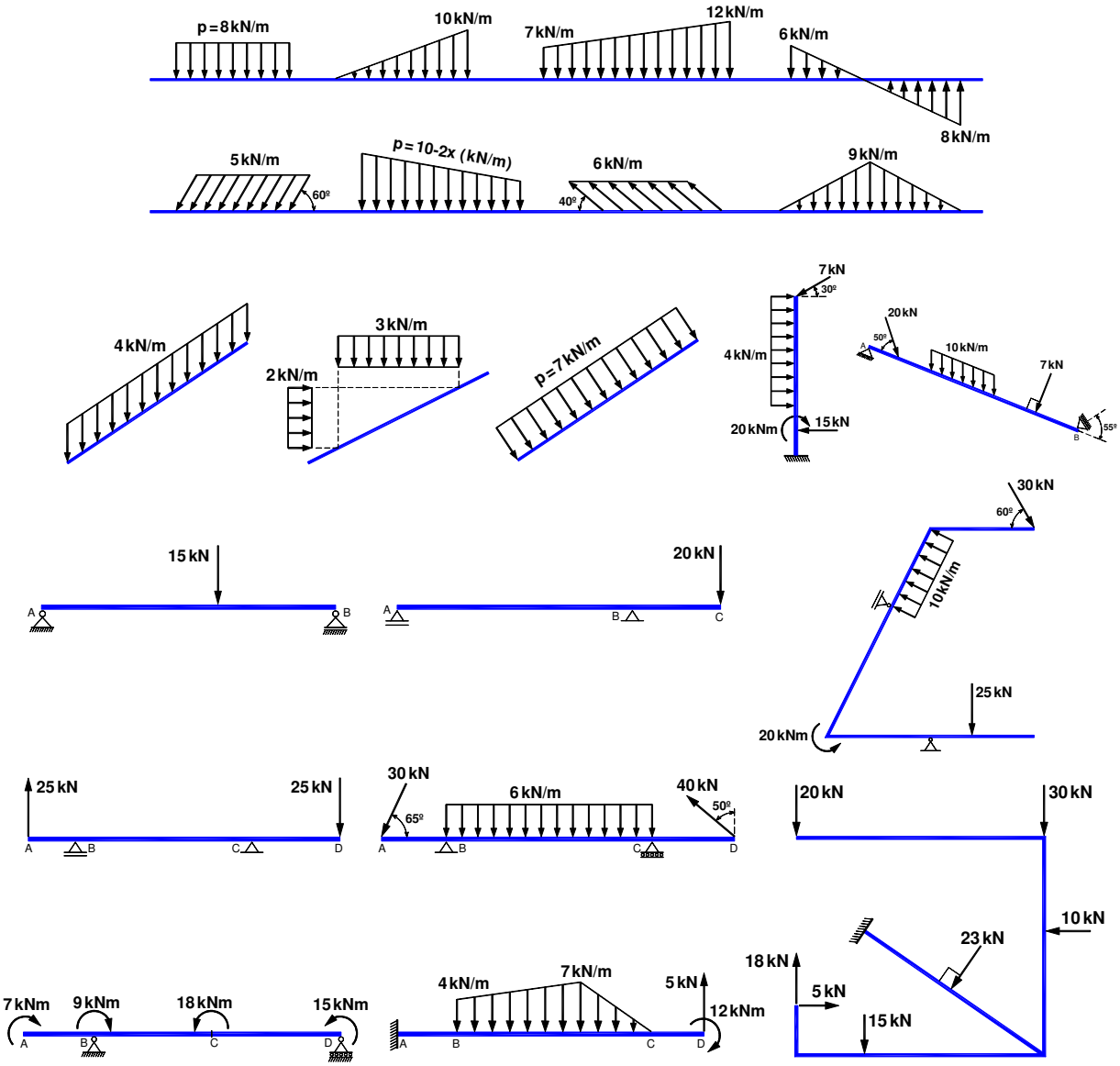
LICENCIATURA EM ENGENHARIA CIVIL

ESTÁTICA



ESTÁTICA DO CORPO RÍGIDO

EQUILÍBRIO DE CORPOS RÍGIDOS

CÁLCULO DE REAÇÕES

EXERCÍCIOS PROPOSTOS E RESOLVIDOS

ISABEL ALVIM TELES

EXERCÍCIO 1

Substitua os carregamentos das figuras por forças resultantes equivalentes no cálculo do equilíbrio de corpos rígidos.

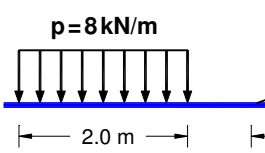


Figura 1

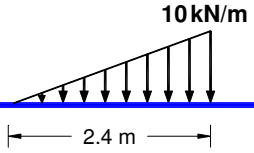
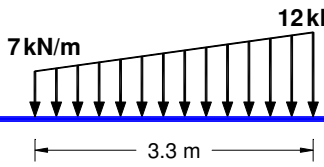


Figura 2



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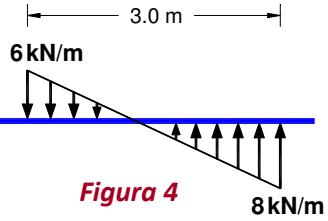
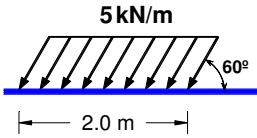


Figura 4



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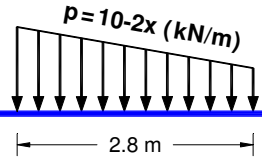

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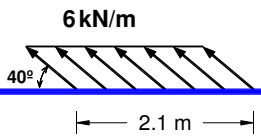

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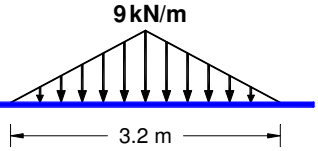

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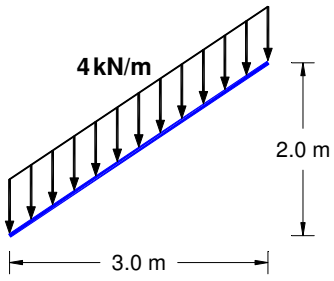


Figura 9

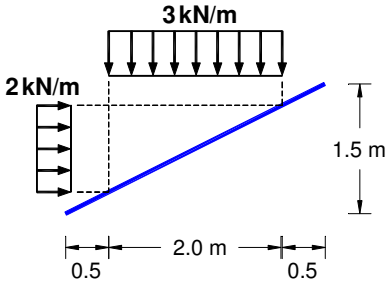


Figura 10

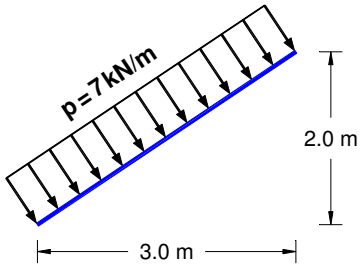
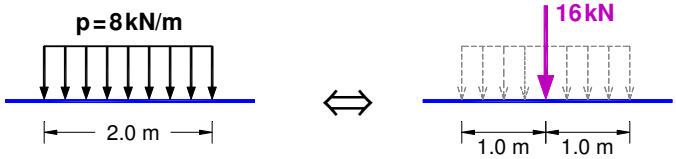


