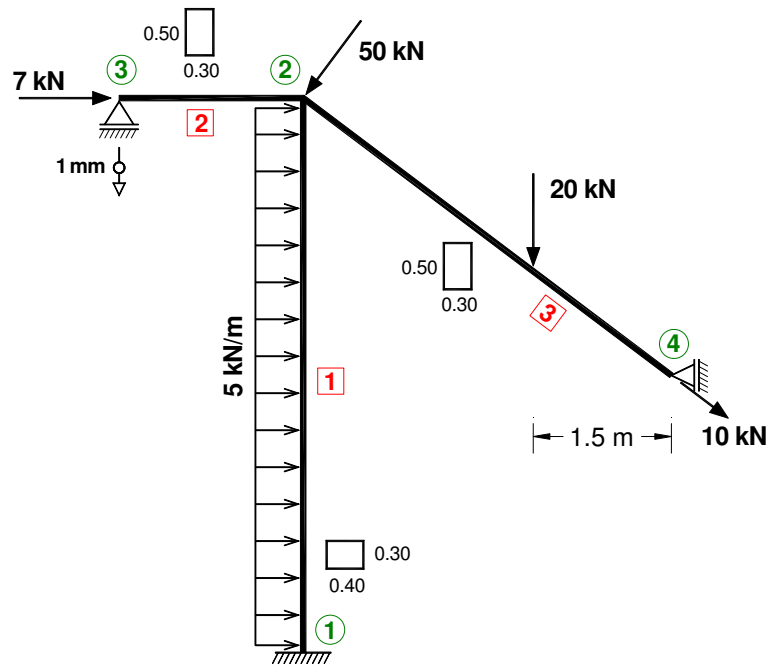


LICENCIATURA EM ENGENHARIA CIVIL

TEORIA DE ESTRUTURAS

MÉTODO DOS DESLOCAMENTOS



ESTRUTURA CONTÍNUA

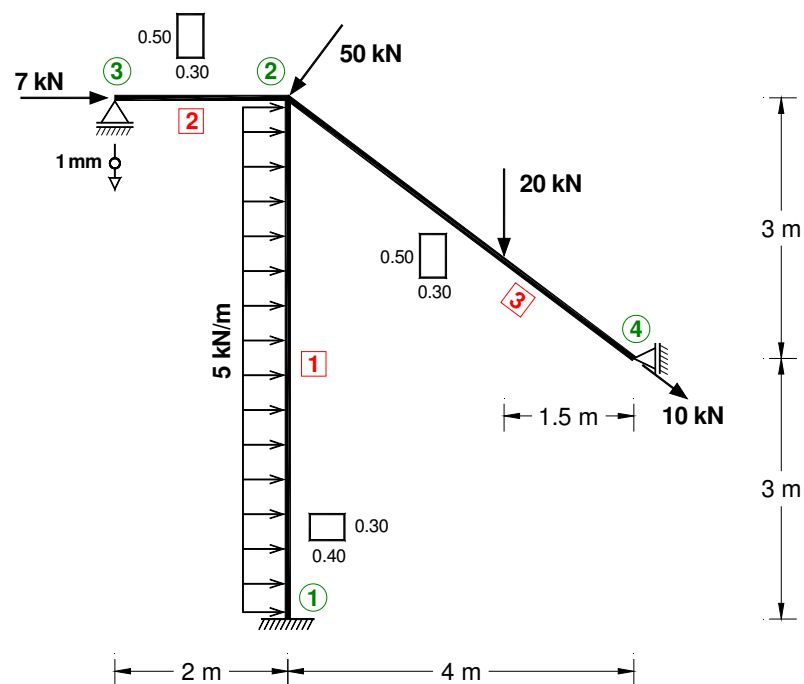
ISABEL ALVIM TELES

EXERCÍCIO PROPOSTO

Considere a estrutura representada na figura, realizada em betão com $E = 30 \text{ GPa}$ e com as secções aí indicadas.

A força de 50 kN aplicada no **nó 2** é perpendicular à **barra 3**.

A força de 10 kN aplicada no **nó 4** é tem a direção da **barra 3**.



Responda às alíneas aplicando o Método dos Deslocamentos.

- a) Determine os deslocamentos dos nós da estrutura;
- b) Determine as reações nos apoios;
- c) Trace os diagramas de esforços da estrutura.

RESOLUÇÃO

• **MATRIZ DE RIGIDEZ ELEMENTAR – SISTEMA DE EIXOS LOCAL**

$$[K_L] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

• **MATRIZ DE TRANSFORMAÇÃO**

$$[T] = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 & 0 & 0 & 0 \\ -\sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 0 & -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

• **MATRIZ DE TRANSFORMAÇÃO TRANSPOSTA**

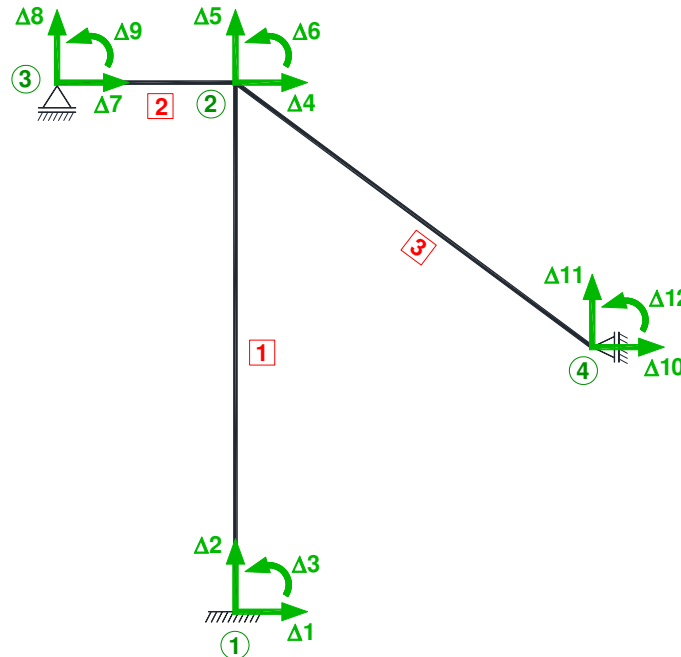
$$[T]^T = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 & 0 & 0 & 0 \\ \sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & 0 & 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

