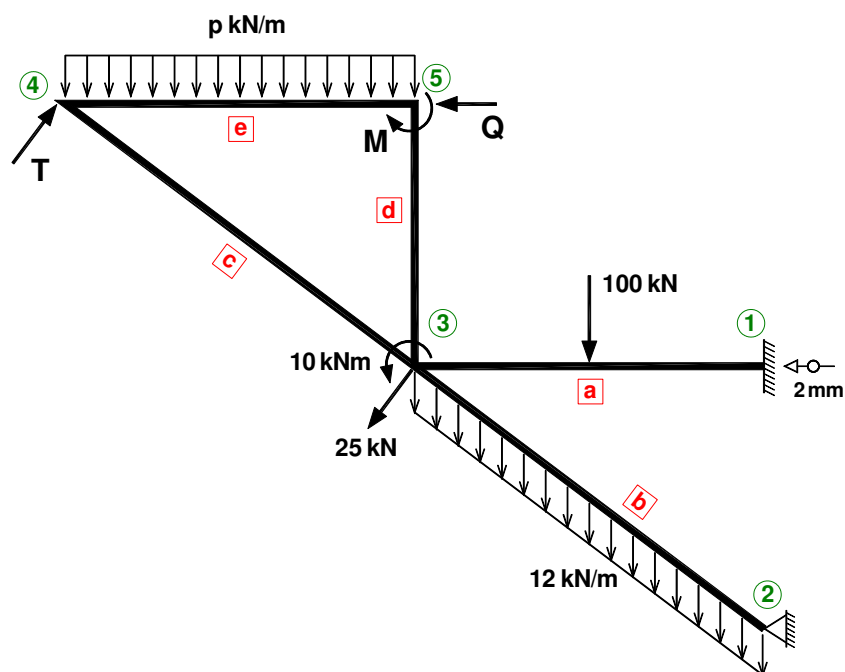


LICENCIATURA EM ENGENHARIA CIVIL

# TEORIA DE ESTRUTURAS

## MÉTODO DOS DESLOCAMENTOS

EXAME DE ÉPOCA ESPECIAL (SETEMBRO) - 2013/2014



ISABEL ALVIM TELES

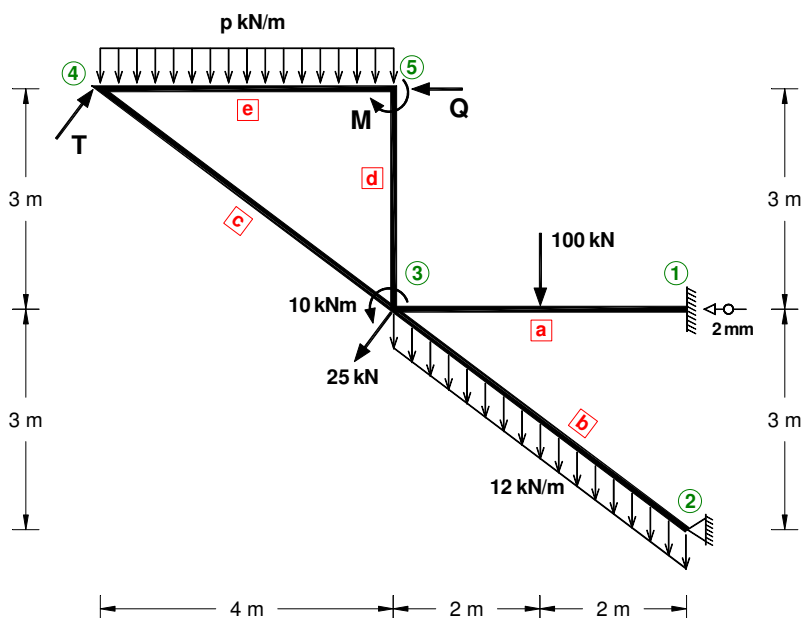
### EXERCÍCIO

Considere a estrutura representada na figura, composta por barras contínuas com distintas secções transversais e materiais constituintes.

Para além das ações a atuar nas barras e nós, a estrutura está ainda submetida a um deslizamento de **2mm** do apoio do **nó 1**, conforme ilustrado na figura.

**Notas:** - a carga uniformemente distribuída de **12 kN/m** a atuar na **barra b** é vertical;  
- a força **T** a atuar no **nó 4** e a força de **25 kN** a atuar no **nó 3** são perpendiculares às **barras c** e **b**.

**Dados:** vetor de solicitação incompleto e matriz de rigidez global da estrutura.



Vector solicitação [ F ]

|    |                |
|----|----------------|
| 1  | F <sub>1</sub> |
| 2  | F <sub>2</sub> |
| 3  | F <sub>3</sub> |
| 4  | F <sub>4</sub> |
| 5  | F <sub>5</sub> |
| 6  | F <sub>6</sub> |
| 7  | F <sub>7</sub> |
| 8  | F <sub>8</sub> |
| 9  | F <sub>9</sub> |
| 10 | 15 kN          |
| 11 | 2 kN           |
| 12 | - 12 kNm       |
| 13 | - 45 kN        |
| 14 | - 18 kN        |
| 15 | - 48 kNm       |

[ K<sub>G</sub> ] Est

|    | 1        | 2        | 3       | 4           | 5           | 6       | 7             | 8             | 9       | 10           | 11          | 12      | 13         | 14       | 15      |
|----|----------|----------|---------|-------------|-------------|---------|---------------|---------------|---------|--------------|-------------|---------|------------|----------|---------|
| 1  | 900 000  | 0        | 0       | 0           | 0           | 0       | -900 000      | 0             | 0       | 0            | 0           | 0       | 0          | 0        | 0       |
| 2  | 0        | 5 062,5  | -10 125 | 0           | 0           | 0       | 0             | -5 062,5      | -10 125 | 0            | 0           | 0       | 0          | 0        | 0       |
| 3  | 0        | -10 125  | 27 000  | 0           | 0           | 0       | 0             | 10 125        | 13 500  | 0            | 0           | 0       | 0          | 0        | 0       |
| 4  | 0        | 0        | 0       | 923 466,24  | -688 711,68 | -7 776  | -923 466,24   | 688 711,68    | -7 776  | 0            | 0           | 0       | 0          | 0        | 0       |
| 5  | 0        | 0        | 0       | -688 711,68 | 521 717,76  | -10 368 | 688 711,68    | -521 717,76   | -10 368 | 0            | 0           | 0       | 0          | 0        | 0       |
| 6  | 0        | 0        | 0       | -7 776      | -10 368     | 43 200  | 7 776         | 10 368        | 21 600  | 0            | 0           | 0       | 0          | 0        | 0       |
| 7  | -900 000 | 0        | 0       | -923 466,24 | 688 711,68  | 7 776   | 2 755 932,48  | -1 377 423,36 | -13 500 | -923 466,24  | 688 711,68  | -7 776  | -9 000     | 0        | -13 500 |
| 8  | 0        | -5 062,5 | 10 125  | 688 711,68  | -521 717,76 | 10 368  | -1 377 423,36 | 1 948 498,02  | 10 125  | 688 711,68   | -521 717,76 | -10 368 | 0          | -900 000 | 0       |
| 9  | 0        | -10 125  | 13 500  | -7 776      | -10 368     | 21 600  | -13 500       | 10 125        | 140 400 | 7 776        | 10 368      | 21 600  | 13 500     | 0        | 13 500  |
| 10 | 0        | 0        | 0       | 0           | 0           | 0       | -923 466,24   | 688 711,68    | 7 776   | 2 123 466,24 | -688 711,68 | 7 776   | -1 200 000 | 0        | 0       |
| 11 | 0        | 0        | 0       | 0           | 0           | 0       | 688 711,68    | -521 717,76   | 10 368  | -688 711,68  | 533 717,76  | 34 368  | 0          | -12 000  | 24 000  |
| 12 | 0        | 0        | 0       | 0           | 0           | 0       | -7 776        | -10 368       | 21 600  | 7 776        | 34 368      | 107 200 | 0          | -24 000  | 32 000  |
| 13 | 0        | 0        | 0       | 0           | 0           | 0       | -9 000        | 0             | 13 500  | -1 200 000   | 0           | 0       | 1 209 000  | 0        | 13 500  |
| 14 | 0        | 0        | 0       | 0           | 0           | 0       | 0             | -900 000      | 0       | 0            | -12 000     | -24 000 | 0          | 912 000  | -24 000 |
| 15 | 0        | 0        | 0       | 0           | 0           | 0       | -13 500       | 0             | 13 500  | 0            | 24 000      | 32 000  | 13 500     | -24 000  | 91 000  |

