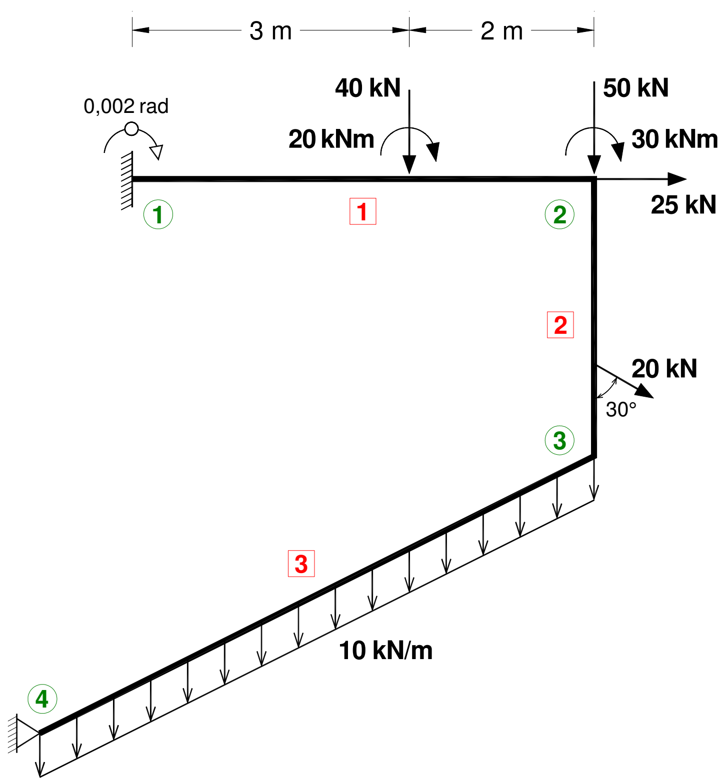
LICENCIATURA EM ENGENHARIA CIVIL

TEORIA DE ESTRUTURAS

MÉTODO DOS DESLOCAMENTOS

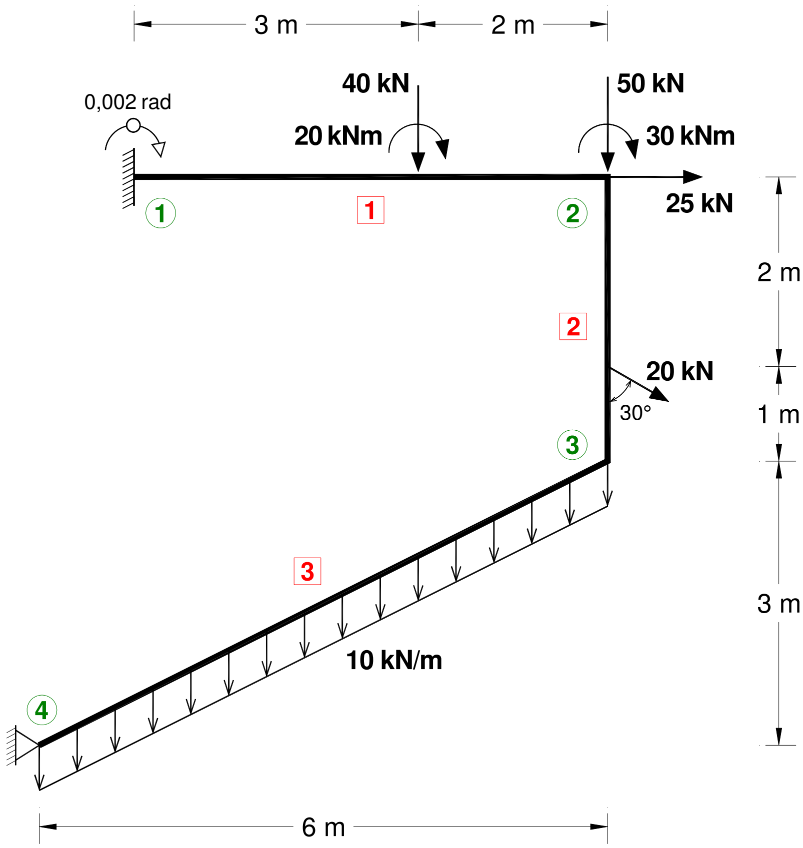


ESTRUTURA CONTÍNUA

ISABEL ALVIM TELES

EXERCÍCIO PROPOSTO

Considere a estrutura representada na figura, realizada em betão com E = 25 GPa e em que todas as barras têm secção 0,30 m x 0,40 m.



Responda às alíneas aplicando o Método dos Deslocamentos.

a) Determine os deslocamentos dos nós da estrutura;

b) Determine as reações nos apoios;

c) Trace os diagramas de esforços da estrutura.