Figura 11

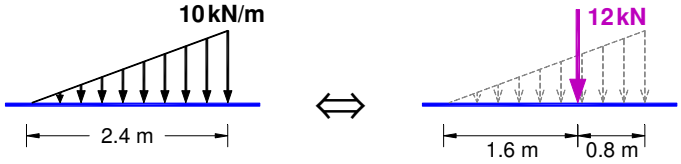
RESOLUÇÃO

Figura 1



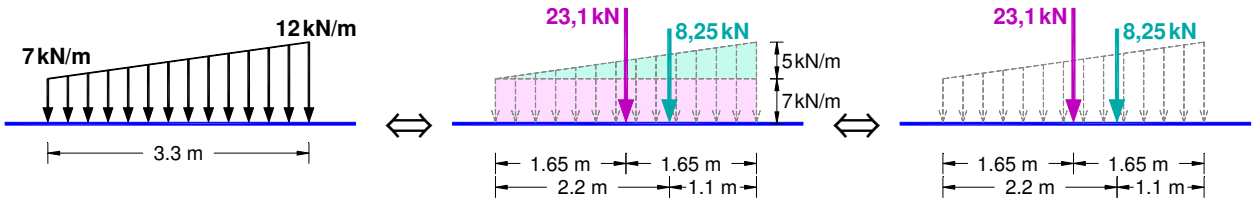
Força equivalente = 8 x 2 = 16 kN

Figura 2



Força equivalente = $\frac{12 \times 2{,}4}{2} = 12$ kN

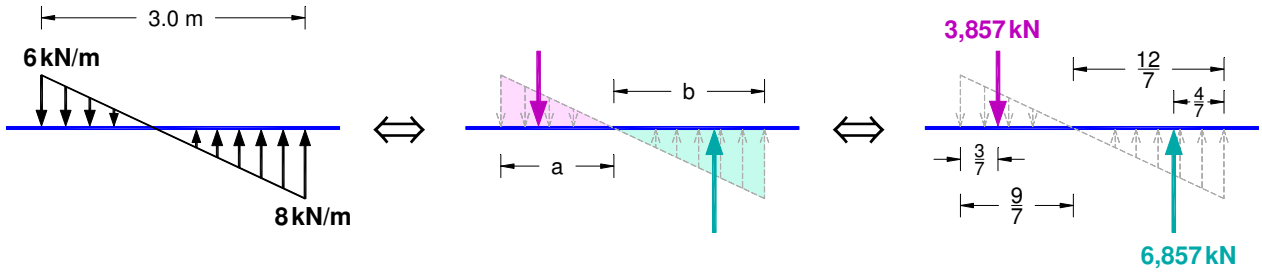
Figura 3



Carga uniforme retangular $\Rightarrow$ Força equivalente = $7 \times 3{,}3 = 23{,}1$ kN

Carga triangular $\Rightarrow$ Força equivalente = $\dfrac{5 \times 3{,}3}{2} = 8{,}25$ kN

Figura 4

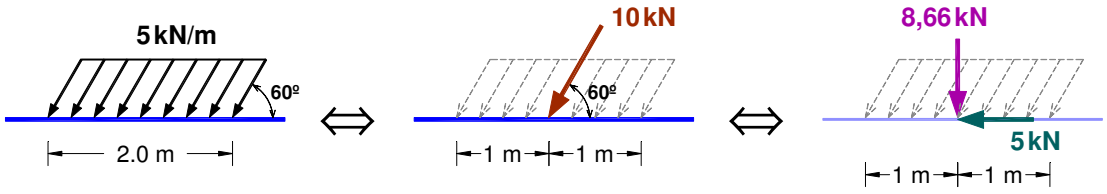


$$\begin{cases} \dfrac{6}{a} = \dfrac{8}{b} \\ a + b = 3 \end{cases} \Rightarrow \begin{cases} a = \dfrac{9}{7} \\ b = \dfrac{12}{7} \end{cases}$$

Carga triangular da esquerda $\Rightarrow$ Força equivalente = $\dfrac{1}{2} \times \dfrac{9}{7} \times 6 = 3{,}857$ kN $\downarrow$

Carga triangular da direita $\Rightarrow$ Força equivalente = $\dfrac{1}{2} \times \dfrac{12}{7} \times 8 = 6{,}857$ kN $\uparrow$

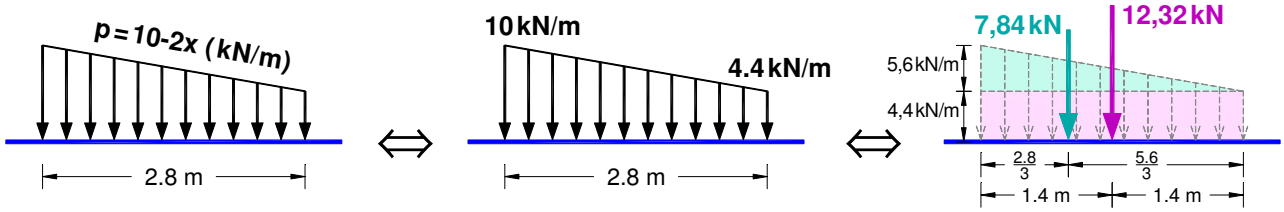
Figura 5



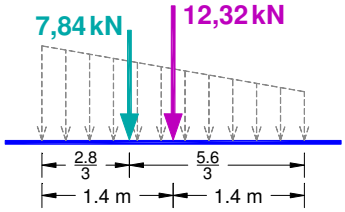
Força equivalente = $5 \times 2 = 10$ kN

$$\begin{cases} F_x = 10 \cos 60^\circ \\ F_y = 10 \operatorname{sen} 60^\circ \end{cases} \Rightarrow \begin{cases} F_x = 5 \text{ kN} \\ F_y = 8{,}66 \text{ kN} \end{cases}$$

Figura 6



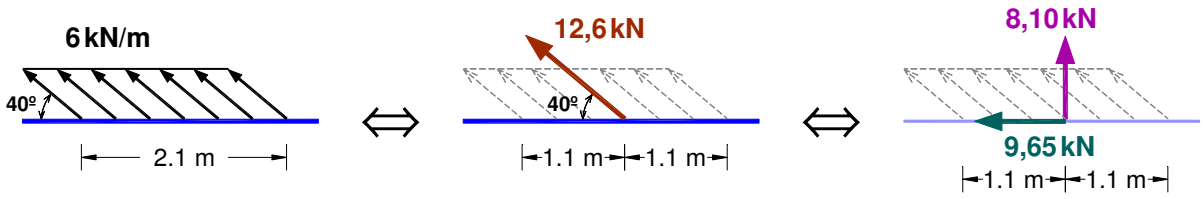
$$p = 10 - 2x \text{ kN/m} \Rightarrow \begin{cases} x = 0 \Rightarrow p = 10 \text{ kN/m} \\ x = 2{,}8 \Rightarrow p = 4{,}4 \text{ kN/m} \end{cases}$$



Carga uniforme retangular $\Rightarrow$ Força equivalente = $4{,}4 \times 2{,}8 = 12{,}32$ kN