Para todas as barras assuma  $i < j$ , sendo  $i$  e  $j$  as numerações dos nós das barras. Os ângulos dos eixos das barras são medidos em torno do nó  $i$ .

Na resolução das alíneas seguintes use a **formulação matricial do Método dos Deslocamentos dos Nós**.

Justifique convenientemente todas as respostas.

a) Sabendo que  $[K_L^b] = [K_L^c]$ , determine  $[K_G^b]$  (matriz de rigidez da **barra b** no sistema de eixos global).

b) Complete o vetor solicitação  $[F]$  anteriormente representado.

c) Recorrendo ao menor número possível de equações, complete o **vetor dos deslocamentos no referencial global  $\Delta G$** .

$[\Delta_G] =$

|                                                 |
|-------------------------------------------------|
| $\Delta 1$                                      |
| $\Delta 2$                                      |
| $\Delta 3$                                      |
| $\Delta 4 = 0$                                  |
| $\Delta 5 = 0$                                  |
| $\Delta 6 = 1,540 \times 10^{-3} \text{ rad}$   |
| $\Delta 7 = -2,204 \text{ mm}$                  |
| $\Delta 8 = -3,138 \text{ mm}$                  |
| $\Delta 9 = 1,449 \times 10^{-4} \text{ rad}$   |
| $\Delta 10$                                     |
| $\Delta 11$                                     |
| $\Delta 12 = 4,568 \times 10^{-4} \text{ rad}$  |
| $\Delta 13 = -3,369 \text{ mm}$                 |
| $\Delta 14 = -3,169 \text{ mm}$                 |
| $\Delta 15 = -1,527 \times 10^{-4} \text{ rad}$ |

d) Considerando o vetor dos deslocamentos da alínea anterior e a matriz de rigidez no referencial local da **barra a** que ao lado se representa, desenhe neste enunciado os diagramas de esforços da referida barra.

$[K_L^a] =$

|          |          |        |          |          |         |
|----------|----------|--------|----------|----------|---------|
| 900 000  | 0        | 0      | -900 000 | 0        | 0       |
| 0        | 5 062,5  | 10 125 | 0        | -5 062,5 | 10 125  |
| 0        | 10 125   | 27000  | 0        | -10 125  | 13 500  |
| -900 000 | 0        | 0      | 900 000  | 0        | 0       |
| 0        | -5 062,5 | -10125 | 0        | 5 062,5  | -10 125 |
| 0        | 10 125   | 13 500 | 0        | -10 125  | 27 000  |



ESFORÇO AXIAL  
(kN)



ESFORÇO TRANSVERSO  
(kN)



MOMENTO FLECTOR  
(kNm)

### RESOLUÇÃO

- MATRIZ DE RIGIDEZ ELEMENTAR – SISTEMA DE EIXOS LOCAL

$$[K_L] = \left[ \begin{array}{ccc|ccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{array} \right]$$

- MATRIZ DE TRANSFORMAÇÃO

$$[T] = \left[ \begin{array}{ccc|ccc} \cos \alpha & \sin \alpha & 0 & 0 & 0 & 0 \\ -\sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 0 & -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

- MATRIZ DE TRANSFORMAÇÃO TRANSPONDA

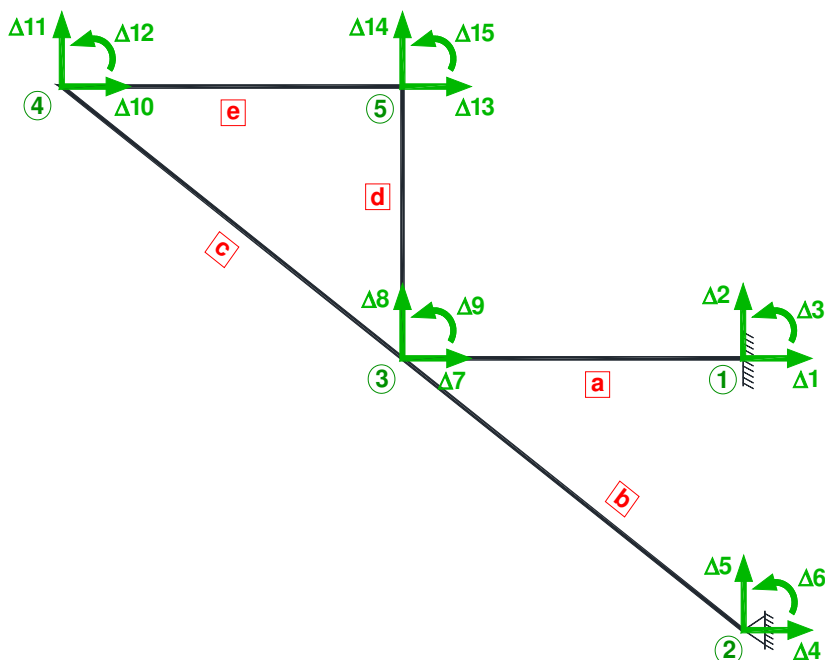
$$[T]^T = \left[ \begin{array}{ccc|ccc} \cos \alpha & -\sin \alpha & 0 & 0 & 0 & 0 \\ \sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & 0 & 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

- MATRIZ DE RIGIDEZ ELEMENTAR – SISTEMA DE EIXOS GLOBAL

$$[K_G] = [T]^T [K_L] [T]$$

ALÍNEA a

• GRAUS DE LIBERDADE DA ESTRUTURA (SISTEMA GLOBAL)



Contribuição da matriz de rigidez global de cada barra para a matriz de rigidez da estrutura:

$$[K_G]^{Est} =$$

|    | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8             | 9 | 10    | 11 | 12 | 13    | 14 | 15 |
|----|---|---|---|---|---|---|---|---------------|---|-------|----|----|-------|----|----|
| 1  |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 2  |   | a |   |   |   |   |   | a             |   |       |    |    |       |    |    |
| 3  |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 4  |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 5  |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 6  |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 7  |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 8  |   | a |   |   |   |   |   | a + b + c + d |   | c     |    |    | d     |    |    |
| 9  |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 10 |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 11 |   |   |   |   |   |   |   | c             |   | c + e |    |    | e     |    |    |
| 12 |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 13 |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |
| 14 |   |   |   |   |   |   |   | d             |   | e     |    |    | d + e |    |    |
| 15 |   |   |   |   |   |   |   |               |   |       |    |    |       |    |    |