RESOLUÇÃO

• MATRIZ DE RIGIDEZ ELEMENTAR – SISTEMA DE EIXOS LOCAL

$$[\mathbf{K_L}] = \left[\begin{array}{ccc|ccc} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ \hline -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{array}\right]$$

• MATRIZ DE TRANSFORMAÇÃO

$$[\mathbf{T}] = \left[\begin{array}{ccc|ccc} \cos\alpha & \text{sen}\,\alpha & 0 & 0 & 0 & 0 \\ -\text{sen}\,\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \cos\alpha & \text{sen}\,\alpha & 0 \\ 0 & 0 & 0 & -\text{sen}\,\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

• MATRIZ DE TRANSFORMAÇÃO TRANSPOSTA

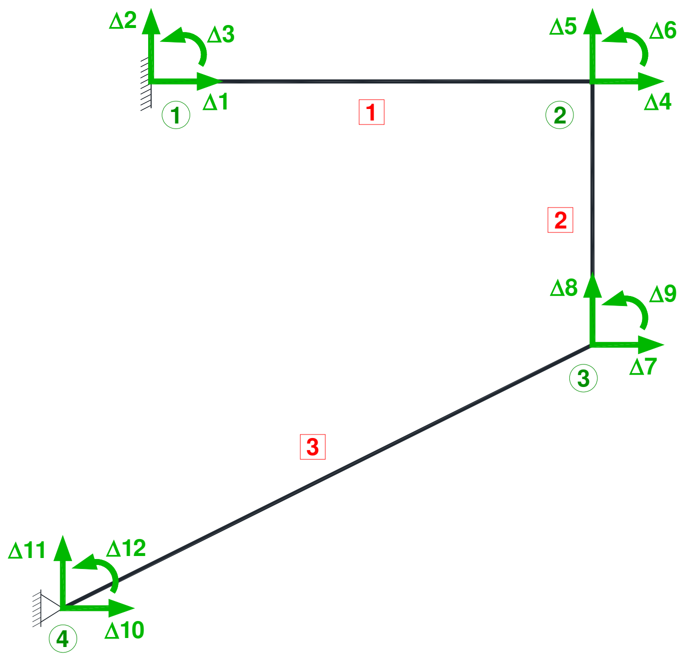
$$[\mathbf{T}]^{\mathbf{T}} = \left[\begin{array}{ccc|ccc} \cos\alpha & -\text{sen}\,\alpha & 0 & 0 & 0 & 0 \\ \text{sen}\,\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \cos\alpha & -\text{sen}\,\alpha & 0 \\ 0 & 0 & 0 & \text{sen}\,\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

• MATRIZ DE RIGIDEZ ELEMENTAR – SISTEMA DE EIXOS GLOBAL

$$[K_G] = [T]^T \, [K_L] \, [T]$$

ALÍNEA a)

• GRAUS DE LIBERDADE DA ESTRUTURA (SISTEMA GLOBAL)



• MATRIZ DE RIGIDEZ LOCAL, MATRIZ DE TRANSFORMAÇÃO E MATRIZ DE RIGIDEZ GLOBAL

BARRA 1 **Nós 1 - 2** Graus de liberdade (sistema global): **Δ1, Δ2, Δ3 - Δ4, Δ5, Δ6**

$E = 25 \times 10^6$ kPa	b = 0.3 m	$A = 0.12\ m^2$	$\alpha = 0^{\circ} \Rightarrow$	$\cos\alpha = 1$
L = 5 m	h = 0.4 m	$I = 0.0016\ m^4$		$\sin\alpha = 0$

$$\left[K_L^1\right] = \left[\begin{array}{ccc|ccc} 600\,000 & 0 & 0 & -600\,000 & 0 & 0 \\ 0 & 3\,840 & 9\,600 & 0 & -3\,840 & 9\,600 \\ 0 & 9\,600 & 32\,000 & 0 & -9\,600 & 16\,000 \\ \hline -600\,000 & 0 & 0 & 600\,000 & 0 & 0 \\ 0 & -3840 & -9\,600 & 0 & 3840 & -9\,600 \\ 0 & 9\,600 & 16\,000 & 0 & -9\,600 & 32\,000 \end{array}\right] \qquad \left[T^1\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[K_G^1\right] = \left[K_L^1\right] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 600\,000 & 0 & 0 & -600\,000 & 0 & 0 \\ 0 & 3\,840 & 9\,600 & 0 & -3\,840 & 9\,600 \\ 0 & 9\,600 & 32\,000 & 0 & -9\,600 & 16\,000 \\ \hline -600\,000 & 0 & 0 & 600\,000 & 0 & 0 \\ 0 & -3840 & -9\,600 & 0 & 3840 & -9\,600 \\ 0 & 9\,600 & 16\,000 & 0 & -9\,600 & 32\,000 \end{array}\right] \end{array}$$

BARRA 2 **Nós 2 - 3** Graus de liberdade (sistema global): **Δ4, Δ5, Δ6 - Δ7, Δ8, Δ9**

E = 25x10^6 kPa	b = 0.3 m	A = 0.12 m^2	α = -90 º ⇒	cos α = 0
L = 3 m	h = 0.4 m	I = 0.0016 m^4		sin α = -1