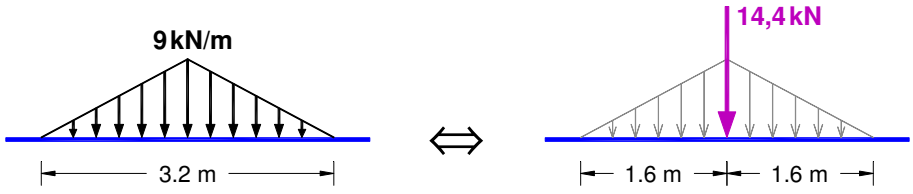
Carga triangular $\Rightarrow$ Força equivalente = $\dfrac{5{,}6 \times 2{,}8}{2} = 7{,}84$ kN

Figura 7



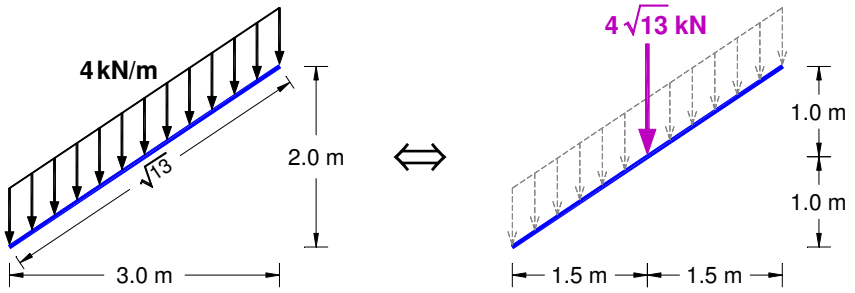
$$\begin{cases} F_x = 12{,}6 \cos 40^\circ \\ F_y = 12{,}6 \operatorname{sen} 40^\circ \end{cases} \Rightarrow \begin{cases} F_x = 9{,}65 \text{ kN} \\ F_y = 8{,}10 \text{ kN} \end{cases}$$

Figura 8



Força equivalente = $\dfrac{3{,}2 \times 9}{2} = 14{,}4$ kN

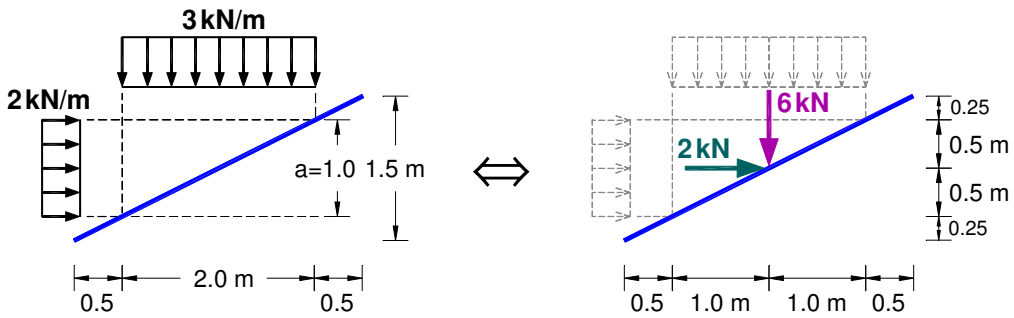
Figura 9



Comprimento da barra = $4{,}4 \times 2{,}8 = \sqrt{2^2+3^2} = \sqrt{13}$ m

Força equivalente = $4\sqrt{13}$ kN $= 14{,}42$ kN $\downarrow$

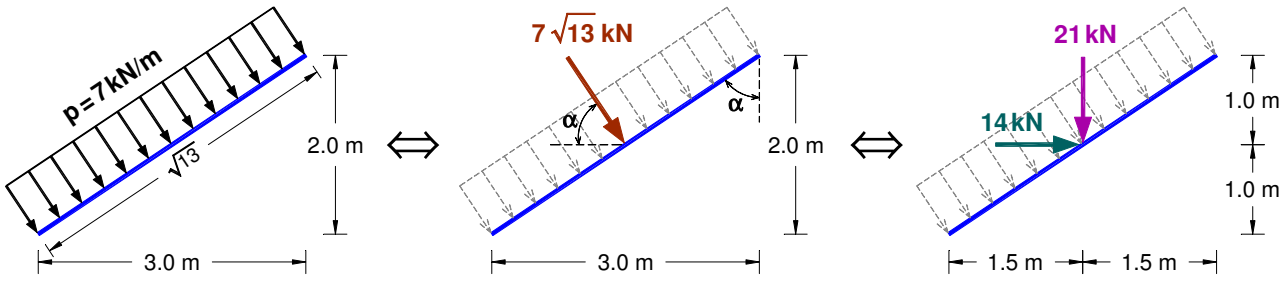
Figura 10



$$\frac{1{,}5}{3} = \frac{a}{2} \Rightarrow a = 1\,m$$

$$\begin{cases} F_x = 2\times 1 = 2\,kN \rightarrow \\ F_y = 3\times 2 = 6\,kN \downarrow \end{cases}$$

Figura 11



Força equivalente = $7\sqrt{13}$ kN $\searrow$

$$\alpha = \text{arctg}\frac{3}{2} = 56{,}31^\circ \Rightarrow \begin{cases} F_x = 7\sqrt{13}\cos\alpha = 14\ kN \rightarrow \\ F_y = 7\sqrt{13}\,\text{sen}\,\alpha = 21\ kN \downarrow \end{cases}$$

EXERCÍCIO 2

Determine as reacções nos apoios das estruturas seguintes.

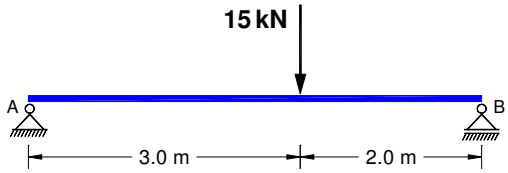


Figura 1

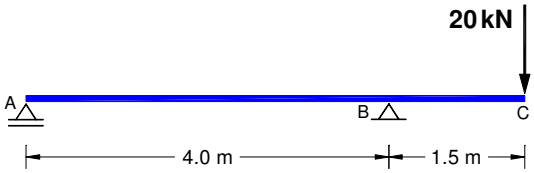


Figura 2

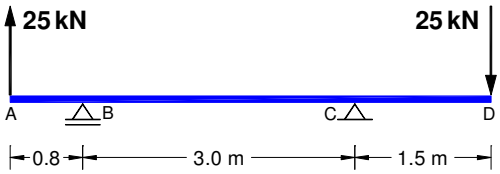


Figura 3

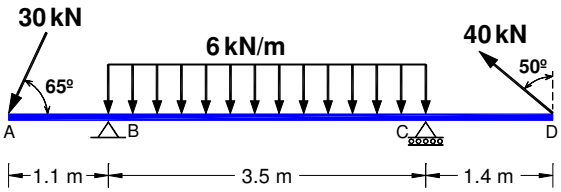


Figura 4

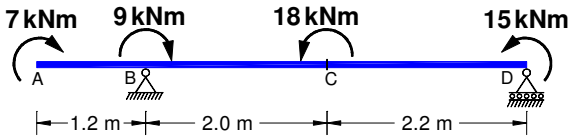


Figura 5

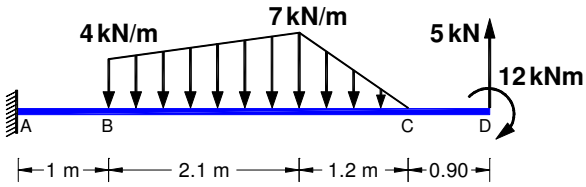
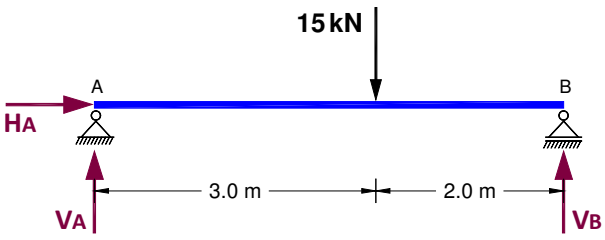
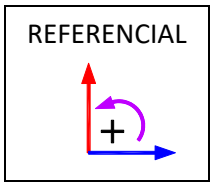