Tal como se pode constatar na figura anterior, três quadrantes da matriz de rigidez global da barra b podem ser “retirados” da matriz de rigidez global.

$$[K_G^b] = \begin{bmatrix} & \textcolor{teal}{4} & \textcolor{teal}{5} & \textcolor{teal}{6} & & \textcolor{teal}{7} & \textcolor{teal}{8} & \textcolor{teal}{9} \\ \textcolor{teal}{4} & 923\,466,24 & -688\,711,68 & -7\,776 & & -923\,466,24 & 688\,711,68 & -7\,776 \\ \textcolor{teal}{5} & -688\,711,68 & 521\,717,76 & -10\,368 & & 688\,711,68 & -521\,717,76 & -10\,368 \\ \textcolor{teal}{6} & -7\,776 & -10\,368 & 43\,200 & & 7\,776 & 10\,368 & 21\,600 \\ \textcolor{teal}{7} & -923\,466,24 & 688\,711,68 & 7\,776 & & K_{7,7}^b & K_{7,8}^b & K_{7,9}^b \\ \textcolor{teal}{8} & 688\,711,68 & -521\,717,76 & 10\,368 & & K_{8,7}^b & K_{8,8}^b & K_{8,9}^b \\ \textcolor{teal}{9} & -7\,776 & -10\,368 & 21\,600 & & K_{9,7}^b & K_{9,8}^b & K_{9,9}^b \end{bmatrix}$$

Falta determinar o quadrante inferior direito, correspondente aos graus de liberdade associados ao nó de maior numeração.

Partindo da informação  $[K_L^b] = [K_L^c]$  e sabendo que a barra b e a barra c têm a mesma inclinação e o mesmo **ângulo  $\alpha$** , então  $[K_G^b] = [K_G^c]$ , ou seja:

$$[K_G^b] = \begin{bmatrix} & \textcolor{teal}{4} & \textcolor{teal}{5} & \textcolor{teal}{6} & & \textcolor{teal}{7} & \textcolor{teal}{8} & \textcolor{teal}{9} \\ \textcolor{teal}{4} & K_{4,4}^b & K_{4,5}^b & K_{4,6}^b & & K_{4,7}^b & K_{4,8}^b & K_{4,9}^b \\ \textcolor{teal}{5} & K_{5,4}^b & K_{5,5}^b & K_{5,6}^b & & K_{5,7}^b & K_{5,8}^b & K_{5,9}^b \\ \textcolor{teal}{6} & K_{6,4}^b & K_{6,5}^b & K_{6,6}^b & & K_{6,7}^b & K_{6,8}^b & K_{6,9}^b \\ \textcolor{teal}{7} & K_{7,4}^b & K_{7,8}^b & K_{7,9}^b & & K_{7,7}^b & K_{7,8}^b & K_{7,9}^b \\ \textcolor{teal}{8} & K_{8,7}^b & K_{8,8}^b & K_{8,9}^b & & K_{8,7}^b & K_{8,8}^b & K_{8,9}^b \\ \textcolor{teal}{9} & K_{9,7}^b & K_{9,8}^b & K_{9,9}^b & & K_{9,7}^b & K_{9,8}^b & K_{9,9}^b \end{bmatrix} = [K_G^c] = \begin{bmatrix} & \textcolor{teal}{7} & \textcolor{teal}{8} & \textcolor{teal}{9} & & \textcolor{teal}{10} & \textcolor{teal}{11} & \textcolor{teal}{12} \\ \textcolor{teal}{7} & K_{7,7}^c & K_{7,8}^c & K_{7,9}^c & & K_{7,10}^c & K_{7,11}^c & K_{7,12}^c \\ \textcolor{teal}{8} & K_{8,7}^c & K_{8,8}^c & K_{8,9}^c & & K_{8,10}^c & K_{8,11}^c & K_{8,12}^c \\ \textcolor{teal}{9} & K_{9,7}^c & K_{9,8}^c & K_{9,9}^c & & K_{9,10}^c & K_{9,11}^c & K_{9,12}^c \\ \textcolor{teal}{10} & K_{10,7}^c & K_{10,8}^c & K_{10,9}^c & & K_{10,10}^c & K_{10,11}^c & K_{10,12}^c \\ \textcolor{teal}{11} & K_{11,7}^c & K_{11,8}^c & K_{11,9}^c & & K_{11,10}^c & K_{11,11}^c & K_{11,12}^c \\ \textcolor{teal}{12} & K_{12,7}^c & K_{12,8}^c & K_{12,9}^c & & K_{12,10}^c & K_{12,11}^c & K_{12,12}^c \end{bmatrix}$$

Donde se conclui que:

$$K_{7,7}^b = K_{10,10}^c$$

$$K_{7,8}^b = K_{10,11}^c$$

$$K_{7,9}^b = K_{10,12}^c$$

$$K_{8,7}^b = K_{11,10}^c$$

$$K_{8,8}^b = K_{11,11}^c$$

$$K_{8,9}^b = K_{11,12}^c$$

$$K_{9,7}^b = K_{12,10}^c$$

$$K_{9,8}^b = K_{12,11}^c$$

$$K_{9,9}^b = K_{12,12}^c$$

Consultando a matriz de rigidez global da estrutura, constata-se que a parte da matriz  $[K_G^c]$  que precisamos de determinar se encontra somada com uma parte da matriz  $[K_G^e]$  (ver figura abaixo). Se conseguirmos calcular a matriz  $[K_G^e]$  então poderemos calcular a parte da matriz  $[K_G^c]$  que precisamos de saber.

$$[K_G]^{Est} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{matrix} & \begin{bmatrix} & & & & & & & a & & & & & & & \\ & a & & & & & & & a & & & & & & \\ & & & & & & & b & & & & & & & \\ & & & & b & & & b & & & & & & & \\ & & & & & & & & a+b+c+d & & c & & d & & \\ & a & & b & & & c & & c+e & & e & & & & \\ & & & & & & d & & e & & d+e & & & & \end{bmatrix} \end{matrix}$$

$$[K_G]^{Est} = \begin{matrix} & \begin{matrix} 10 & 11 & 12 \end{matrix} \\ \begin{matrix} 10 \\ 11 \\ 12 \end{matrix} & \begin{bmatrix} 2\,123\,466,24 & -688\,711,68 & 7\,776 \\ -688\,711,68 & 533\,717,76 & 34\,368 \\ 7\,776 & 34\,368 & 107\,200 \end{bmatrix} \end{matrix} = \begin{matrix} & \begin{matrix} 10 & 11 & 12 \end{matrix} \\ \begin{matrix} 10 \\ 11 \\ 12 \end{matrix} & \begin{bmatrix} K_{10,10}^c + K_{10,10}^e & K_{10,11}^c + K_{10,11}^e & K_{10,12}^c + K_{10,12}^e \\ K_{11,10}^c + K_{11,10}^e & K_{11,11}^c + K_{11,11}^e & K_{11,12}^c + K_{11,12}^e \\ K_{12,10}^c + K_{12,10}^e & K_{12,11}^c + K_{12,11}^e & K_{12,12}^c + K_{12,12}^e \end{bmatrix} \end{matrix}$$