• **MATRIZ DE RIGIDEZ ELEMENTAR – SISTEMA DE EIXOS GLOBAL**

$$[K_G] = [T]^T [K_L] [T]$$

ALÍNEA a)

• GRAUS DE LIBERDADE DA ESTRUTURA (SISTEMA GLOBAL)



• MATRIZ DE RIGIDEZ LOCAL, MATRIZ DE TRANSFORMAÇÃO E MATRIZ DE RIGIDEZ GLOBAL

BARRA 1	Nós 1 - 2	Graus de liberdade (sistema global): Δ1, Δ2, Δ3 - Δ4, Δ5, Δ6			
$E = 30 \times 10^6 \text{ kPa}$	$b = 0.3 \text{ m}$	$A = 0.12 \text{ m}^2$	$\alpha = 90^\circ \Rightarrow$	$\cos \alpha = 0$	
$L = 6 \text{ m}$	$h = 0.4 \text{ m}$	$I = 0.002 \text{ m}^4$		$\sin \alpha = 1$	

$$\left[K_L^1 \right] = \begin{bmatrix} 600\,000 & 0 & 0 & -600\,000 & 0 & 0 \\ 0 & 2\,666.67 & 8\,000 & 0 & -2\,666.67 & 8\,000 \\ 0 & 8\,000 & 32\,000 & 0 & -8\,000 & 16\,000 \\ \hline -600\,000 & 0 & 0 & 600\,000 & 0 & 0 \\ 0 & -2\,666.67 & -8\,000 & 0 & 2\,666.67 & -8\,000 \\ 0 & 8\,000 & 16\,000 & 0 & -8\,000 & 32\,000 \end{bmatrix}$$

$$\left[T^1 \right] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[K_G^1 \right] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 2\,666.67 & 0 & -8\,000 & -2\,666.67 & 0 & -8\,000 \\ 0 & 600\,000 & 0 & 0 & -600\,000 & 0 \\ -8\,000 & 0 & 32\,000 & 8\,000 & 0 & 16\,000 \\ \hline -2\,666.67 & 0 & 8\,000 & 2\,666.67 & 0 & 8\,000 \\ 0 & -600\,000 & 0 & 0 & 600\,000 & 0 \\ -8\,000 & 0 & 16\,000 & 8\,000 & 0 & 32\,000 \end{bmatrix} \end{matrix}$$

BARRA 2 Nós 2 - 3 Graus de liberdade (sistema global): $\Delta 4, \Delta 5, \Delta 6 - \Delta 7, \Delta 8, \Delta 9$

$E = 30 \times 10^6 \text{ kPa}$	$b = 0.3 \text{ m}$	$A = 0.15 \text{ m}^2$	$\alpha = 180^\circ \Rightarrow$	$\cos \alpha = -1$
$L = 2 \text{ m}$	$h = 0.5 \text{ m}$	$I = 0.003 \text{ m}^4$		$\sin \alpha = 0$

$$\left[K_L^2 \right] = \begin{bmatrix} 2\,250\,000 & 0 & 0 & -2\,250\,000 & 0 & 0 \\ 0 & 140\,625 & 140\,625 & 0 & -140\,625 & 140\,625 \\ 0 & 140\,625 & 187\,500 & 0 & -140\,625 & 93\,750 \\ -2\,250\,000 & 0 & 0 & 2\,250\,000 & 0 & 0 \\ 0 & -140\,625 & -140\,625 & 0 & 140\,625 & -140\,625 \\ 0 & 140\,625 & 93\,750 & 0 & -140\,625 & 187\,500 \end{bmatrix} \quad \left[T^2 \right] = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[K_G^2 \right] = \begin{bmatrix} & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 2\,250\,000 & 0 & 0 & -2\,250\,000 & 0 & 0 \\ 5 & 0 & 140\,625 & -140\,625 & 0 & -140\,625 & -140\,625 \\ 6 & 0 & -140\,625 & 187\,500 & 0 & 140\,625 & 93\,750 \\ 7 & -2\,250\,000 & 0 & 0 & 2\,250\,000 & 0 & 0 \\ 8 & 0 & -140\,625 & 140\,625 & 0 & 140\,625 & 140\,625 \\ 9 & 0 & -140\,625 & 93\,750 & 0 & 140\,625 & 187\,500 \end{bmatrix}$$

BARRA 3 Nós 2 - 4 Graus de liberdade (sistema global): $\Delta 4, \Delta 5, \Delta 6 - \Delta 10, \Delta 11, \Delta 12$

$E = 30 \times 10^6 \text{ kPa}$	$b = 0.3 \text{ m}$	$A = 0.15 \text{ m}^2$	$\alpha = -36,87^\circ \Rightarrow$	$\cos \alpha = 0.8$
$L = 5 \text{ m}$	$h = 0.5 \text{ m}$	$I = 0.003 \text{ m}^4$		$\sin \alpha = -0.6$

$$\left[K_L^3 \right] = \begin{bmatrix} 900\,000 & 0 & 0 & -900\,000 & 0 & 0 \\ 0 & 9\,000 & 22\,500 & 0 & -9\,000 & 22\,500 \\ 0 & 22\,500 & 75\,000 & 0 & -22\,500 & 37\,500 \\ -900\,000 & 0 & 0 & 900\,000 & 0 & 0 \\ 0 & -9\,000 & -22\,500 & 0 & 9\,000 & -22\,500 \\ 0 & 22\,500 & 37\,500 & 0 & -22\,500 & 75\,000 \end{bmatrix} \quad \left[T^3 \right] = \begin{bmatrix} 0.8 & -0.6 & 0 & 0 & 0 & 0 \\ 0.6 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & -0.6 & 0 \\ 0 & 0 & 0 & 0.6 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[K_G^3 \right] = \begin{bmatrix} & 4 & 5 & 6 & 10 & 11 & 12 \\ 4 & 579\,240 & -427\,680 & 13\,500 & -579\,240 & 427\,680 & 13\,500 \\ 5 & -427\,680 & 329\,760 & 18\,000 & 427\,680 & -329\,760 & 18\,000 \\ 6 & 13\,500 & 18\,000 & 75\,000 & -13\,500 & -18\,000 & 37\,500 \\ 10 & -579\,240 & 427\,680 & -13\,500 & 579\,240 & -427\,680 & -13\,500 \\ 11 & 427\,680 & -329\,760 & -18\,000 & -427\,680 & 329\,760 & -18\,000 \\ 12 & 13\,500 & 18\,000 & 37\,500 & -13\,500 & -18\,000 & 75\,000 \end{bmatrix}$$