Figura 6

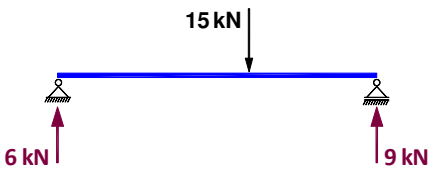
RESOLUÇÃO

Figura 1





$$\begin{cases} \sum F_x = 0 \Rightarrow H_A = 0 \\ \sum F_y = 0 \Rightarrow V_A + V_B = 15 \\ \sum M_A = 0 \Rightarrow V_B \times 5 - 15 \times 3 = 0 \end{cases} \Rightarrow \begin{cases} H_A = 0 \\ V_A = 6 \text{ kN} \uparrow \\ V_B = 9 \text{ kN} \uparrow \end{cases}$$



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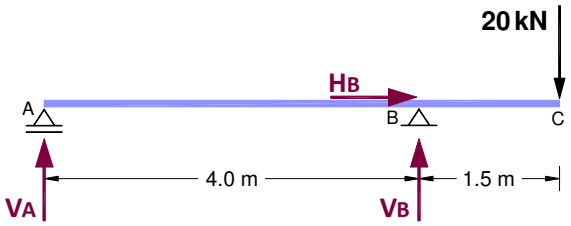
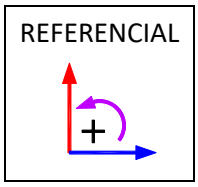
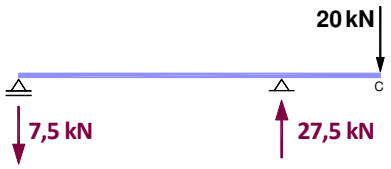




$$\begin{cases} \sum F_x = 0 \Rightarrow H_B = 0 \\ \sum F_y = 0 \Rightarrow V_A + V_B = 20 \\ \sum M_A = 0 \Rightarrow V_B \times 4 - 20 \times 5{,}5 = 0 \end{cases} \Rightarrow \begin{cases} H_A = 0 \\ V_A = -7{,}5 \text{ kN} \downarrow \\ V_B = 27{,}5 \text{ kN} \uparrow \end{cases}$$



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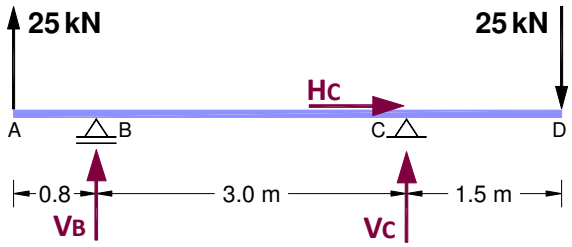
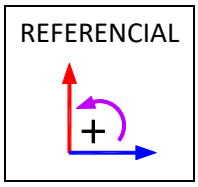




$$\begin{cases} \sum F_x = 0 \Rightarrow H_C = 0 \\ \sum F_y = 0 \Rightarrow V_B + V_C + 25 - 25 = 0 \\ \sum M_B = 0 \Rightarrow V_C \times 3 - 25 \times 0{,}8 - 25 \times 4{,}5 = 0 \end{cases} \Rightarrow \begin{cases} H_C = 0 \\ V_B = -\dfrac{132{,}5}{3} = -44{,}17 \text{ kN} \downarrow \\ V_C = \dfrac{132{,}5}{3} = 44{,}17 \text{ kN} \uparrow \end{cases}$$

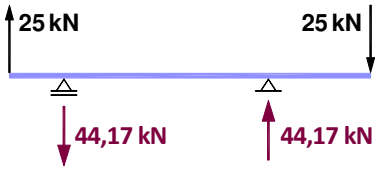
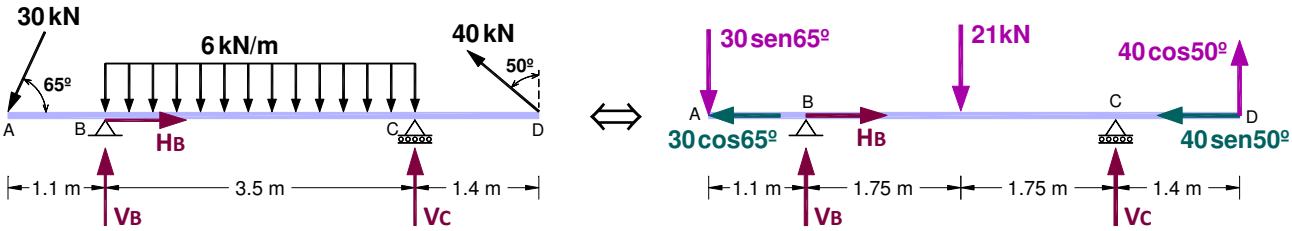
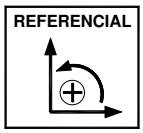


Figura 4





$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_B = 0 \end{cases} \Rightarrow \begin{cases} -30\cos 65^\circ + H_B - 40\,\text{sen}\,50^\circ = 0 \\ V_B + V_C - 30\,\text{sen}\,65^\circ - 21 + 40\cos 50^\circ = 0 \\ V_C \times 3{,}5 + 30\,\text{sen}\,65^\circ \times 1{,}1 - 21 \times 1{,}75 + 40\cos 50^\circ \times 4{,}9 = 0 \end{cases}$$

$$\begin{cases} H_B = 43{,}32\ \text{kN} \rightarrow \\ V_B = 56{,}52\ \text{kN} \uparrow \\ V_C = -34{,}04\ \text{kN} \downarrow \end{cases}$$

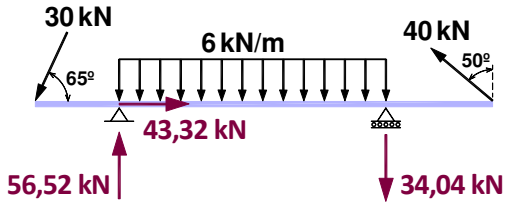
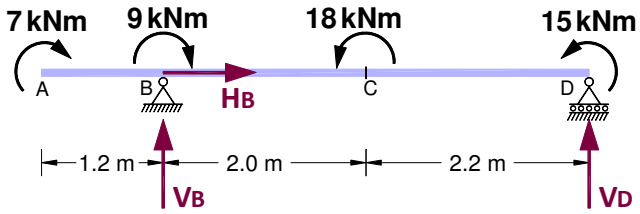
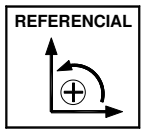


Figura 5





$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_B = 0 \end{cases} \Rightarrow \begin{cases} H_B = 0 \\ V_B + V_D = 0 \\ V_D \times 4{,}2 - 7 - 9 + 18 + 15 = 0 \end{cases} \Rightarrow \begin{cases} H_B = 0 \\ V_B = 4{,}05\ \text{kN} \uparrow \\ V_D = -4{,}05\ \text{kN} \downarrow \end{cases}$$