A **barra e** é horizontal com o nó menor (*i*) à esquerda  $\Rightarrow [K_L^e] = [K_G^e]$

$$[K_L^e] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} = \begin{matrix} & \begin{matrix} 10 & 11 & 12 & 13 & 14 & 15 \end{matrix} \\ \begin{matrix} 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{matrix} & \begin{bmatrix} K_{10,10}^e & 0 & 0 & -1\,200\,000 & 0 & 0 \\ 0 & K_{11,11}^e & K_{11,12}^e & 0 & -12\,000 & 24\,000 \\ 0 & K_{12,11}^e & K_{12,12}^e & 0 & -24\,000 & 32\,000 \\ -1\,200\,000 & 0 & 0 & K_{13,13}^e & 0 & 0 \\ 0 & -12\,000 & -24\,000 & 0 & K_{14,13}^e & K_{14,15}^e \\ 0 & 24\,000 & 32\,000 & 0 & K_{15,14}^e & K_{15,15}^e \end{bmatrix} \end{matrix}$$

$$K_{10,10}^e = -K_{10,13}^e = 1\,200\,000$$

$$K_{11,11}^e = -K_{11,14}^e = 12\,000$$

$$K_{12,11}^e = -K_{12,14}^e = 24\,000$$

$$K_{11,12}^e = K_{11,15}^e = 24\,000$$

$$K_{12,12}^e = 2 \times K_{12,15}^e = 64\,000$$

$$[K_G^e] = \begin{array}{c|ccc|ccc} & 10 & 11 & 12 & 13 & 14 & 15 \\ \hline 10 & 1\,200\,000 & 0 & 0 & -1\,200\,000 & 0 & 0 \\ 11 & 0 & 12\,000 & 24\,000 & 0 & -12\,000 & 24\,000 \\ 12 & 0 & 24\,000 & 64\,000 & 0 & -24\,000 & 32\,000 \\ \hline 13 & -1\,200\,000 & 0 & 0 & 1\,200\,000 & 0 & 0 \\ 14 & 0 & -12\,000 & -24\,000 & 0 & 12\,000 & -24\,000 \\ 15 & 0 & 24\,000 & 32\,000 & 0 & -24\,000 & 64\,000 \end{array}$$

$$[K_G]^{Est} = \begin{array}{c|ccc} & 10 & 11 & 12 \\ \hline 10 & 2\,123\,466,24 & -688\,711,68 & 7\,776 \\ 11 & -688\,711,68 & 533\,717,76 & 34\,368 \\ 12 & 7\,776 & 34\,368 & 107\,200 \end{array} = \begin{array}{c|ccc} & 10 & 11 & 12 \\ \hline 10 & K_{10,10}^c + K_{10,10}^e & K_{10,11}^c + K_{10,11}^e & K_{10,12}^c + K_{10,12}^e \\ 11 & K_{11,10}^c + K_{11,10}^e & K_{11,11}^c + K_{11,11}^e & K_{11,12}^c + K_{11,12}^e \\ 12 & K_{12,10}^c + K_{12,10}^e & K_{12,11}^c + K_{12,11}^e & K_{12,12}^c + K_{12,12}^e \end{array}$$

$$2\,123\,466,24 = K_{10,10}^c + 1\,200\,000 \Rightarrow K_{10,10}^c = 923\,466,24 \Rightarrow K_{7,7}^d = 923\,466,24$$

$$-688\,711,68 = K_{10,11}^c + 0 \Rightarrow K_{10,11}^c = -688\,711,68 = K_{11,10}^c \Rightarrow K_{7,8}^d = K_{8,7}^d = -688\,711,68$$

$$7\,776 = K_{10,12}^c + 0 \Rightarrow K_{10,12}^c = 7\,776 = K_{12,10}^c \Rightarrow K_{7,9}^d = K_{9,7}^d = 7\,776$$

$$533\,717,76 = K_{11,11}^c + 12\,000 \Rightarrow K_{11,11}^c = 521\,717,76 \Rightarrow K_{8,8}^d = 521\,717,76$$

$$34\,368 = K_{11,12}^c + 24\,000 \Rightarrow K_{11,12}^c = 10\,368 = K_{12,11}^c \Rightarrow K_{8,9}^d = K_{9,8}^d = 10\,368$$

$$107\,200 = K_{12,12}^c + 64\,000 \Rightarrow K_{12,12}^c = 43\,200 \Rightarrow K_{9,9}^d = 43\,200$$

$$[K_G^d] = \begin{bmatrix} & \textcolor{teal}{4} & \textcolor{teal}{5} & \textcolor{teal}{6} & & \textcolor{teal}{7} & \textcolor{teal}{8} & \textcolor{teal}{9} \\ \textcolor{teal}{4} & 923\,466,24 & -688\,711,68 & -7\,776 & & -923\,466,24 & 688\,711,68 & -7\,776 \\ \textcolor{teal}{5} & -688\,711,68 & 521\,717,76 & -10\,368 & & 688\,711,68 & -521\,717,76 & -10\,368 \\ \textcolor{teal}{6} & -7\,776 & -10\,368 & 43\,200 & & 7\,776 & 10\,368 & 21\,600 \\ \textcolor{teal}{7} & -923\,466,24 & 688\,711,68 & 7\,776 & & 923\,466,24 & -688\,711,68 & 7\,776 \\ \textcolor{teal}{8} & 688\,711,68 & -521\,717,76 & 10\,368 & & -688\,711,68 & 521\,717,76 & 10\,368 \\ \textcolor{teal}{9} & -7\,776 & -10\,368 & 21\,600 & & 7\,776 & 10\,368 & 43\,200 \end{bmatrix}$$

### ALÍNEA b

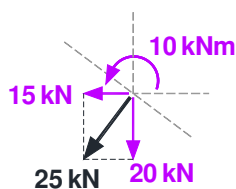
#### • VETOR DE SOLICITAÇÃO

Falta calcular as Forças correspondentes aos **nós 1, 2 e 3**.

$$[F_G] = \begin{bmatrix} \textcolor{teal}{1} & F1 \\ \textcolor{teal}{2} & F2 \\ \textcolor{teal}{3} & F3 \\ \textcolor{teal}{4} & F4 \\ \textcolor{teal}{5} & F5 \\ \textcolor{teal}{6} & F6 \\ \textcolor{teal}{7} & F7 \\ \textcolor{teal}{8} & F8 \\ \textcolor{teal}{9} & F9 \\ \textcolor{teal}{10} & 15\text{ kN} \\ \textcolor{teal}{11} & 2\text{ kN} \\ \textcolor{teal}{12} & -12\text{ kNm} \\ \textcolor{teal}{13} & -45\text{ kN} \\ \textcolor{teal}{14} & -18\text{ kN} \\ \textcolor{teal}{15} & -48\text{ kNm} \end{bmatrix}$$

### Solicitação aplicada nos nós (sistema global)

|                       | Nó 3 |
|-----------------------|------|
| $\textcolor{teal}{7}$ | -15  |
| $\textcolor{teal}{8}$ | -20  |
| $\textcolor{teal}{9}$ | 10   |