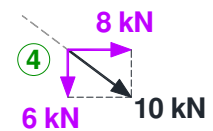
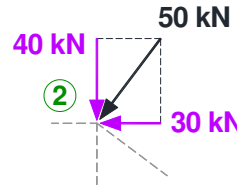
• **VETOR DE SOLICITAÇÃO**

Solicitação aplicada nos nós (sistema global)

Nó 2	
4	-30
5	-40
6	0

Nó 3	
7	7
8	0
9	0

Nó 4	
10	8
11	-6
12	0



Solicitação aplicada nas barras – Forças equivalentes nodais

$$F_{Global} = T^T \cdot F_{Local}$$

BARRA 1	Nós 1 - 2	Graus de liberdade - sistema local: $\Delta L1, \Delta L2, \Delta L3 - \Delta L4, \Delta L5, \Delta L6$
		Graus de liberdade - sistema global: $\Delta G1, \Delta G2, \Delta G3 - \Delta G4, \Delta G5, \Delta G6$

$p_y = -5 \text{ kN/m}$

$$F_1 = 0$$

$$F_2 = \frac{p_y \cdot L}{2} = \frac{-5 \times 6}{2} = -15 \text{ kN}$$

$$F_3 = \frac{p_y \cdot L^2}{12} = \frac{-5 \times 6^2}{12} = -15 \text{ kNm}$$

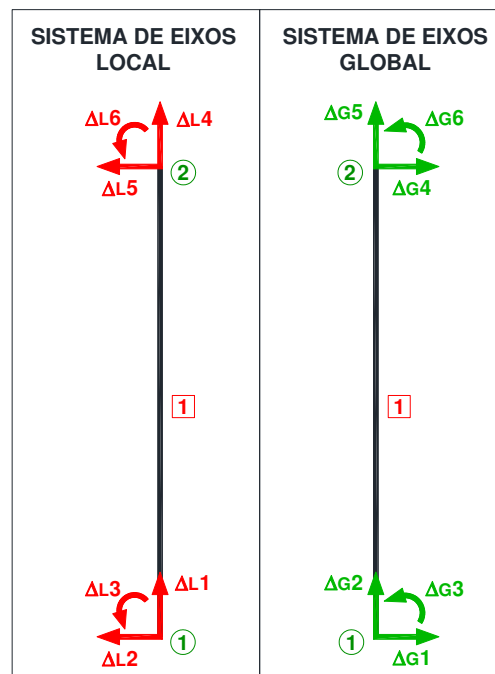
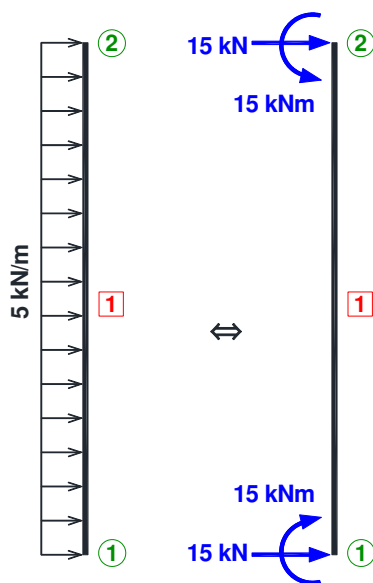
$$F_4 = 0$$

$$F_5 = \frac{p_y \cdot L}{2} = \frac{-5 \times 6}{2} = -15 \text{ kN}$$

$$F_6 = -\frac{p_y \cdot L^2}{12} = -\frac{-5 \times 6^2}{12} = 15 \text{ kNm}$$

$p_y = -5 \text{ kN/m}$
$F_{L-EN} - LOCAL$

F_1	0
F_2	-15
F_3	-15
F_4	0
F_5	-15
F_6	15



É fácil relacionar o sistema de eixos local com o global:

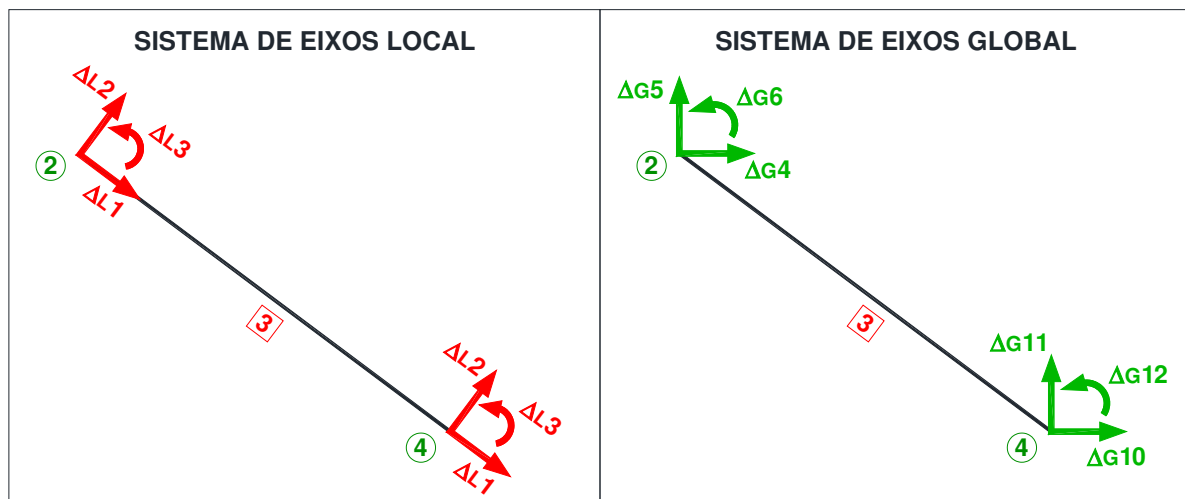
- $\Delta L1 \Leftrightarrow \Delta G2$
- $\Delta L2 \Leftrightarrow -\Delta G1$
- $\Delta L3 \Leftrightarrow \Delta G3$
- $\Delta L4 \Leftrightarrow \Delta G5$
- $\Delta L5 \Leftrightarrow -\Delta G4$
- $\Delta L6 \Leftrightarrow \Delta G6$