$$\left[K_L^2\right] = \begin{bmatrix} 1000\,000 & 0 & 0 & -1000\,000 & 0 & 0 \\ 0 & 17\,777{,}78 & 26\,666{,}67 & 0 & -17\,777{,}78 & 26\,666{,}67 \\ 0 & 26\,666{,}67 & 53\,333{,}33 & 0 & -26\,666{,}67 & 26\,666{,}67 \\ -1000\,000 & 0 & 0 & 1000\,000 & 0 & 0 \\ 0 & -17\,777{,}78 & -26\,666{,}67 & 0 & 17\,777{,}78 & -26\,666{,}67 \\ 0 & 26\,666{,}67 & 26\,666{,}67 & 0 & -26\,666{,}67 & 53\,333{,}33 \end{bmatrix} \qquad \left[T^2\right] = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[K_G^2\right] = \left[T^2\right]^T \left[K_L^2\right] \left[T^2\right] = \begin{array}{c} \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} \begin{array}{c} \begin{array}{cccccc} 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} 17\,777{,}78 & 0 & 26\,666{,}67 & -17\,777{,}78 & 0 & 26\,666{,}67 \\ 0 & 1000000 & 0 & 0 & -1000000 & 0 \\ 26\,666{,}67 & 0 & 53\,333{,}33 & -26\,666{,}67 & 0 & 26\,666{,}67 \\ -17\,777{,}78 & 0 & -26\,666{,}67 & 17\,777{,}78 & 0 & -26\,666{,}67 \\ 0 & -1000000 & 0 & 0 & 1000000 & 0 \\ 26\,666{,}67 & 0 & 26\,666{,}67 & -26\,666{,}67 & 0 & 53\,333{,}33 \end{bmatrix} \end{array}$$

BARRA 3 **Nós 3 - 4** Graus de liberdade (sistema global): **Δ7, Δ8, Δ9 - Δ10, Δ11, Δ12**

E = 25x10^6 kPa	b = 0.3 m	A = 0.12 m^2	α = -153,435 º ⇒	cos α = -0.8944
L = $3\sqrt{5}$ m	h = 0.4 m	I = 0.0016 m^4		sin α = -0.4472

$$\left[K_L^3\right] = \begin{bmatrix} 447\,213{,}60 & 0 & 0 & -447\,213{,}60 & 0 & 0 \\ 0 & 1\,590{,}09 & 5\,333{,}33 & 0 & -1\,590{,}09 & 5\,333{,}33 \\ 0 & 5\,333{,}33 & 23\,851{,}39 & 0 & -5\,333{,}33 & 11\,925{,}70 \\ -447\,213{,}60 & 0 & 0 & 447\,213{,}60 & 0 & 0 \\ 0 & -1\,590{,}09 & -5\,333{,}33 & 0 & 1\,590{,}09 & -5\,333{,}33 \\ 0 & 5\,333{,}33 & 11\,925{,}70 & 0 & -5\,333{,}33 & 23\,851{,}39 \end{bmatrix} \qquad \left[T^3\right] = \begin{bmatrix} -0{,}8944 & -0{,}4472 & 0 & 0 & 0 & 0 \\ 0{,}4472 & -0{,}8944 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0{,}8944 & -0{,}4472 & 0 \\ 0 & 0 & 0 & 0{,}4472 & -0{,}8944 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[K_G^3\right] = \left[T^3\right]^T \left[K_L^3\right] \left[T^3\right] = \begin{array}{c} \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccccc} 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\ \begin{bmatrix} 358\,088{,}89 & 178\,249{,}40 & 2\,385{,}14 & -358\,088{,}89 & -178\,249{,}40 & 2\,385{,}14 \\ 178\,249{,}40 & 90\,714{,}79 & -4\,770{,}28 & -178\,249{,}40 & -90\,714{,}79 & -4\,770{,}28 \\ 2\,385{,}14 & -4\,770{,}28 & 23\,851{,}39 & -2\,385{,}14 & 4\,770{,}28 & 11\,925{,}70 \\ -358\,088{,}89 & -178\,249{,}40 & -2\,385{,}14 & 358\,088{,}89 & 178\,249{,}40 & -2\,385{,}14 \\ -178\,249{,}40 & -90\,714{,}79 & 4\,770{,}28 & 178\,249{,}40 & 90\,714{,}79 & 4\,770{,}28 \\ 2\,385{,}14 & -4\,770{,}28 & 11\,925{,}70 & -2\,385{,}14 & 4\,770{,}28 & 23\,851{,}39 \end{bmatrix} \end{array}$$

• VETOR DE SOLICITAÇÃO

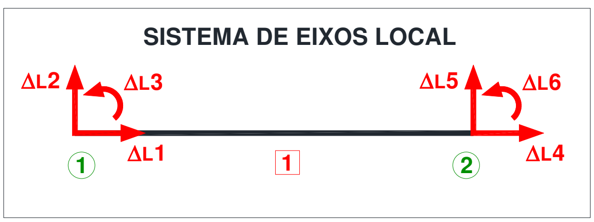
Solicitação aplicada nos nós (sistema global)

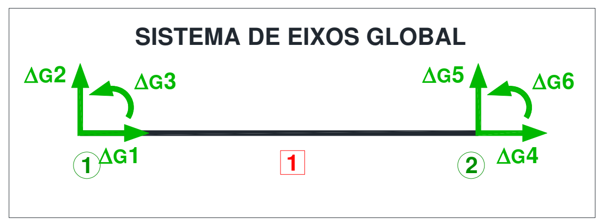
	Nó 2
4	25
5	- 50
6	- 30

Solicitação aplicada nas barras – Forças equivalentes nodais

$$F_{Global} = T^T \cdot F_{Local}$$

BARRA 1	Nós 1 - 2	Graus de liberdade - sistema local: ΔL1, ΔL2, ΔL3 - ΔL4, ΔL5, ΔL6 Graus de liberdade - sistema global: ΔG1, ΔG2, ΔG3 - ΔG4, ΔG5, ΔG6