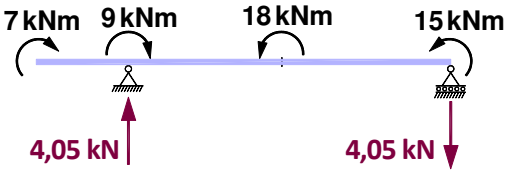
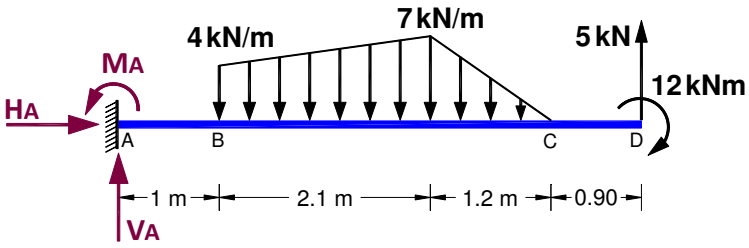
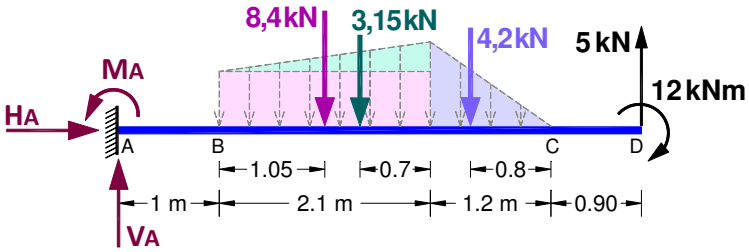
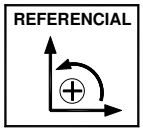


Figura 6







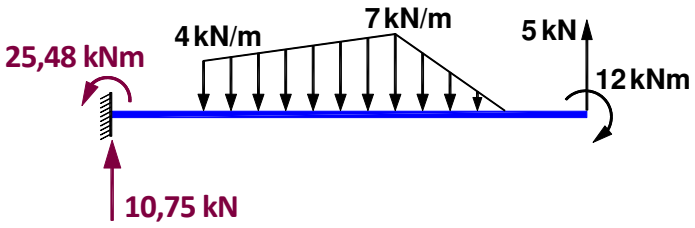
Carga uniforme retangular $\Rightarrow$ Força equivalente = $4 \times 2{,}1 = 8{,}4$ kN

Carga triangular $\Rightarrow$ Força equivalente = $\dfrac{(7-4)\times 2{,}1}{2} = 3{,}15$ kN

Carga triangular $\Rightarrow$ Força equivalente = $\dfrac{7\times 1{,}2}{2} = 4{,}2$ kN

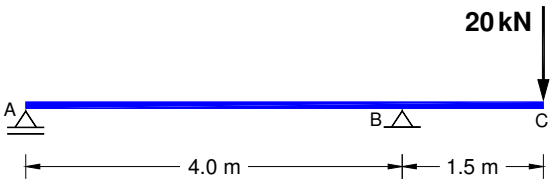
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_A = 0 \end{cases} \Rightarrow \begin{cases} H_A = 0 \\ V_A - 8{,}4 - 3{,}15 - 4{,}2 + 5 = 0 \\ M_A - 8{,}4\times 2{,}05 - 3{,}15\times(3{,}1-0{,}7) - 4{,}2\times(4.3-0.8) + 5\times 5{,}2 - 12 = 0 \end{cases}$$

$$\begin{cases} H_A = 0 \\ V_A = 10{,}75 \text{ kN} \uparrow \\ M_A = 25{,}48 \text{ kNm} \circlearrowleft \end{cases}$$

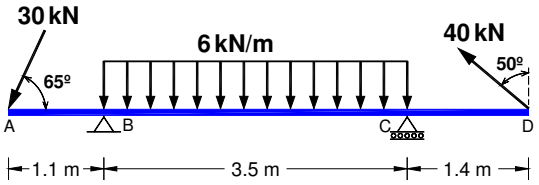


EXERCÍCIO 3

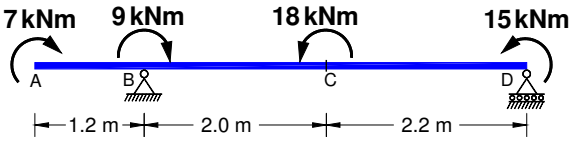
a) Determine o ponto de aplicação da força vertical de **25 kN** (↓), que actuando conjuntamente com a força de 20 kN representada na figura, conduz a uma reacção nula no apoio **A.**



b) Determine a grandeza e sentido do momento a aplicar em **C**, que actuando conjuntamente com o restante carregamento ilustrado na figura, reduz a reacção vertical em **B** para **30 kN** (↑).

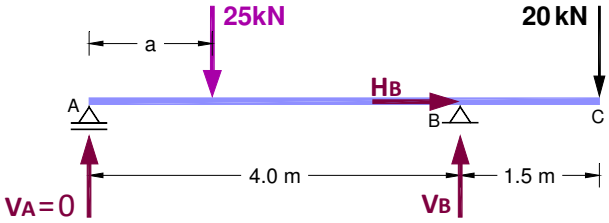


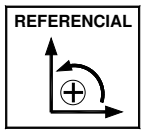
c) Determine a grandeza e sentido da carga vertical uniformemente distribuída a aplicar no tramo **AB** que actuando conjuntamente com o restante carregamento ilustrado na figura, leva a que a reacção no apoio **B** seja uma força vertical de **22 kN** (↑).



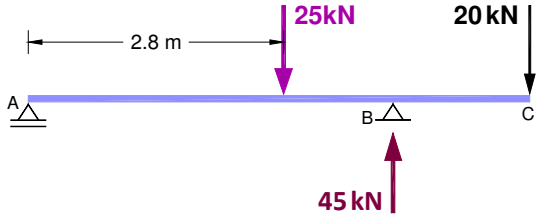
RESOLUÇÃO

Alínea a)

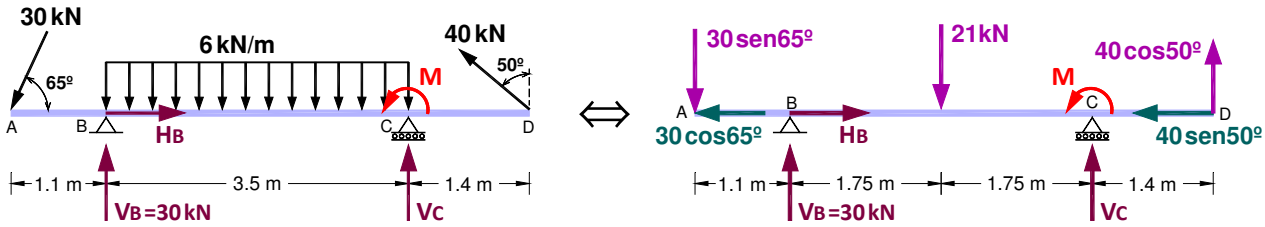


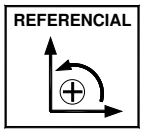


$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_B = 0 \end{cases} \Rightarrow \begin{cases} H_A = 0 \\ V_B - 25 - 20 = 0 \\ 25\times(4-a) - 20\times 1{,}2 = 0 \end{cases} \Rightarrow \begin{cases} H_A = 0 \\ V_B = 45 \text{ kN} \uparrow \\ a = 2{,}8 \text{ m} \end{cases}$$