Barra 1 - Nós 1 - 2 F_{L-EN} - LOCAL	\Rightarrow	Barra 1 - Nós 1 - 2 F_{G-EN} - GLOBAL																								
<table border="1" style="margin: auto;"> <tr><td>F_1</td><td>0</td></tr> <tr><td>F_2</td><td>-15</td></tr> <tr><td>F_3</td><td>-15</td></tr> <tr><td>F_4</td><td>0</td></tr> <tr><td>F_5</td><td>-15</td></tr> <tr><td>F_6</td><td>15</td></tr> </table>	F_1	0	F_2	-15	F_3	-15	F_4	0	F_5	-15	F_6	15		<table border="1" style="margin: auto;"> <tr><td>F_1</td><td>15</td></tr> <tr><td>F_2</td><td>0</td></tr> <tr><td>F_3</td><td>-15</td></tr> <tr><td>F_4</td><td>15</td></tr> <tr><td>F_5</td><td>0</td></tr> <tr><td>F_6</td><td>15</td></tr> </table>	F_1	15	F_2	0	F_3	-15	F_4	15	F_5	0	F_6	15
F_1	0																									
F_2	-15																									
F_3	-15																									
F_4	0																									
F_5	-15																									
F_6	15																									
F_1	15																									
F_2	0																									
F_3	-15																									
F_4	15																									
F_5	0																									
F_6	15																									

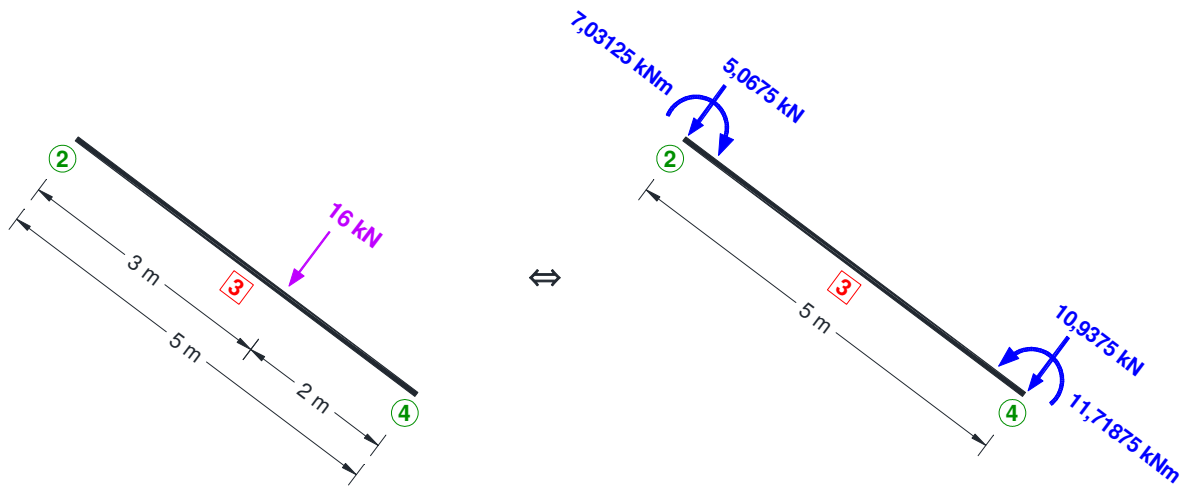
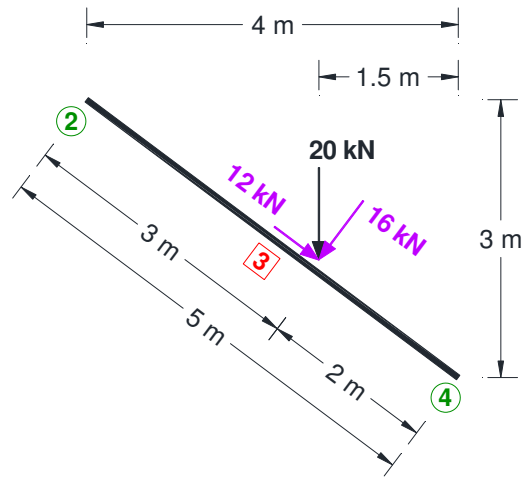
Vamos confirmar fazendo: $F_{Global} = T^T \cdot F_{Local}$

	F_{G-EN}	=	T^T	*	F_{L-EN}		Barra 1 - Nós 1 - 2				
							F_{G-EN} - GLOBAL				
1	F_1	0	-1	0	0	0	0	1	F_1	=	15 kN
2	F_2	1	0	0	0	0	0	2	F_2	=	0 kN
3	F_3	0	0	1	0	0	0	3	F_3	=	-15 kNm
4	F_4	0	0	0	0	-1	0	4	F_4	=	15 kN
5	F_5	0	0	0	1	0	0	5	F_5	=	0 kN
6	F_6	0	0	0	0	0	1	6	F_6	=	15 kNm

BARRA 3 Nós 2 - 4 Graus de liberdade - sistema local: $\Delta L1, \Delta L2, \Delta L3$ - $\Delta L4, \Delta L5, \Delta L6$
 Graus de liberdade - sistema global: $\Delta G4, \Delta G5, \Delta G6$ - $\Delta G10, \Delta G11, \Delta G12$