$P_Y = -40$ kN

$$F_{//esq} = 0$$

$$F_{\perp esq} = \frac{P_y \cdot b^2}{L^3}(L + 2a) = \frac{-40 \times 2^2}{5^3}(5 + 2 \times 3) = -14{,}08 \text{ kN}$$

$$F_{M\text{-}esq} = \frac{P_y \cdot a \cdot b^2}{L^2} = \frac{-40 \times 3 \times 2^2}{5^2} = -19{,}2 \text{ kNm}$$

$$F_{//dir} = 0$$

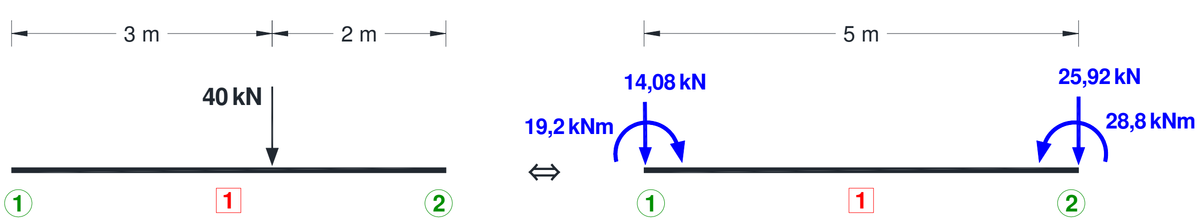
$$F_{\perp dir} = \frac{P_y \cdot a^2}{L^3}(L + 2b) = \frac{-40 \times 3^2}{5^3}(5 + 2 \times 2) = -25{,}92 \text{ kN}$$

$$F_{M\text{-}dir} = -\frac{P_y \cdot a^2 \cdot b}{L^2} = -\frac{-40 \times 3^2 \times 2}{5^2} = 28{,}8 \text{ kNm}$$

P_Y = - 40 kN

$F_{L\text{-}EN}$ - LOCAL

F_1	=	0
F_2		-14,08
F_3		-19,2
F_4		0
F_5		-25,92
F_6		28,8

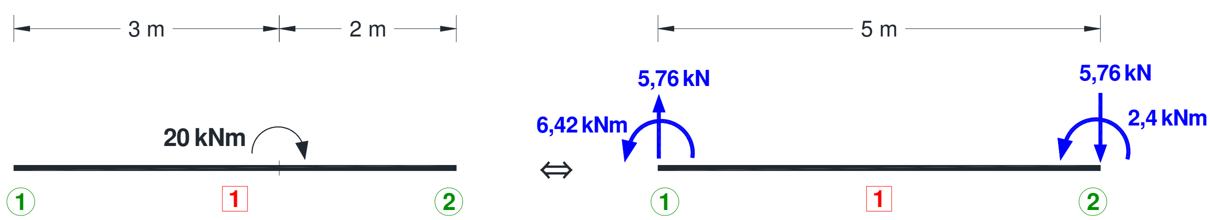


M_Z = -20 kNm	$F_{//esq} = 0$
	$F_{\perp esq} = -\frac{6\ M_Z}{L^3}\,a\,.\,b = -\frac{6\times(-20)}{5^3}\times 3\times 2 = 5{,}76\ \text{kN}$
	$F_{M\text{-}esq} = -\frac{M_Z\,.\,b}{L}\left(2-\frac{3\,b}{L}\right) = -\frac{-20\times 2}{5}\left(2-\frac{3\times 2}{5}\right) = 6{,}4\ \text{kNm}$
	$F_{//dir} = 0$
	$F_{\perp dir} = \frac{6\ M_Z}{L^3}\,a\,.\,b = \frac{6\times(-20)}{5^3}\times 3\times 2 = -5{,}76\ \text{kN}$
	$F_{M\text{-}dir} = -\frac{M_Z\,.\,a}{L}\left(2-\frac{3\,a}{L}\right) = -\frac{-20\times 3}{5}\left(2-\frac{3\times 3}{5}\right) = 2{,}4\ \text{kNm}$

M_Z = - 20 kNm

$F_{L\text{-}EN}$ - LOCAL

F_1	=	0
F_2		5,76
F_3		6,4
F_4		0
F_5		-5,76
F_6		2,4



P_Y = -40 kN + M_Z = -20 kNm

$F_{L\text{-}EN}$ - LOCAL

F_1	=	0 + 0
F_2		-14,08 + 5,76
F_3		-19,2 + 6,4
F_4		0 + 0
F_5		-25,92 – 5,76
F_6		28,8 + 2,4

⇒

Barra 1 - Nós 1 - 2

$F_{L\text{-}EN}$ - LOCAL

F_1	=	0
F_2		-8,32
F_3		-12,8
F_4		0
F_5		-31,68
F_6		31,2

É fácil relacionar o sistema de eixos local com o global:

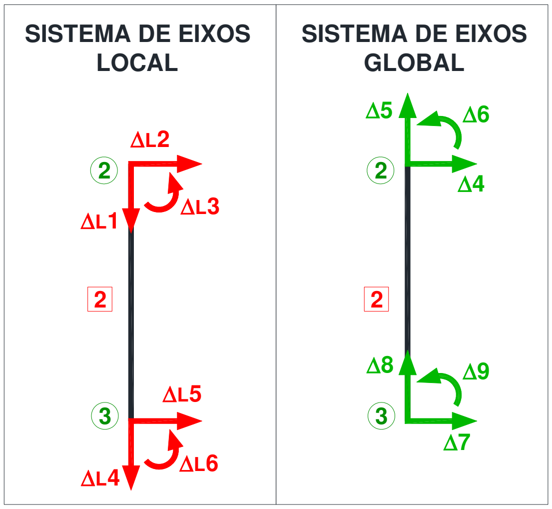
ΔL1 ⇔ ΔG1
ΔL2 ⇔ ΔG2
ΔL3 ⇔ ΔG3
ΔL4 ⇔ ΔG4
ΔL5 ⇔ ΔG5
ΔL6 ⇔ ΔG6

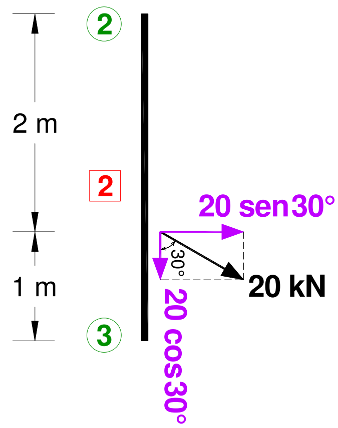
Barra 1 - Nós 1 - 2

$F_{G\text{-}EN}$ - GLOBAL

F_1	=	0
F_2		-8,32
F_3		-12,8
F_4		0
F_5		-31,68
F_6		31,2

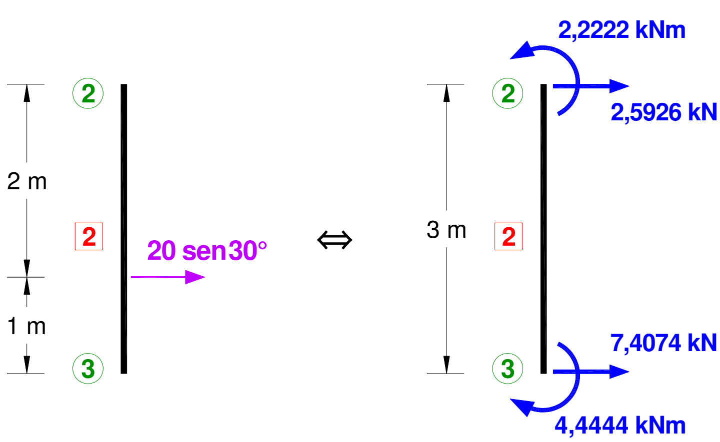
BARRA 2	**Nós 2 - 3**	Graus de liberdade - sistema local: **ΔL1, ΔL2, ΔL3 - ΔL4, ΔL5, ΔL6** Graus de liberdade - sistema global: **ΔG1, ΔG2, ΔG3 - ΔG4, ΔG5, ΔG6**





Força 20 kN $\Rightarrow \begin{cases} P_y = 20 \text{ sen } 30^o \text{ kN} & (\perp \text{ à barra}) \\ P_x = 20 \cos 30^o \text{ kN} & (// \text{ à barra}) \end{cases}$

$P_Y =$ 20 sen 30° kN	$F_{//esq} = 0$
	$F_{\perp esq} = \dfrac{P_y \cdot b^2}{L^3}(L+2a) = \dfrac{20 \text{ sen } 30^o \times 1^2}{3^3}(3+2\times 2) = 2{,}5926 \text{ kN}$
	$F_{M\text{-}esq} = \dfrac{P_y \cdot a \cdot b^2}{L^2} = \dfrac{20 \text{ sen } 30^o \times 2 \times 1^2}{3^2} = 2{,}2222 \text{ kNm}$
	$F_{//dir} = 0$
	$F_{\perp dir} = \dfrac{P_y \cdot a^2}{L^3}(L+2b) = \dfrac{20 \text{ sen } 30^o \times 2^2}{3^3}(3+2\times 1) = 7{,}4074 \text{ kN}$
	$F_{M\text{-}dir} = -\dfrac{P_y \cdot a^2 \cdot b}{L^2} = -\dfrac{20 \text{ sen } 30^o \times 2^2 \times 1}{3^2} = -4{,}4444 \text{ kNm}$

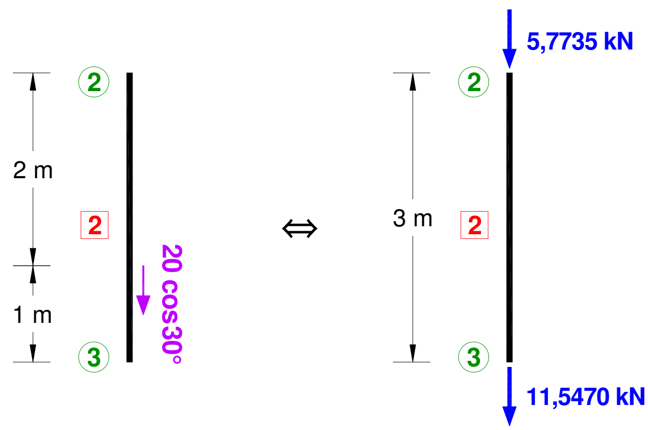


$P_Y =$ 20 sen 30° kN

$F_{L\text{-}EN}$ - LOCAL

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2{,}5926 \\ 2{,}2222 \\ 0 \\ 7{,}4074 \\ -4{,}4444 \end{Bmatrix}$$