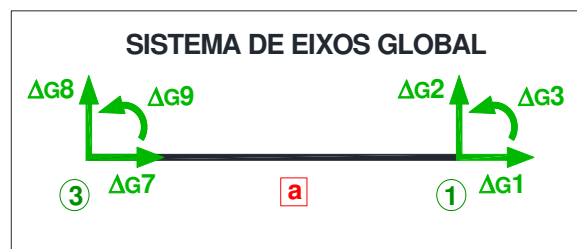
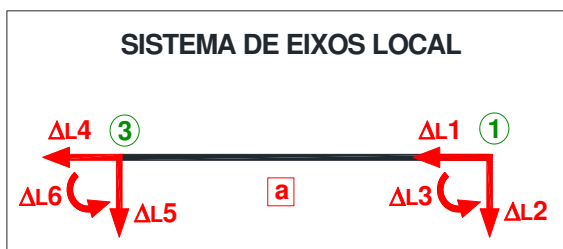
Solicitação aplicada nas barras – Forças equivalentes nodais

**BARRA a**

**Nós 1 - 3**

Graus de liberdade - sistema local:  $\Delta L1, \Delta L2, \Delta L3$  -  $\Delta L4, \Delta L5, \Delta L6$

Graus de liberdade - sistema global:  $\Delta G1, \Delta G2, \Delta G3$  -  $\Delta G7, \Delta G8, \Delta G9$



$P_Y = -100 \text{ kN}$

$$F_{//eq} = 0$$

$$F_{\perp eq} = \frac{P_Y \cdot b^2}{L^3} (L + 2a) = \frac{-100 \times 2^2}{4^3} (4 + 2 \times 2) = -50 \text{ kN}$$

$$F_{M-esq} = \frac{P_Y \cdot a \cdot b^2}{L^2} = \frac{-100 \times 2 \times 2^2}{4^2} = -50 \text{ kNm}$$

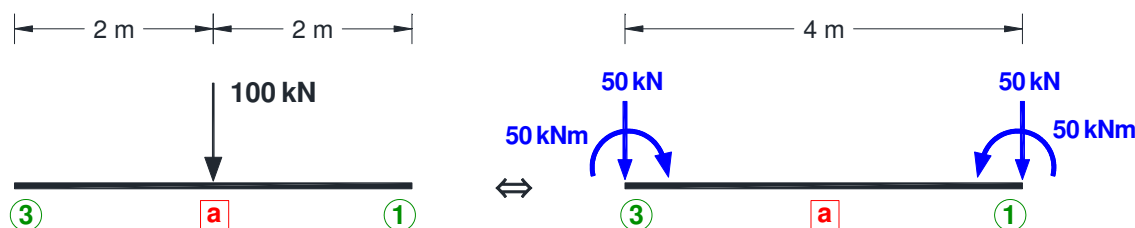
$$F_{//dir} = 0$$

$$F_{\perp dir} = \frac{P_Y \cdot a^2}{L^3} (L + 2b) = \frac{-100 \times 2^2}{4^3} (4 + 2 \times 2) = -50 \text{ kN}$$

$$F_{M-dir} = -\frac{P_Y \cdot a^2 \cdot b}{L^2} = -\frac{-100 \times 2^2 \times 2}{4^2} = 50 \text{ kNm}$$

|                           |
|---------------------------|
| $P_Y = -100 \text{ kN}$   |
| $F_{L-EN} - \text{LOCAL}$ |

|       |     |
|-------|-----|
| $F_1$ | 0   |
| $F_2$ | 50  |
| $F_3$ | 50  |
| $F_4$ | 0   |
| $F_5$ | 50  |
| $F_6$ | -50 |



É fácil relacionar o sistema de eixos local com o global:

$\Delta L1 \Leftrightarrow -\Delta G1$

$\Delta L2 \Leftrightarrow -\Delta G2$

$\Delta L3 \Leftrightarrow \Delta G3$

$\Delta L4 \Leftrightarrow -\Delta G7$

$\Delta L5 \Leftrightarrow -\Delta G8$

$\Delta L6 \Leftrightarrow \Delta G9$

**Barra 1 - Nós 1 - 3**

$F_{G-EN} - \text{GLOBAL}$

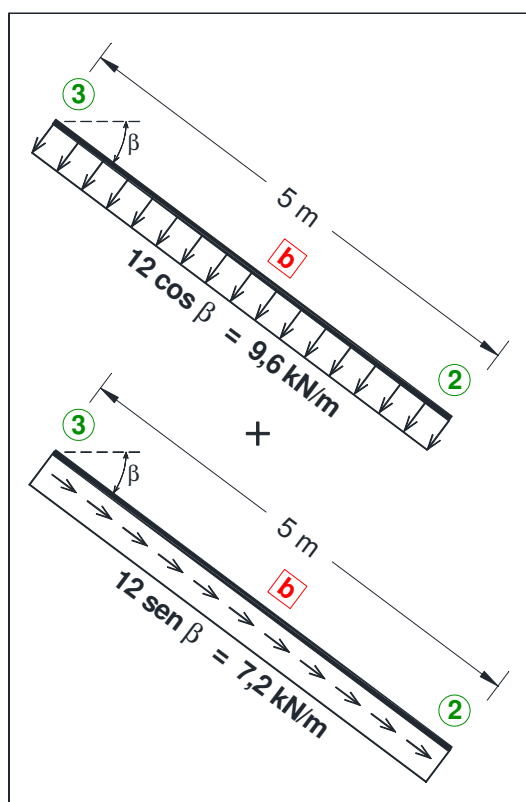
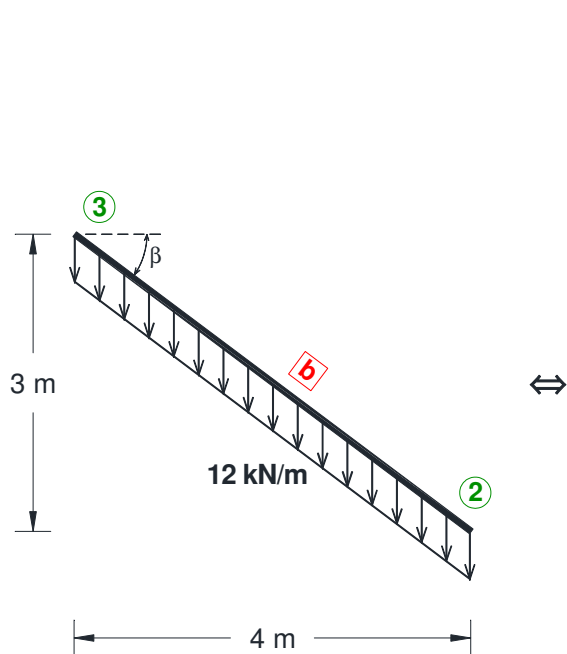
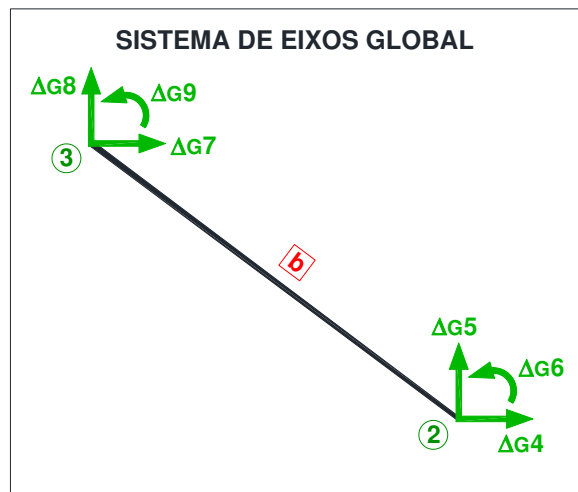
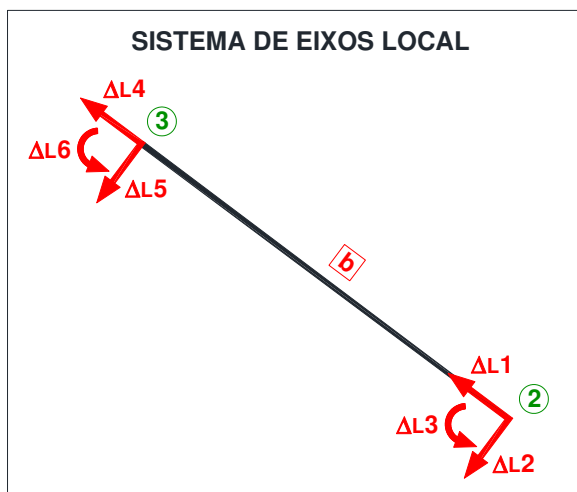
|       |     |
|-------|-----|
| $F_1$ | 0   |
| $F_2$ | -50 |
| $F_3$ | 50  |
| $F_7$ | 0   |
| $F_8$ | -50 |
| $F_9$ | -50 |