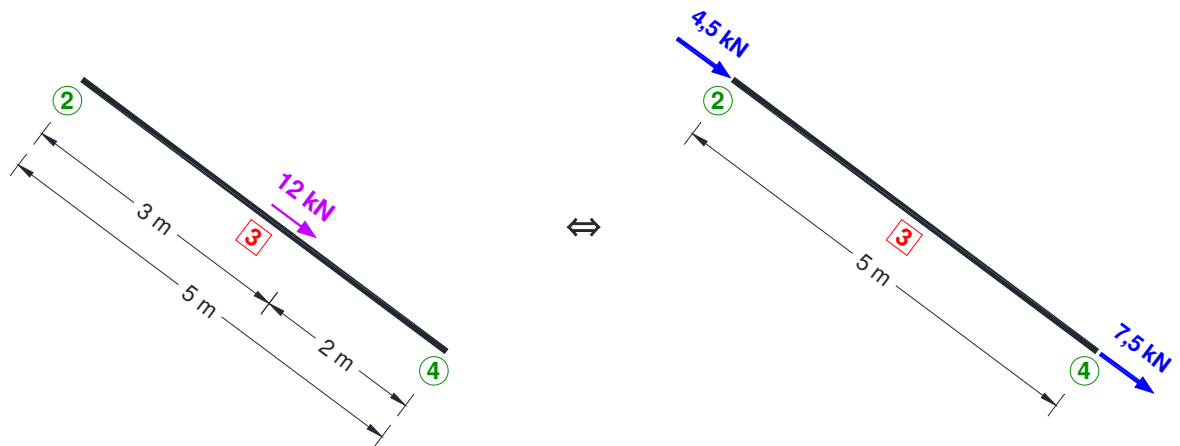
Força 20 kN \Rightarrow $\begin{cases} P_y = -16 \text{ kN} \swarrow (\perp \text{ à barra}) \\ P_x = +12 \text{ kN} \searrow (\parallel \text{ à barra}) \end{cases}$



$P_y = -16 \text{ kN}$	$F_1 = 0$
	$F_2 = \frac{P_y \cdot b^2}{L^3} (L+2a) = \frac{-16 \times 1,875^2}{5^3} (5+2 \times 3,125) = -5,0675 \text{ kN}$
	$F_3 = \frac{P_y \cdot a \cdot b^2}{L^2} = \frac{-16 \times 3,125 \times 1,875^2}{5^2} = -7,03125 \text{ kNm}$
	$F_4 = 0$
	$F_5 = \frac{P_y \cdot a^2}{L^3} (L+2b) = \frac{-16 \times 3,125^2}{5^3} (5+2 \times 1,875) = -10,9375 \text{ kN}$
	$F_6 = -\frac{P_y \cdot a^2 \cdot b}{L^2} = -\frac{-16 \times 3,125^2 \times 1,875}{5^2} = 11,71875 \text{ kNm}$

$P_y = -16 \text{ kN}$
$F_{L-EN} - LOCAL$

F_1	=	0
F_2		-5.0625
F_3		-7.03125
F_4		0
F_5		-10.9375
F_6		11.71875



$P_x = 12 \text{ kN}$

$$F_1 = \frac{P_x \cdot b}{L} = \frac{12 \times 1,875}{5} = 4,5 \text{ kN}$$

$$F_2 = 0$$

$$F_3 = 0$$

$$F_4 = \frac{P_x \cdot a}{L} = \frac{12 \times 3,125}{5} = 7,5 \text{ kN}$$

$$F_5 = 0$$

$$F_6 = 0$$

$P_x = 12 \text{ kN}$

$F_{L-EN} - LOCAL$

F ₁	=	4.5
F ₂	=	0
F ₃	=	0
F ₄	=	7.5
F ₅	=	0
F ₆	=	0

$P_y = -16 \text{ kN} + P_x = 12 \text{ kN}$

$F_{L-EN} - LOCAL$

F ₁	=	0 + 4.5
F ₂	=	-5.0625
F ₃	=	-7.03125
F ₄	=	0 + 7.5
F ₅	=	-10.9375
F ₆	=	11.71875

Barra 3 - Nós 2 - 4

$F_{L-EN} - LOCAL$

F ₁	=	4.5
F ₂	=	-5.0625
F ₃	=	-7.03125
F ₄	=	7.5
F ₅	=	-10.9375
F ₆	=	11.71875

$F_{G-EN} = T^T * F_{L-EN}$

4	F ₄	0.8	0.6	0	0	0	0	1	4.5
5	F ₅	-0.6	0.8	0	0	0	0	2	-5.0625
6	F ₆	0	0	1	0	0	0	3	-7.03125
10	F ₁₀	0	0	0	0.8	0.6	0	4	7.5
11	F ₁₁	0	0	0	-0.6	0.8	0	5	-10.9375
12	F ₁₂	0	0	0	0	0	1	6	11.71875

Barra 3 - Nós 2 - 4

$F_{G-EN} - GLOBAL$

4	F ₄	=	0.5625 kN
5	F ₅	=	-6.75 kN
6	F ₆	=	-7.03125 kNm
10	F ₁₀	=	-0.5625 kN
11	F ₁₁	=	-13.25 kN
12	F ₁₂	=	11.71875 kNm

**Vetor solicitação
sistema de eixos global**

$$\begin{matrix}
 1 \\
 2 \\
 3 \\
 4 \\
 5 \\
 6 \\
 7 \\
 8 \\
 9 \\
 10 \\
 11 \\
 12
 \end{matrix}
 \left[\begin{array}{c}
 15 \\
 0 \\
 -15 \\
 \hline
 -30 + 15 + 0.5625 \\
 -40 - 6.75 \\
 15 - 7.03125 \\
 \hline
 7 \\
 0 \\
 0 \\
 \hline
 8 - 0.5625 \\
 -6 - 13.25 \\
 11.71875
 \end{array} \right]
 =
 \begin{matrix}
 1 \\
 2 \\
 3 \\
 4 \\
 5 \\
 6 \\
 7 \\
 8 \\
 9 \\
 10 \\
 11 \\
 12
 \end{matrix}
 \left[\begin{array}{c}
 15 \\
 0 \\
 -15 \\
 \hline
 -14.4375 \\
 -46.75 \\
 7.96875 \\
 \hline
 7 \\
 0 \\
 0 \\
 \hline
 7.4375 \\
 -19.25 \\
 11.71875
 \end{array} \right]$$

Vetor das reações sistema de eixos global	Vetor dos deslocamentos sistema de eixos global
$ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{matrix} \left[\begin{array}{c} \mathbf{R1} \\ \mathbf{R2} \\ \mathbf{R3} \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ \mathbf{R8} \\ 0 \\ \hline \mathbf{R10} \\ 0 \\ 0 \end{array} \right] $	$ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{matrix} \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline \mathbf{\Delta4} \\ \mathbf{\Delta5} \\ \mathbf{\Delta6} \\ \hline \mathbf{\Delta7} \\ -0.001 \\ \mathbf{\Delta9} \\ \hline 0 \\ \mathbf{\Delta11} \\ \mathbf{\Delta12} \end{array} \right] $