Alínea b)

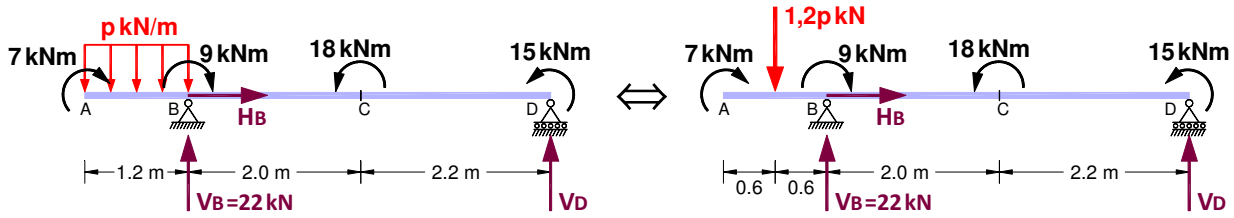


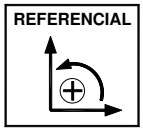


$$\sum M_C = 0 \quad \Rightarrow \quad -30\times3{,}5+30\,\text{sen}\,65^\circ\times4{,}6+21\times1{,}75+40\cos 50^\circ\times1{,}4+M=0$$

$$M = -92{,}82\ \text{kNm} \quad \Rightarrow \quad M = 92{,}82\ \text{kNm}$$

Alínea c)





$$\sum M_D = 0 \quad \Rightarrow \quad -7-9+18+15-22\times4{,}2+1{,}2p\times4{,}8=0$$

$$p = 13{,}09\ \text{kN/m} \quad \downarrow\downarrow\downarrow\downarrow\downarrow$$

EXERCÍCIO 4

Determine as reacções nos apoios das estruturas seguintes.

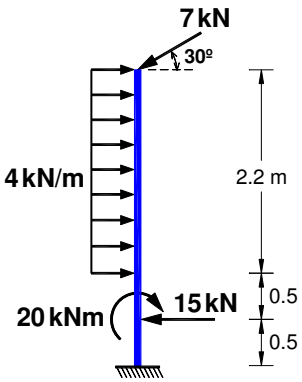


Figura 1

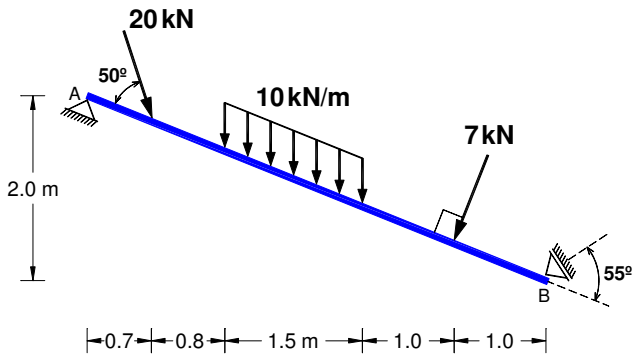


Figura 2

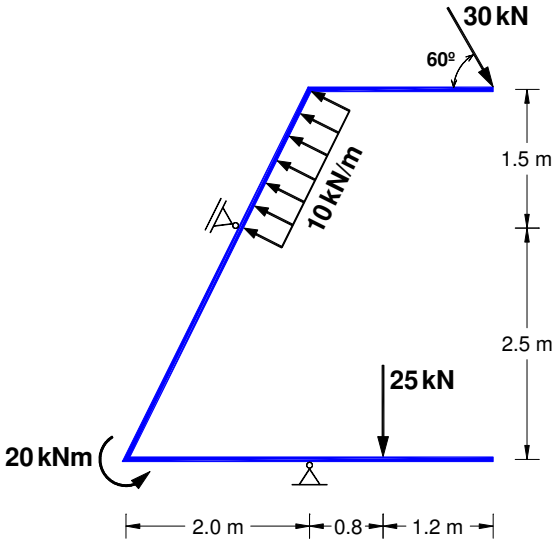


Figura 3

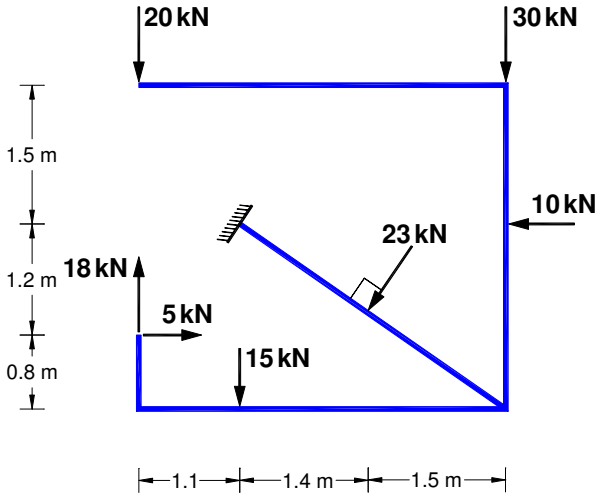
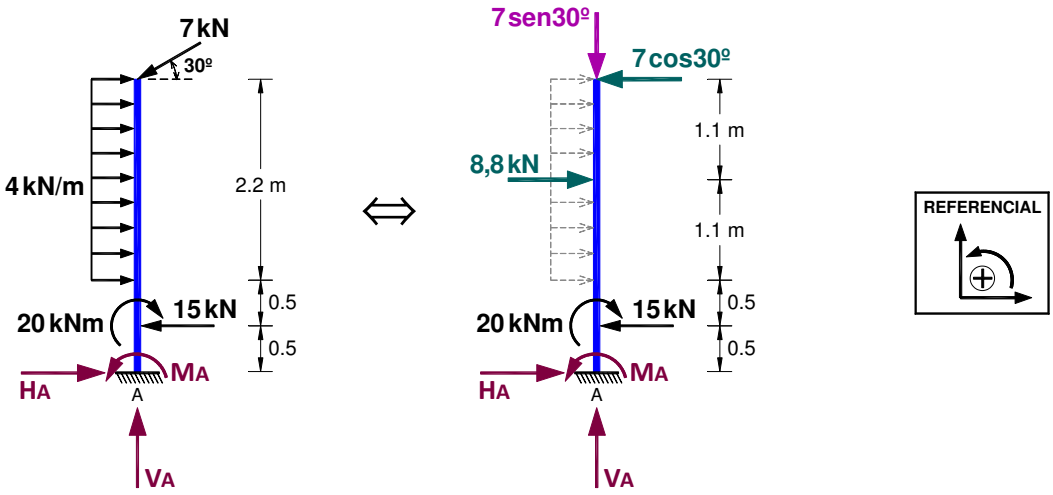