**BARRA b**

**Nós 2 - 3**

Graus de liberdade - sistema local:  $\Delta L1, \Delta L2, \Delta L3$  -  $\Delta L4, \Delta L5, \Delta L6$

Graus de liberdade - sistema global:  $\Delta G4, \Delta G5, \Delta G6$  -  $\Delta G7, \Delta G8, \Delta G9$



Comprimento da barra:  $L = 5 \text{ m}$

$$\beta \Rightarrow \begin{cases} \cos \beta = \frac{4}{5} = 0,8 \\ \sin \beta = \frac{3}{5} = 0,6 \end{cases}$$

## TEORIA DE ESTRUTURAS

ISABEL ALVIM TELES

$p_y = -9,6 \text{ kN/m}$

$$F_{//\text{esq}} = 0$$

$$F_{\perp\text{esq}} = \frac{p_y \cdot L}{2} = \frac{-9,6 \times 5}{2} = -24 \text{ kN}$$

$$F_{M\text{-esq}} = \frac{p_y \cdot L^2}{12} = \frac{-9,6 \times 5^2}{12} = -20 \text{ kNm}$$

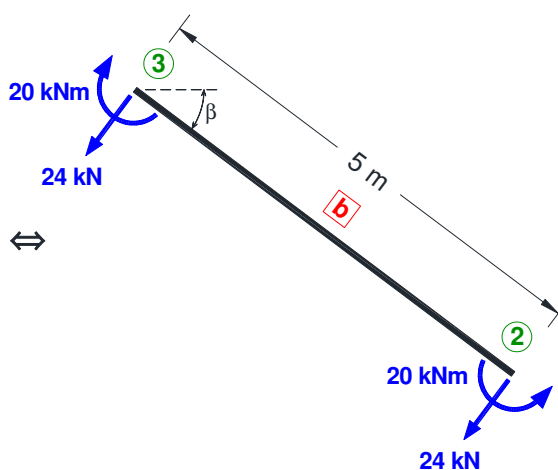
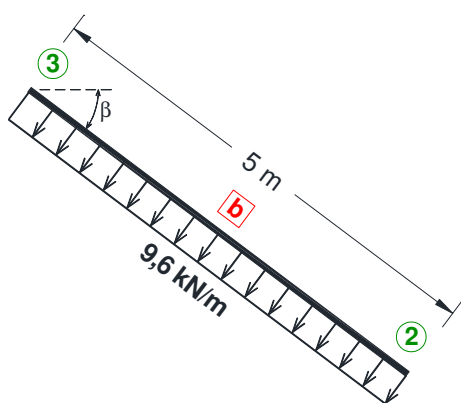
$$F_{//\text{dir}} = 0$$

$$F_{\perp\text{dir}} = \frac{p_y \cdot L}{2} = \frac{-9,6 \times 5}{2} = -24 \text{ kN}$$

$$F_{M\text{-dir}} = -\frac{p_y \cdot L^2}{12} = -\frac{-9,6 \times 5^2}{12} = 20 \text{ kNm}$$

$p_y = -9,6 \text{ kN/m}$   
 $F_{L\text{-}EN} - \text{LOCAL}$

|       |   |     |
|-------|---|-----|
| $F_1$ |   | 0   |
| $F_2$ |   | 24  |
| $F_3$ |   | 20  |
| $F_4$ | = | 0   |
| $F_5$ |   | 24  |
| $F_6$ |   | -20 |



$p_x = 7,2 \text{ kN/m}$

$$F_{//\text{esq}} = \frac{p_x \cdot L}{2} = \frac{7,2 \times 5}{2} = 18 \text{ kN}$$

$$F_{\perp\text{esq}} = 0$$

$$F_{M\text{-esq}} = 0$$

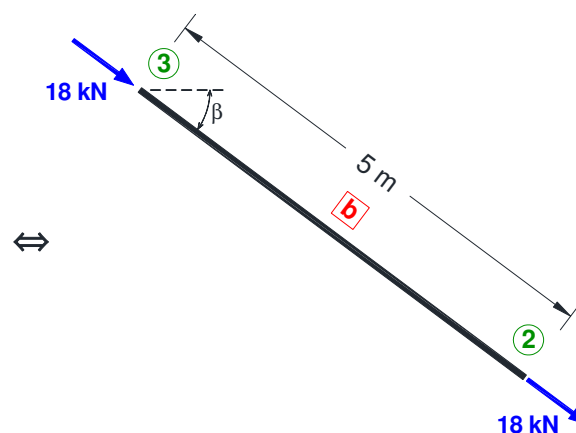
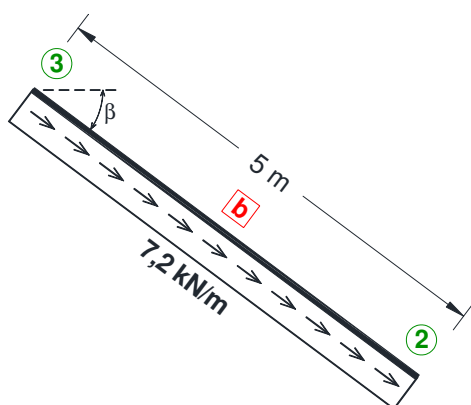
$$F_{//\text{dir}} = \frac{p_x \cdot L}{2} = \frac{7,2 \times 5}{2} = 18 \text{ kN}$$

$$F_{\perp\text{dir}} = 0$$

$$F_{M\text{-dir}} = 0$$

$p_x = 7,2 \text{ kN/m}$   
 $F_{L\text{-}EN} - \text{LOCAL}$

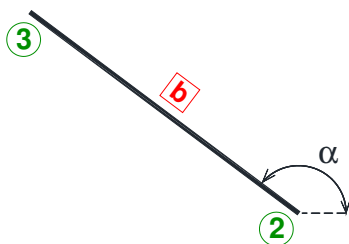
|       |   |     |
|-------|---|-----|
| $F_1$ |   | -18 |
| $F_2$ |   | 0   |
| $F_3$ |   | 0   |
| $F_4$ | = | -18 |
| $F_5$ |   | 0   |
| $F_6$ |   | 0   |