• **MATRIZ DE RIGIDEZ GLOBAL DA ESTRUTURA**

$$[K_G]^{Est} \cdot [\Delta_G] = [F_G] + [R_G]$$

$$\begin{array}{c}
 4 \quad 5 \quad 6 \quad 7 \quad 9 \quad 11 \quad 12 \quad 1 \quad 2 \quad 3 \quad 8 \quad 10 \\
 \begin{array}{c}
 4 \\
 5 \\
 6 \\
 7 \\
 9 \\
 11 \\
 12 \\
 1 \\
 2 \\
 3 \\
 8 \\
 10
 \end{array}
 \left[\begin{array}{cc}
 & \\
 & \\
 & \\
 & \\
 & \\
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 & \\
 & \\
 & \\
 & \\
 & \\
 &
 \end{array} \right]
 \begin{array}{c}
 * \\
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 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{c}
 \\
 \\
 \\
 \Delta_{Li} \\
 \\
 \\
 \\
 \\
 \Delta_F \\
 \\
 \\
 \\
 \end{array}
 =
 \begin{array}{c}
 \\
 \\
 \\
 F_{Li} \\
 \\
 \\
 \\
 \\
 F_F \\
 \\
 \\
 \\
 \end{array}
 +
 \begin{array}{c}
 \\
 \\
 \\
 0 \\
 \\
 \\
 \\
 \\
 R_F \\
 \\
 \\
 \\
 \end{array}
 \end{array}$$

The matrix is partitioned into four quadrants: K_{LL} (top-left), K_{LF} (top-right), K_{FL} (bottom-left), and K_{FF} (bottom-right). The displacement vector Δ_G is partitioned into Δ_{Li} (top) and Δ_F (bottom). The force vector F_G is partitioned into F_{Li} (top) and F_F (bottom). The reaction vector R_G is zero for the top part and R_F for the bottom part.

$$\begin{cases}
 [K_{LL}] \cdot [\Delta_{Li}] + [K_{LF}] \cdot [\Delta_F] = [F_{Li}] \\
 [K_{FL}] \cdot [\Delta_{Li}] + [K_{FF}] \cdot [\Delta_F] = [F_F] + [R_F]
 \end{cases}
 \Rightarrow [\Delta_{Li}]$$

Na página seguinte estão representados o sistema $[K_G]^{Est} \cdot [\Delta_G] = [F_G] + [R_G]$ na forma inicial, com os graus de liberdade ordenados de forma crescente do 1 ao 12, e na forma rearranjada, separando graus de liberdade sem restrições de deslocamento (livres): 4, 5, 6, 7, 9, 11 e 12 e graus de liberdade com restrições de deslocamento (fixos): 1, 2, 3, 8 e 10.

TEORIA DE ESTRUTURAS

* $K_G - Estrutura$												
	1	2	3	4	5	6	7	8	9	10	11	12
1	2666,67	0	-8 000	-2666,67	0	-8 000	0	0	0	0	0	0
2	0	600 000	0	0	-600 000	0	0	0	0	0	0	0
3	-8 000	0	32 000	8 000	0	16 000	0	0	0	0	0	0
4	-2 666,67	0	8 000	2831906,67	-427 680	21 500	-2250 000	0	0	-579 240	427 680	13 500
5	0	-600 000	0	-427 680	1070 385	-122 625	0	-140 625	-140 625	427 680	-329 760	18 000
6	-8 000	0	16 000	21 500	-122 625	294 500	0	140 625	93 750	-13 500	-18 000	37 500
7	0	0	0	-2250 000	0	0	2250 000	0	0	0	0	0
8	0	0	0	0	-140 625	140 625	0	140 625	140 625	0	0	0
9	0	0	0	0	-140 625	93 750	0	140 625	187 500	0	0	0
10	0	0	0	-579 240	427 680	-13 500	0	0	0	579 240	-427 680	-13 500
11	0	0	0	427 680	-329 760	-18 000	0	0	0	-427 680	329 760	-18 000
12	0	0	0	13 500	18 000	37 500	0	0	0	-13 500	-18 000	75 000

Δ_G	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	Δ_4	0	0	0	0	0	0	0	0	0	0	0
5	Δ_5	0	0	0	0	0	0	0	0	0	0	0
6	Δ_6	0	0	0	0	0	0	0	0	0	0	0
7	Δ_7	0	0	0	0	0	0	0	0	0	0	0
8	Δ_8	0	0	0	0	0	0	0	0	0	0	0
9	Δ_9	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	Δ_{11}	0	0	0	0	0	0	0	0	0	0	0
12	Δ_{12}	0	0	0	0	0	0	0	0	0	0	0

F_G	1	2	3	4	5	6	7	8	9	10	11	12
1	15	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	-15	0	0	0	0	0	0	0	0	0	0	0
4	-14,4375	0	0	0	0	0	0	0	0	0	0	0
5	-46,75	0	0	0	0	0	0	0	0	0	0	0
6	7,96875	0	0	0	0	0	0	0	0	0	0	0
7	7	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	7,4375	0	0	0	0	0	0	0	0	0	0	0
11	-19,25	0	0	0	0	0	0	0	0	0	0	0
12	11,71875	0	0	0	0	0	0	0	0	0	0	0

R_G	1	2	3	4	5	6	7	8	9	10	11	12
R1	0	0	0	0	0	0	0	0	0	0	0	0
R2	0	0	0	0	0	0	0	0	0	0	0	0
R3	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
R8	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
R10	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0