Figura 4

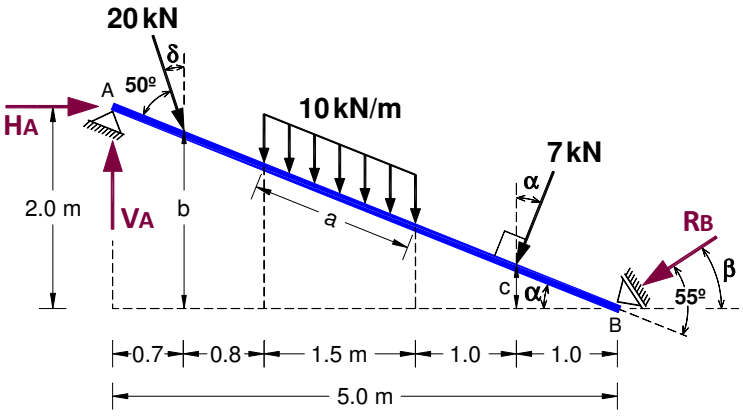
RESOLUÇÃO

Figura 1



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_A = 0 \end{cases} \Rightarrow \begin{cases} H_A - 15 + 8{,}8 - 7\cos 30^\circ = 0 \\ V_A - 7\,\text{sen}30^\circ = 0 \\ M_A - 20 + 15\times 0{,}5 - 8{,}8\times 2{,}1 + 7\cos 30^\circ \times 3{,}2 = 0 \end{cases} \Rightarrow \begin{cases} H_A = 12{,}26\ \text{kN} \rightarrow \\ V_A = 3{,}5\ \text{kN} \uparrow \\ M_A = 11{,}58\ \text{kNm} \circlearrowleft \end{cases}$$

Figura 2



$\alpha = \text{arctg}\frac{2}{5} = 21{,}801^\circ$

$\beta = 55^\circ - \alpha = 33{,}199^\circ$

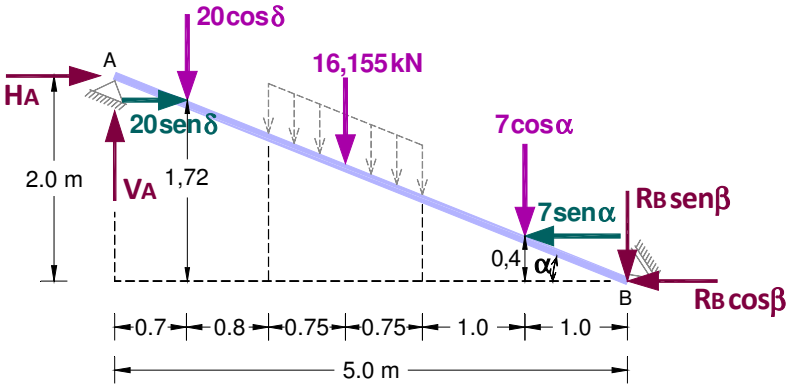
$\delta = 90^\circ - \alpha - 50^\circ = 18{,}199^\circ$

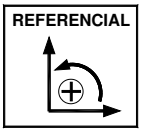
$a = \frac{1{,}5}{\cos\alpha} = \sqrt{2{,}61} = 1{,}6155$ m

Força equivalente = 10 x 1,6155 = 16,155 kN

$b = 4{,}3\,\text{tg}\alpha = 1{,}72$ m

$c = 1\,\text{tg}\alpha = 0{,}40$ m





$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_A = 0 \end{cases} \Rightarrow \begin{cases} H_A + 20\,\text{sen}\,18{,}199^\circ - 7\,\text{sen}\,21{,}801^\circ - R_B \cos 33{,}199^\circ = 0 \\ V_A - 20\cos 18{,}199^\circ - 16{,}155 - 7\cos 21{,}801^\circ - R_B\,\text{sen}\,33{,}199^\circ = 0 \\ 20\,\text{sen}\,18{,}199^\circ \times (2-1{,}72) - 20\cos 18{,}199^\circ \times 0{,}7 - 16{,}155 \times 2{,}25 - 7\cos 21{,}801^\circ \times 4 + \\ \qquad - 7\,\text{sen}\,21{,}801^\circ \times (2-0{,}4) - R_B\,\text{sen}\,33{,}199^\circ \times 5 - R_B \cos 33{,}199^\circ \times 2 = 0 \end{cases}$$

$$\begin{cases} H_A = -18{,}453\ \text{kN} \leftarrow \\ V_A = 31{,}965\ \text{kN} \uparrow \\ R_B = -17{,}695\ \text{kN} \nearrow \end{cases}$$

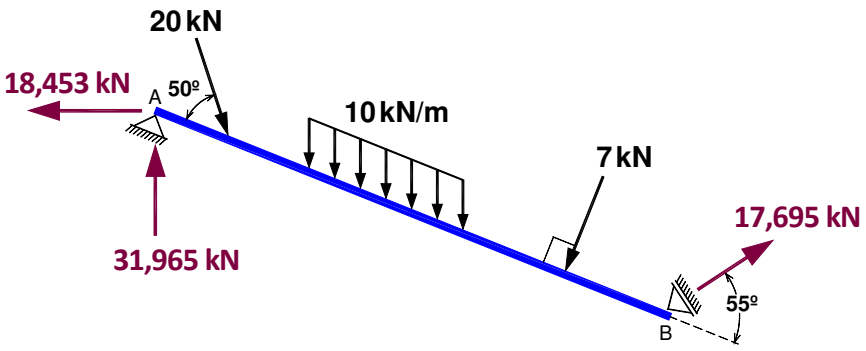
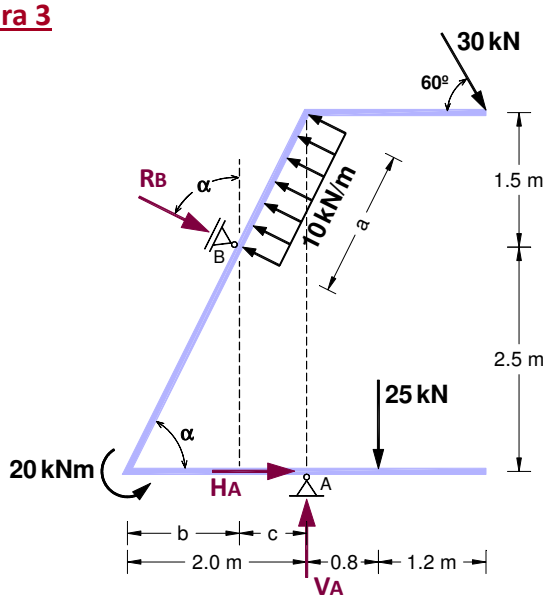


Figura 3



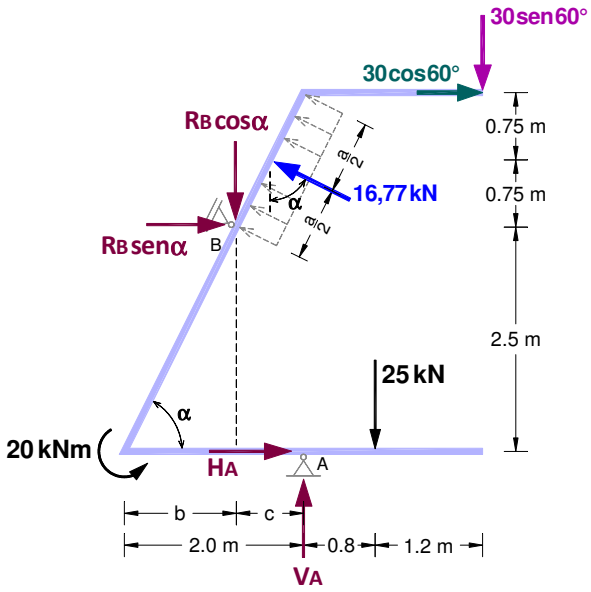
$$\alpha = \text{arctg}\frac{4}{2} = 63{,}435^\circ$$