|                                                    |         |                                      |     |
|----------------------------------------------------|---------|--------------------------------------|-----|
| $p_y = -9,6 \text{ kN/m} + p_x = 7,2 \text{ kN/m}$ |         | <b>Barra b - Nós 2 - 3</b>           |     |
| <b><math>F_{L-EN}</math> - LOCAL</b>               |         | <b><math>F_{L-EN}</math> - LOCAL</b> |     |
| $F_1$                                              | 0 - 18  | $F_1$                                | -18 |
| $F_2$                                              | 24 + 0  | $F_2$                                | 24  |
| $F_3$                                              | 20 + 0  | $F_3$                                | 20  |
| $F_4$                                              | 0 - 18  | $F_4$                                | -18 |
| $F_5$                                              | 24 + 0  | $F_5$                                | 24  |
| $F_6$                                              | -20 + 0 | $F_6$                                | -20 |

$$F_{Global} = T^T \cdot F_{Local}$$

$$\alpha \Rightarrow \begin{cases} \cos \alpha = -0,8 \\ \sin \alpha = 0,6 \end{cases}$$



|                                       |       |                         |      |   |      |      |   |   |                              |   |                                       |         |
|---------------------------------------|-------|-------------------------|------|---|------|------|---|---|------------------------------|---|---------------------------------------|---------|
| <b><math>F_{G-EN}</math></b>          |       | <b><math>T^T</math></b> |      |   |      |      |   | * | <b><math>F_{L-EN}</math></b> |   | <b>Barra b - Nós 2 - 3</b>            |         |
| <b><math>F_{G-EN}</math> - GLOBAL</b> |       |                         |      |   |      |      |   |   |                              |   | <b><math>F_{G-EN}</math> - GLOBAL</b> |         |
| 4                                     | $F_4$ | -0,8                    | -0,6 | 0 | 0    | 0    | 0 | 1 | -18                          | 4 | $F_4$                                 | 0 kN    |
| 5                                     | $F_5$ | 0,6                     | -0,8 | 0 | 0    | 0    | 0 | 2 | 24                           | 5 | $F_5$                                 | -30 kN  |
| 6                                     | $F_6$ | 0                       | 0    | 1 | 0    | 0    | 0 | 3 | 20                           | 6 | $F_6$                                 | 20 kNm  |
| 7                                     | $F_7$ | 0                       | 0    | 0 | -0,8 | -0,6 | 0 | 4 | -18                          | 7 | $F_7$                                 | 0 kN    |
| 8                                     | $F_8$ | 0                       | 0    | 0 | 0,6  | -0,8 | 0 | 5 | 24                           | 8 | $F_8$                                 | -30 kN  |
| 9                                     | $F_9$ | 0                       | 0    | 0 | 0    | 0    | 1 | 6 | -20                          | 9 | $F_9$                                 | -20 kNm |

**Vetor solicitação:**  $[F_G] =$

|    |         |    |               |    |         |
|----|---------|----|---------------|----|---------|
| 1  | F1      | 1  | 0             | 1  | 0       |
| 2  | F2      | 2  | -50           | 2  | -50 kN  |
| 3  | F3      | 3  | 50            | 3  | 50 kNm  |
| 4  | F4      | 4  | 0             | 4  | 0       |
| 5  | F5      | 5  | -30           | 5  | -30 kN  |
| 6  | F6      | 6  | 20            | 6  | 20 kNm  |
| 7  | F7      | 7  | -15 + 0       | 7  | -15 kN  |
| 8  | F8      | 8  | -20 - 50 - 30 | 8  | -100 kN |
| 9  | F9      | 9  | 10 - 50 - 20  | 9  | -60 kNm |
| 10 | 15 kN   | 10 | 15 kN         | 10 | 15 kN   |
| 11 | 2 kN    | 11 | 2 kN          | 11 | 2 kN    |
| 12 | -12 kNm | 12 | -12 kNm       | 12 | -12 kNm |
| 13 | -45 kN  | 13 | -45 kN        | 13 | -45 kN  |
| 14 | -18 kN  | 14 | -18 kN        | 14 | -18 kN  |
| 15 | -48 kNm | 15 | -48 kNm       | 15 | -48 kNm |

### ALÍNEA c

Encastramento do nó 1:  $\Delta 1 = -0,002 \text{ m}$  ← (deslizamento horizontal do apoio)

$$\Delta 2 = 0$$

$$\Delta 3 = 0$$

Determinação de  $\Delta 10$  e  $\Delta 11$

$$[K_G^{\text{Est}}] \times [\Delta] = [F] + [R]$$

No esquema em baixo estão assinalados (sombreados) na matriz de rigidez global os valores não nulos.

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 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 |       |   |       |   |       |   |       |   |       |   |       |    |          |    |          |    |          |    |          |    |          |    |          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |       |   |       |   |       |   |       |   |       |   |   |   |   |   |   |   |   |    |   |    |   |    |   |    |   |    |   |    |   |
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|  |  |  |  |  |  |  |  |  |  |  |  |  |  | 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 10 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 11 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 12 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 14 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 15 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\times$ <table border="1"> <tr><td>1</td><td><math>\Delta 1</math></td></tr> <tr><td>2</td><td><math>\Delta 2</math></td></tr> <tr><td>3</td><td><math>\Delta 3</math></td></tr> <tr><td>4</td><td><math>\Delta 4</math></td></tr> <tr><td>5</td><td><math>\Delta 5</math></td></tr> <tr><td>6</td><td><math>\Delta 6</math></td></tr> <tr><td>7</td><td><math>\Delta 7</math></td></tr> <tr><td>8</td><td><math>\Delta 8</math></td></tr> <tr><td>9</td><td><math>\Delta 9</math></td></tr> <tr><td>10</td><td><math>\Delta 10</math></td></tr> <tr><td>11</td><td><math>\Delta 11</math></td></tr> <tr><td>12</td><td><math>\Delta 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| 9                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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 |       |   |       |   |       |   |       |   |       |   |       |    |          |    |          |    |          |    |          |    |          |    |          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |       |   |       |   |       |   |       |   |       |   |   |   |   |   |   |   |   |    |   |    |   |    |   |    |   |    |   |    |   |
| 8                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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| 9                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      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| 10                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     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| 11                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     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| 12                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     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| 13                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     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| 14                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     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 |       |   |       |   |       |   |       |   |       |   |       |    |          |    |          |    |          |    |          |    |          |    |          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |       |   |       |   |       |   |       |   |       |   |   |   |   |   |   |   |   |    |   |    |   |    |   |    |   |    |   |    |   |
| 15                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     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Como não sabemos as reações (desconhecemos os valores  $R_1$  a  $R_5$ ), só podemos escolher equações das linhas 6 a 15.

Para evitar resolver um sistema de duas equações com duas incógnitas, deverão ser escolhidas equações que permitam determinar independentemente  $\Delta 10$  e  $\Delta 11$ .