* $K_G - Estrutura$												
	4	5	6	7	8	9	10	11	12	1	2	3
4	2831906,67	-427 680	21 500	-2250 000	13 500	427 680	-2 666,67	0	8 000	0	0	-579 240
5	-427 680	1070 385	-122 625	0	-140 625	-329 760	18 000	0	-600 000	0	-140 625	427 680
6	21 500	-122 625	294 500	0	93 750	-18 000	37 500	-8 000	16 000	140 625	-13 500	-13 500
7	-2250 000	0	0	2250 000	0	0	0	0	0	0	0	0
8	0	-140 625	93 750	0	187 500	0	0	0	0	140 625	0	0
9	427 680	-329 760	-18 000	0	329 760	-18 000	0	0	0	-427 680	0	0
10	13 500	18 000	37 500	0	-18 000	75 000	0	0	0	-13 500	0	0
11	-2 666,67	0	-8 000	0	0	2 666,67	0	-8 000	0	0	0	0
12	0	-600 000	0	0	0	0	600 000	0	0	0	0	0
1	8 000	0	16 000	0	0	-8 000	0	32 000	0	0	0	0
2	0	-140 625	140 625	0	140 625	0	0	140 625	0	140 625	0	0
3	0	427 680	-427 680	0	0	0	0	0	0	0	0	0
4	-579 240	427 680	-13 500	0	-427 680	-13 500	0	0	0	0	0	579 240

Δ_G	4	5	6	7	8	9	10	11	12	1	2	3
4	Δ_4	0	0	0	0	0	0	0	0	0	0	0
5	Δ_5	0	0	0	0	0	0	0	0	0	0	0
6	Δ_6	0	0	0	0	0	0	0	0	0	0	0
7	Δ_7	0	0	0	0	0	0	0	0	0	0	0
8	Δ_8	0	0	0	0	0	0	0	0	0	0	0
9	Δ_9	0	0	0	0	0	0	0	0	0	0	0
10	Δ_{10}	0	0	0	0	0	0	0	0	0	0	0
11	Δ_{11}	0	0	0	0	0	0	0	0	0	0	0
12	Δ_{12}	0	0	0	0	0	0	0	0	0	0	0

F_G	4	5	6	7	8	9	10	11	12	1	2	3
4	-14,4375	0	0	0	0	0	0	0	0	0	0	0
5	-46,75	0	0	0	0	0	0	0	0	0	0	0
6	7,96875	0	0	0	0	0	0	0	0	0	0	0
7	7	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	-19,25	0	0	0	0	0	0	0	0	0	0	0
11	11,71875	0	0	0	0	0	0	0	0	0	0	0
12	15	0	0	0	0	0	0	0	0	0	0	0

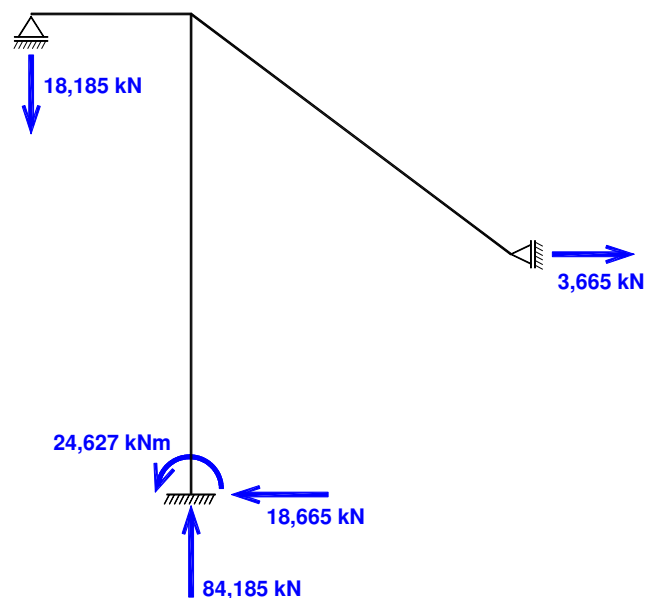
R_G	4	5	6	7	8	9	10	11	12	1	2	3
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

Deslocamentos dos nós da estrutura

$\Delta 1 = 0$	
$\Delta 2 = 0$	
$\Delta 3 = 0$	
$\Delta 4 = 8,61 \times 10^{-4} \text{ m}$	→
$\Delta 5 = -1,40 \times 10^{-4} \text{ m}$	↓
$\Delta 6 = 1,71 \times 10^{-4} \text{ rad}$	↻
$\Delta 7 = 8,64 \times 10^{-4} \text{ m}$	→
$\Delta 8 = -1,00 \times 10^{-3} \text{ m}$	↓
$\Delta 9 = 5,59 \times 10^{-4} \text{ rad}$	↻
$\Delta 10 = 0$	
$\Delta 11 = -1,33 \times 10^{-3} \text{ m}$	↓
$\Delta 12 = -3,69 \times 10^{-4} \text{ rad}$	↻

Reacções nos apoios

$R1 = H1 = -18,665 \text{ kN}$	←
$R2 = V1 = 84,185 \text{ kN}$	↑
$R3 = M1 = 24,627 \text{ kNm}$	↻
$R8 = V2 = -18,185 \text{ kN}$	↓
$R10 = H3 = 3,665 \text{ kN}$	→



Esforços nas barras

$$[F_L] = [K_L] \cdot [\Delta_L] - [F_{L-EN}]$$

$$[\Delta_L] = [T] \cdot [\Delta_G]$$

$$[F_L] = [K_L] \cdot [T] \cdot [\Delta_G] - [F_{L-EN}]$$

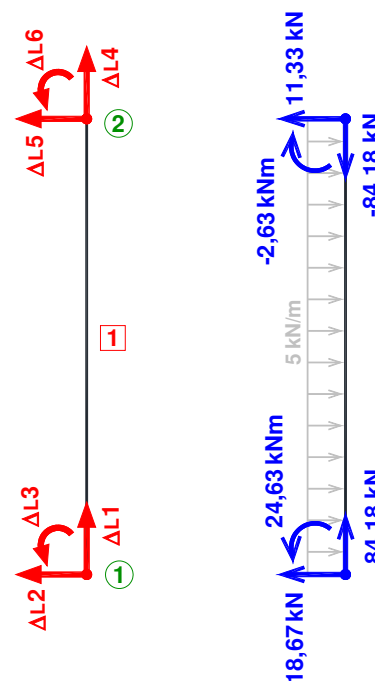
BARRA 1 Nós 1 - 2

- $\Delta L1 \Leftrightarrow \Delta G2 = 0$
- $\Delta L2 \Leftrightarrow -\Delta G1 = 0$
- $\Delta L3 \Leftrightarrow \Delta G3 = 0$
- $\Delta L4 \Leftrightarrow \Delta G5 = -1,40 \times 10^{-4}$
- $\Delta L5 \Leftrightarrow -\Delta G4 = -8,61 \times 10^{-4}$
- $\Delta L6 \Leftrightarrow \Delta G6 = 1,71 \times 10^{-4}$