$P_X = 20 \cos 30° \text{ kN}$	$F_{//esq} = \frac{P_X \cdot b}{L} = \frac{20 \cos 30^{o} \times 1}{3} = 5{,}7735 \text{ kN}$
	$F_{\perp esq} = 0$
	$F_{M\text{-}esq} = 0$
	$F_{//dir} = \frac{P_X \cdot a}{L} = \frac{20 \cos 30^{o} \times 2}{3} = 11{,}5470 \text{ kN}$
	$F_{\perp dir} = 0$
	$F_{M\text{-}dir} = 0$



$P_X = 20 \cos 30° \text{ kN}$		
$F_{L\text{-}EN}$ - LOCAL		
F_1	=	5,7735
F_2		0
F_3		0
F_4		11,5470
F_5		0
F_6		0

$P_Y = 20 \cos 30° \text{ kN} + P_X = 20 \cos 30° \text{ kN}$		
$F_{L\text{-}EN}$ - LOCAL		
F_1	=	0 + 5,7735
F_2		2,5926 + 0
F_3		2,2222 + 0
F_4		0 + 11,5470
F_5		7,4074 + 0
F_6		-4,4444 + 0

⇒

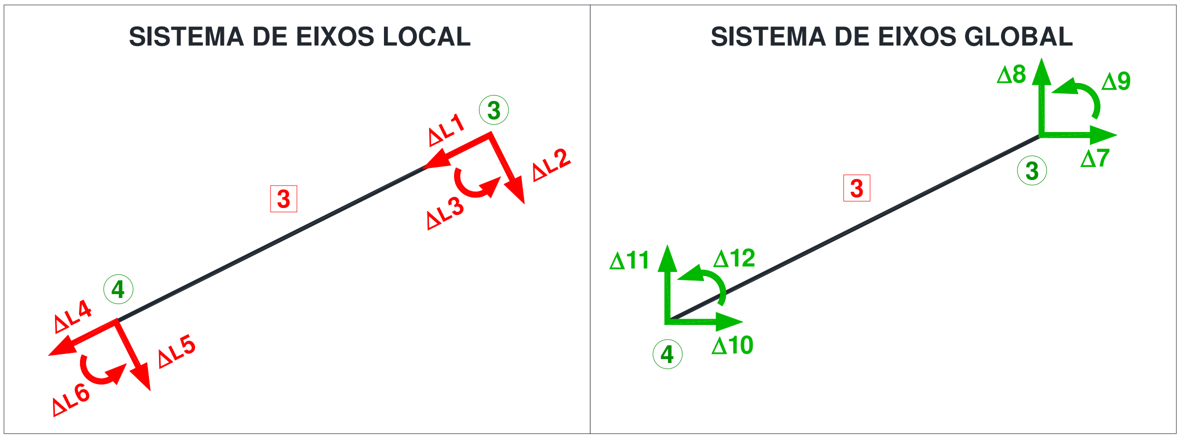
Barra 2 - Nós 2 - 3		
$F_{L\text{-}EN}$ - LOCAL		
F_1	=	5,7735
F_2		2,5926
F_3		2,2222
F_4		11,5470
F_5		7,4074
F_6		-4,4444

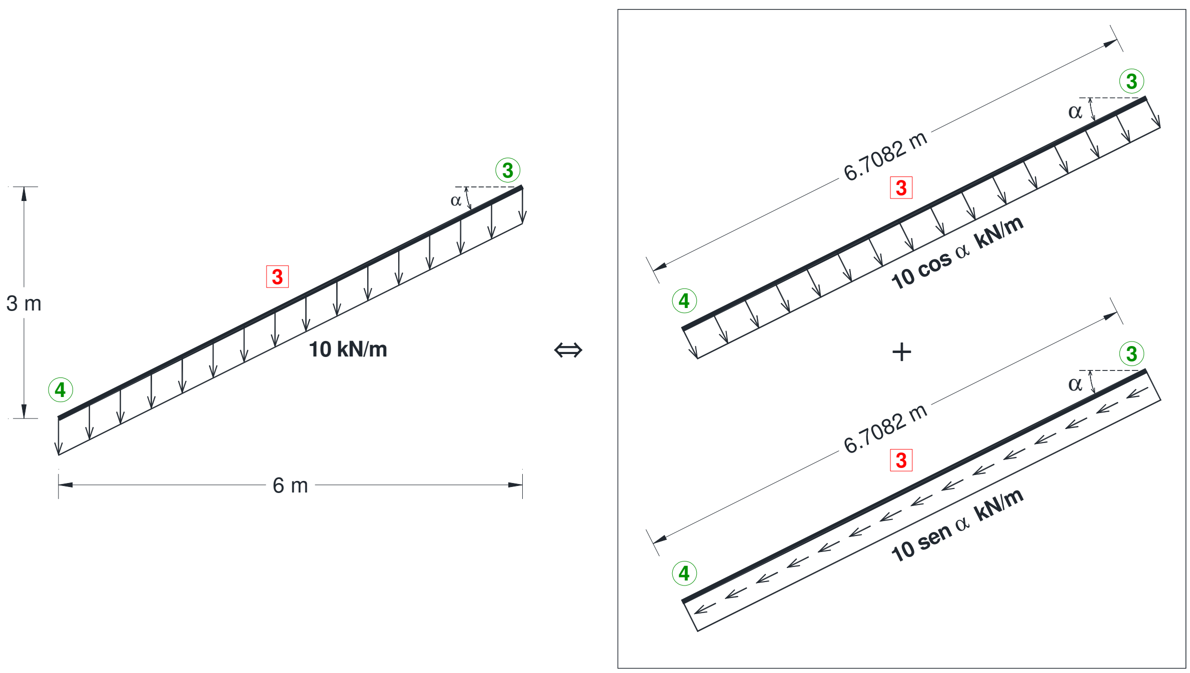
É fácil relacionar o sistema de eixos local com o global:

ΔL1 ⇔ - ΔG5	**ΔG4 ⇔ ΔL2**
ΔL2 ⇔ ΔG4	**ΔG5 ⇔ - ΔL1**
ΔL3 ⇔ ΔG6	**ΔG6 ⇔ ΔL3**
ΔL4 ⇔ - ΔG8	**ΔG7 ⇔ ΔL5**
ΔL5 ⇔ ΔG7	**ΔG8 ⇔ - ΔL4**
ΔL6 ⇔ ΔG9	**ΔG9 ⇔ ΔL6**

Barra 2 - Nós 2 - 3		
$F_{G\text{-}EN}$ - GLOBAL		
F_4	=	2,5926
F_5		-5,7735
F_6		2,2222
F_7		7,4074
F_8		-11,5470
F_9		-4,4444

BARRA 3	**Nós 3 - 4**	Graus de liberdade - sistema local: **ΔL1, ΔL2, ΔL3 - ΔL4, ΔL5, ΔL6** Graus de liberdade - sistema global: **ΔG7, ΔG8, ΔG9 - ΔG10, ΔG11, ΔG12**





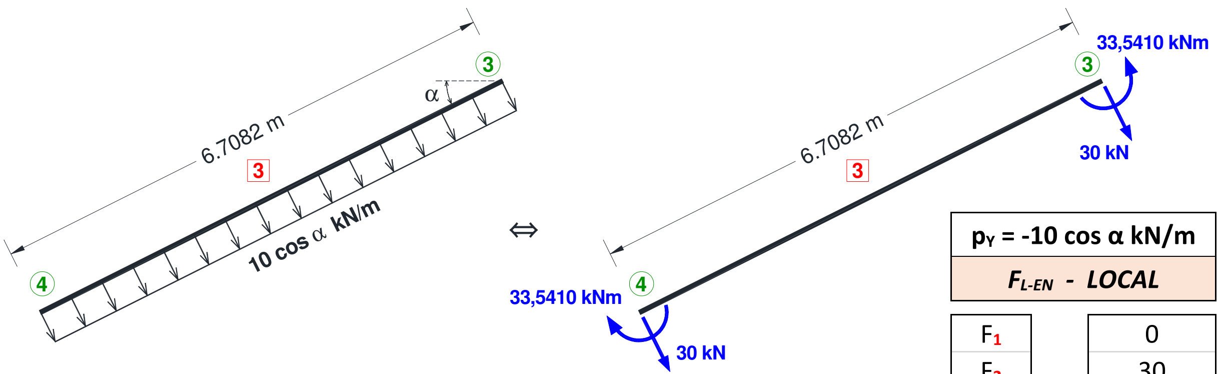
Comprimento da barra: $L = 3\sqrt{5} \;\; = 6{,}7082 \;\; \text{m}$

$$\alpha = \text{arctg}\,\frac{3}{6} \;\Rightarrow\; \alpha = \; 26{,}565\,^{o} \;\Rightarrow\; \begin{cases} \cos\,\alpha \; = 0{,}8944 \\ \text{sen}\;\;\alpha \; = 0{,}4472 \end{cases}$$

$10 \cos \alpha \;=\; 8{,}9443$ kN/m

$10\ \text{sen}\ \alpha \;=\; 4{,}4721$ kN/m

p_Y = -10 cos α kN/m	$F_{//esq} = 0$
	$F_{\perp esq} = \frac{p_y \cdot L}{2} = \frac{-10\cos\alpha \times 3\sqrt{5}}{2} = -30\ \text{kN}$
	$F_{M\text{-}esq} = \frac{p_y \cdot L^2}{12} = \frac{-10\cos\alpha \times (3\sqrt{5})^2}{12} = -33{,}5410\ \text{kNm}$
	$F_{//dir} = 0$
	$F_{\perp dir} = \frac{p_y \cdot L}{2} = \frac{-10\cos\alpha \times 3\sqrt{5}}{2} = -30\ \text{kN}$
	$F_{M\text{-}dir} = -\frac{p_y \cdot L^2}{12} = -\frac{-10\cos\alpha \times (3\sqrt{5})^2}{12} = 33{,}5410\ \text{kNm}$