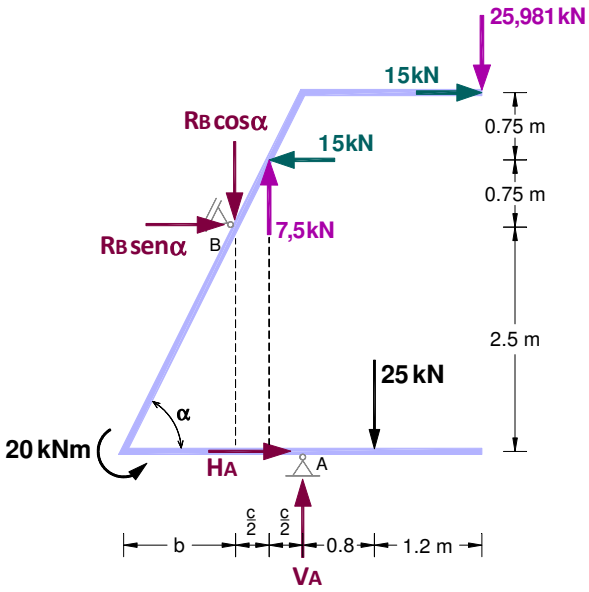
$$a = \frac{1{,}5}{\text{sen}\,\alpha} = 1{,}677 \text{ m}$$

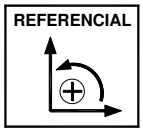
Força equivalente = 10 x 1,677 = 16,77 kN

$$b = \frac{2{,}5}{\text{tg}\,\alpha} = 1{,}25 \text{ m}$$

$$c = \frac{1{,}5}{\text{tg}\,\alpha} = 0{,}75 \text{ m}$$







Força equivalente = 10 x a = 16,77 kN $\Rightarrow$ $\begin{cases} F_x = 16{,}77\,\text{sen}\,\alpha = 15 \text{ kN} \leftarrow \\ F_y = 16{,}77\cos\alpha = 7{,}5 \text{ kN} \uparrow \end{cases}$

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_B = 0 \end{cases} \Rightarrow \begin{cases} H_A + R_B\,\text{sen}\,63{,}435^\circ - 15 + 15 = 0 \\ V_A - R_B\cos 63{,}435^\circ - 25 + 7{,}5 - 25{,}981 = 0 \\ 20 + H_A \times 2{,}5 + V_A \times 0{,}75 - 25 \times 1{,}55 + 16{,}77 \times \frac{1{,}677}{2} - 15 \times 1{,}5 - 25{,}981 \times 2{,}75 = 0 \end{cases}$$

$$\begin{cases} H_A = 31{,}07 \text{ kN} \rightarrow \\ V_A = 27{,}95 \text{ kN} \uparrow \\ R_B = -34{,}74 \text{ kN} \nwarrow \end{cases}$$

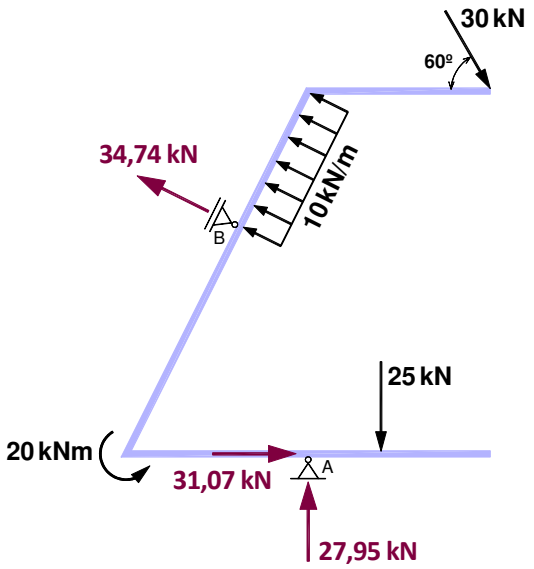
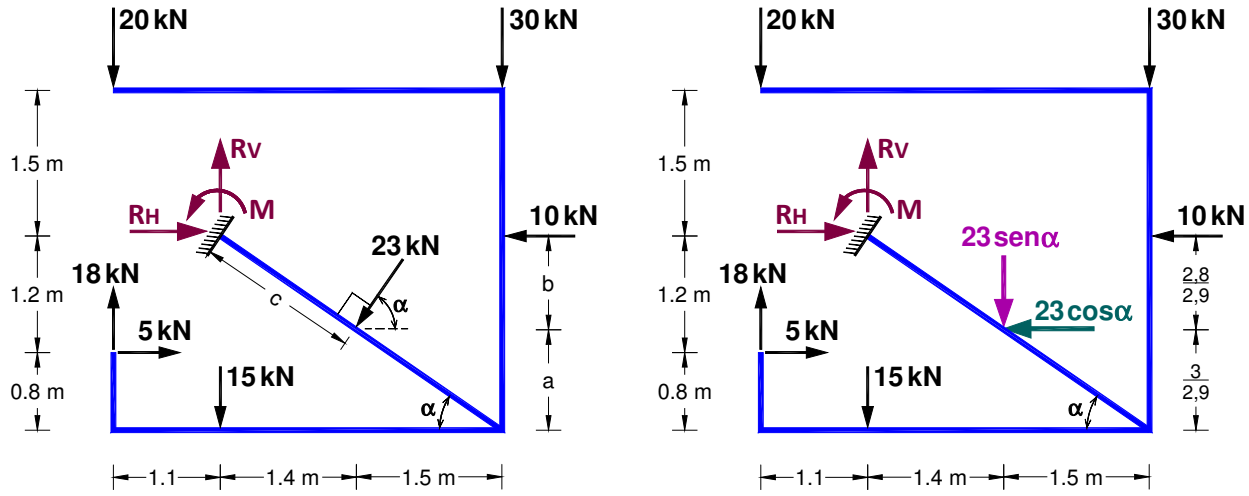


Figura 4

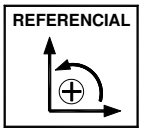


$$\alpha = \text{arctg}\frac{2}{2,9} = 34,592^{\circ}$$

$$a = \frac{3}{2,9} = 1,034 \text{ m}$$

$$b = 2 - a = \frac{2,8}{2,9} = 0,966 \text{ m}$$

$$c = \frac{1,4}{\cos\alpha} = 1,701 \text{ m}$$



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_{apoio} = 0 \end{cases} \Rightarrow \begin{cases} R_H + 5 - 23\,\text{sen}\,34{,}592^\circ - 10 = 0 \\ R_V + 18 - 15 - 23\cos 34{,}592^\circ - 30 - 20 = 0 \\ M - 23 \times \dfrac{1{,}4}{\cos 34{,}592^\circ} - 18 \times 1{,}1 + 5 \times 1{,}2 + 20 \times 1{,}1 - 30 \times 2{,}9 = 0 \end{cases}$$

$$\begin{cases} R_H = 18{,}06 \text{ kN} \rightarrow \\ R_V = 65{,}93 \text{ kN} \uparrow \\ M = 117{,}92 \text{ kNm} \circlearrowleft \end{cases}$$