F_L	K_L						*	Δ_L	-	F_{L-EN}
F_1	600 000	0	0	-600 000	0	0	1	0	0	
F_2	0	2 666.67	8 000	0	-2 666.67	8 000	2	0	-15	
F_3	0	8 000	32 000	0	-8 000	16 000	3	0	-15	
F_4	-600 000	0	0	600 000	0	0	4	-1.40E-04	0	
F_5	0	-2 666.67	-8 000	0	2 666.67	-8 000	5	-8.61E-04	-15	
F_6	0	8 000	16 000	0	-8 000	32 000	6	1.71E-04	15	

Barra 1 - Nós 1 - 2
 F_L - LOCAL

F_1	84.185 kN
F_2	18.665 kN
F_3	24.627 kNm
F_4	-84.185 kN
F_5	11.335 kN
F_6	-2.634 kNm



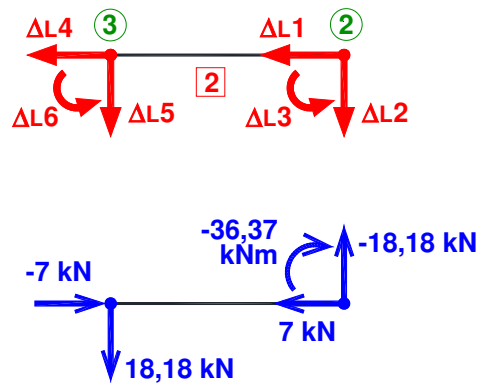
BARRA 2 Nós 2 - 3

$$\begin{aligned}
 \Delta L1 &\leftrightarrow -\Delta G4 = -8,61 \times 10^{-4} \\
 \Delta L2 &\leftrightarrow -\Delta G5 = 1,40 \times 10^{-4} \\
 \Delta L3 &\leftrightarrow \Delta G6 = 1,71 \times 10^{-4} \\
 \Delta L4 &\leftrightarrow -\Delta G7 = -8,64 \times 10^{-4} \\
 \Delta L5 &\leftrightarrow -\Delta G8 = 1,00 \times 10^{-3} \\
 \Delta L6 &\leftrightarrow \Delta G9 = 5,59 \times 10^{-4}
 \end{aligned}$$

F_L	=	K_L						*	Δ_L	-	F_{L-EN}
F_1		2250 000	0	0	2250 000	0	0	1	-8.61E-04		0
F_2		0	140 625	140 625	0	140 625	140 625	2	1.40E-04		0
F_3		0	140 625	187 500	0	140 625	187 500	3	1.71E-04		0
F_4		-2250 000	0	0	-2250 000	0	0	4	-8.64E-04		0
F_5		0	-140 625	-140 625	0	-140 625	-140 625	5	0.001		0
F_6		0	140 625	93 750	0	140 625	93 750	6	5.59E-04		0

Barra 2 - Nós 2 - 3
 F_L - LOCAL

F_1	=	7.000 kN
F_2		-18.185 kN
F_3		-36.370 kNm
F_4		-7.000 kN
F_5		18.185 kN
F_6		0.000 kNm



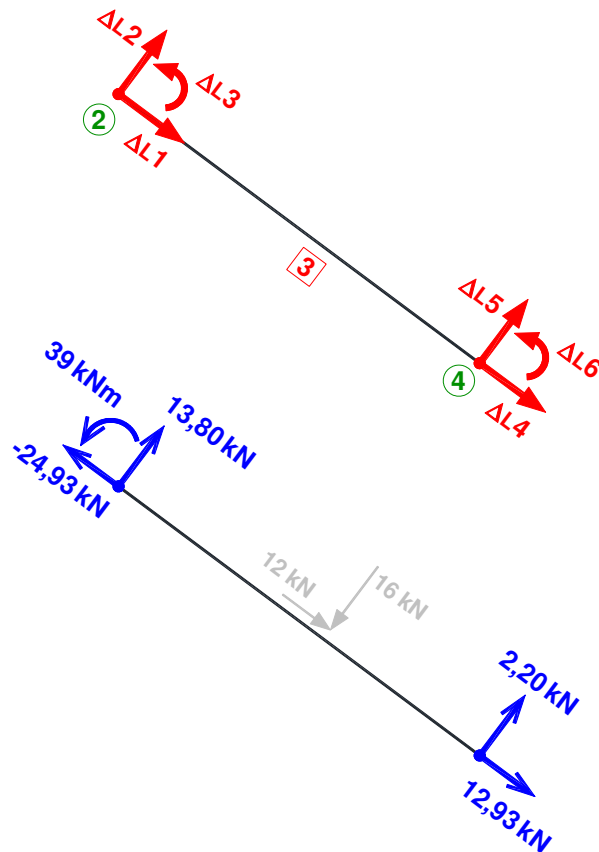
BARRA 3 Nós 2 - 4

$$F_L = K_L * T * \Delta_G - F_{L-EN}$$

F_1	900 000	0	0	-900 000	0	0	0.8	-0.6	0	0	0	0	4	8.61E-04	4.5
F_2	0	9 000	22 500	0	-9 000	22 500	0.6	0.8	0	0	0	0	5	-1.40E-04	-5.0625
F_3	0	22 500	75 000	0	-22 500	37 500	0	0	1	0	0	0	6	1.71E-04	-7.03125
F_4	-900 000	0	0	900 000	0	0	0	0	0	0.8	-0.6	0	10	0	7.5
F_5	0	-9 000	-22 500	0	9 000	-22 500	0	0	0	0.6	0.8	0	11	-1.33E-03	-10.9375
F_6	0	22 500	37 500	0	-22 500	75 000	0	0	0	0	0	1	12	-3.69E-04	11.7188

Barra 3 - Nós 2 - 4
 F_L - LOCAL

F_1	-24.932 kN
F_2	13.801 kN
F_3	39.004 kNm
F_4	12.932 kN
F_5	2.199 kN
F_6	0.000 kNm



Diagramas de esforços