Equação correspondente ao grau de liberdade 13  $\Rightarrow \Delta 10$

$$K_{13,7} \times \Delta 7 + K_{13,9} \times \Delta 9 + K_{13,10} \times \Delta 10 + K_{13,13} \times \Delta 13 + K_{13,15} \times \Delta 15 = F_{13}$$

$$-9\,000 \times \Delta 7 + 13\,500 \times \Delta 9 - 1\,200\,000 \times \Delta 10 + 1\,209\,000 \times \Delta 13 + 13\,500 \times \Delta 15 = -45$$

$$-9\,000 \times (-2,204 \times 10^{-3}) + 13\,500 \times 1,449 \times 10^{-4} - 1\,200\,000 \times \Delta 10 + 1\,209\,000 \times (-3,369 \times 10^{-3}) + 13\,500 \times (-1,527 \times 10^{-4}) = -45$$

$$\Delta 10 = -3,34 \times 10^{-3} \text{ m} = -3,34 \text{ mm} \leftarrow$$

Equação correspondente ao grau de liberdade 14  $\Rightarrow \Delta_{11}$

$$K_{14,8} \times \Delta_8 + K_{13,11} \times \Delta_{11} + K_{14,12} \times \Delta_{12} + K_{14,14} \times \Delta_{14} + K_{14,15} \times \Delta_{15} = F_{14}$$

$$-900\,000 \times \Delta_8 - 12\,000 \times \Delta_{11} - 24\,000 \times \Delta_{12} + 912\,000 \times \Delta_{14} - 24\,000 \times \Delta_{15} = -18$$

$$-900\,000 \times (-3,138 \times 10^{-3}) - 12\,000 \times \Delta_{11} - 24\,000 \times 4,568 \times 10^{-4} + 912\,000 \times (-3,169 \times 10^{-3}) - 24\,000 \times (-1,527 \times 10^{-4}) = -18$$

$$\Delta_{11} = -4,60 \times 10^{-3} \text{ m} = -4,60 \text{ mm} \downarrow$$

### ALÍNEA d

**Barra a**  $[F_L] = [K_L] \cdot [\Delta_L] - [F_{L-EN}]$

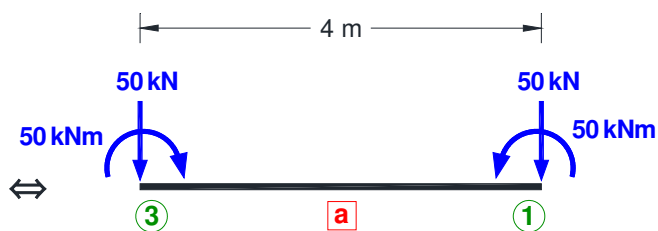
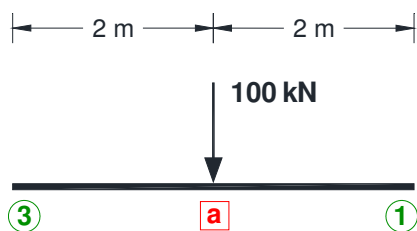
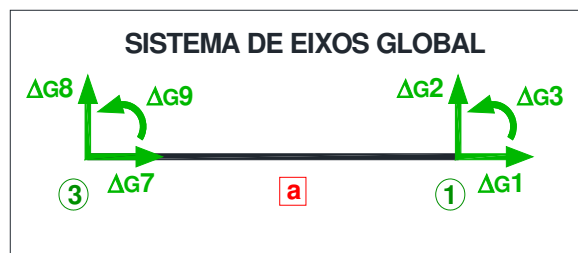
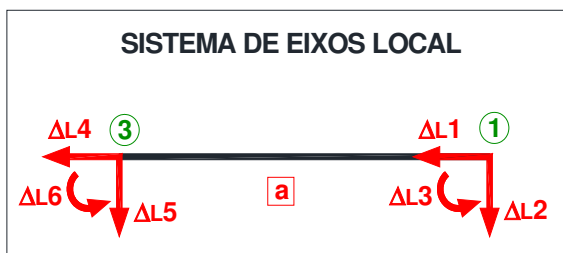
O vetor das forças equivalentes nodais no referencial local  $[F_{L-EN}]$  já foi calculado na alínea a).

**BARRA a**

**Nós 1 - 3**

Graus de liberdade - sistema local:  $\Delta_{L1}, \Delta_{L2}, \Delta_{L3} - \Delta_{L4}, \Delta_{L5}, \Delta_{L6}$

Graus de liberdade - sistema global:  $\Delta_{G1}, \Delta_{G2}, \Delta_{G3} - \Delta_{G7}, \Delta_{G8}, \Delta_{G9}$



|                           |
|---------------------------|
| $P_Y = -100 \text{ kN}$   |
| $F_{L-EN} - \text{LOCAL}$ |

|       |     |
|-------|-----|
| $F_1$ | 0   |
| $F_2$ | 50  |
| $F_3$ | 50  |
| $F_4$ | 0   |
| $F_5$ | 50  |
| $F_6$ | -50 |

É fácil relacionar o sistema de eixos local com o global:

$$\begin{aligned} \Delta L1 &\Leftrightarrow -\Delta G1 & \Delta L1 &= 2 \times 10^{-3} \\ \Delta L2 &\Leftrightarrow -\Delta G2 & \Delta L2 &= 0 \\ \Delta L3 &\Leftrightarrow \Delta G3 & \Delta L3 &= 0 \\ \Delta L4 &\Leftrightarrow -\Delta G7 & \Delta L4 &= 2,204 \times 10^{-3} \\ \Delta L5 &\Leftrightarrow -\Delta G8 & \Delta L5 &= 3,138 \times 10^{-3} \\ \Delta L6 &\Leftrightarrow \Delta G9 & \Delta L6 &= 1,449 \times 10^{-4} \end{aligned}$$

$$\Rightarrow [\Delta_L] = \begin{bmatrix} 2 \times 10^{-3} \\ 0 \\ 0 \\ 2,204 \times 10^{-3} \\ 3,138 \times 10^{-3} \\ 1,449 \times 10^{-4} \end{bmatrix}$$

$$[F_L] = [K_L] \times [\Delta_L] - [F_{L-EN}]$$

$$[F_L] = \begin{bmatrix} 900\,000 & 0 & 0 & -900\,000 & 0 & 0 \\ 0 & 5\,062,5 & 10\,125 & 0 & -5\,062,5 & 10\,125 \\ 0 & 10\,125 & 27\,000 & 0 & -10\,125 & 13\,500 \\ -900\,000 & 0 & 0 & 900\,000 & 0 & 0 \\ 0 & -5\,062,5 & -10\,125 & 0 & 5\,062,5 & -10\,125 \\ 0 & 10\,125 & 13\,500 & 0 & -10\,125 & 27\,000 \end{bmatrix} \times \begin{bmatrix} 2 \times 10^{-3} \\ 0 \\ 0 \\ 2,204 \times 10^{-3} \\ 3,138 \times 10^{-3} \\ 1,449 \times 10^{-4} \end{bmatrix} - \begin{bmatrix} 0 \\ 50 \\ 50 \\ 0 \\ 50 \\ -50 \end{bmatrix} = \begin{bmatrix} -183,728 \\ -64,418 \\ -79,815 \\ 183,728 \\ -35,582 \\ 22,142 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

**Nota:** Os cálculos foram realizados com a ajuda do programa *Excel*, pelo que os resultados finais apresentados poderão ter algumas diferenças (de arredondamentos) em relação aos obtidos com os algarismos significativos indicados nesta resolução.