p_Y = -10 cos α kN/m

$F_{L\text{-}EN}$ - LOCAL

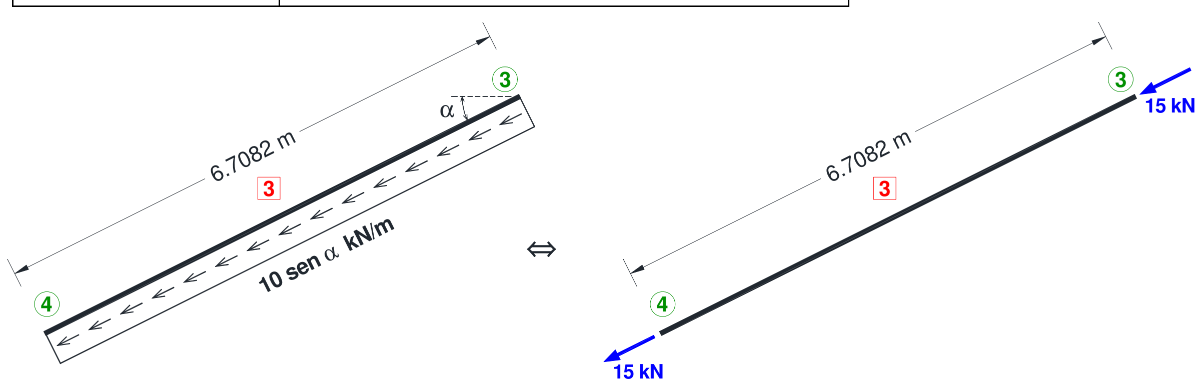
F_1		0
F_2		30
F_3		33,5410
F_4	=	0
F_5		30
F_6		-33,5410

p_X = -10 sen α kN/m	$F_{//esq} = \frac{p_x \cdot L}{2} = \frac{-10\,\text{sen}\,\alpha \times 3\sqrt{5}}{2} = -15\ \text{kN}$
	$F_{\perp esq} = 0$
	$F_{M\text{-}esq} = 0$
	$F_{//dir} = \frac{p_x \cdot L}{2} = \frac{-10\,\text{sen}\,\alpha \times 3\sqrt{5}}{2} = -15\ \text{kN}$
	$F_{\perp dir} = 0$
	$F_{M\text{-}dir} = 0$

p_X = -10 sen α kN/m

$F_{L\text{-}EN}$ - LOCAL

F_1		15
F_2		0
F_3		0
F_4	=	15
F_5		0
F_6		0



p_Y = -10 cos α kN/m + p_X = -10 cos α kN/m	
$F_{L\text{-}EN}$ - LOCAL	
F_1	0 + 15
F_2	30 + 0
F_3	33,5410 + 0
F_4	0 + 15
F_5	30 + 0
F_6	-33,5410 + 0

⇒

Barra 3 - Nós 3 - 4	
$F_{L\text{-}EN}$ - LOCAL	
F_1	15
F_2	30
F_3	33,5410
F_4	15
F_5	30
F_6	-33,5410

$$F_{Global} = T^T \cdot F_{Local}$$

$F_{G\text{-}EN}$ = T^T * $F_{L\text{-}EN}$

	$F_{G\text{-}EN}$	T^T							$F_{L\text{-}EN}$
7	F_7	-0,8944	0,4472	0	0	0	0	1	15
8	F_8	-0,4472	-0,8944	0	0	0	0	2	30
9	F_9	0	0	1	0	0	0	3	33,5410
10	F_{10}	0	0	0	-0,8944	0,4472	0	4	15
11	F_{11}	0	0	0	-0,4472	-0,8944	0	5	30
12	F_{12}	0	0	0	0	0	1	6	-33,5410

Barra 3 - Nós 3 - 4		
	$F_{G\text{-}EN}$ - GLOBAL	
7	F_7	0 kN
8	F_8	-33,5410 kN
9	F_9	33,5410 kNm
10	F_{10}	0 kN
11	F_{11}	-33,5410 kN
12	F_{12}	-33,5410 kNm

Vetor solicitação - sistema de eixos global

$$[F_G] = \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12 \end{array} \begin{bmatrix} 0 \\ -8,32 \\ -12,8 \\ 25+0+2,5926 \\ -50-31,68-5,7735 \\ -30+31,20+2,2222 \\ 7,4074+0 \\ -11,5470+33,5410 \\ -4,4444+33,5410 \\ 0 \\ -33,5410 \\ -33,5410 \end{bmatrix} = \begin{bmatrix} 0 \\ -8,32 \\ -12,8 \\ 27,5926 \\ -87,4535 \\ 3,4222 \\ 7,4074 \\ -45,088 \\ 29,0966 \\ 0 \\ -33,5410 \\ -33,5410 \end{bmatrix} \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12 \end{array}$$

Vetor das reações sistema de eixos global	Vetor dos deslocamentos sistema de eixos global
$$[R_G] = \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12 \end{array} \begin{bmatrix} \mathbf{R1}\\ \mathbf{R2}\\ \mathbf{R3}\\ \hline 0\\0\\0\\ \hline 0\\0\\0\\ \hline \mathbf{R10}\\ \mathbf{R11}\\ 0 \end{bmatrix}$$	$$[\Delta_G] = \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12 \end{array} \begin{bmatrix} 0\\0\\ -0{,}002\\ \hline \boldsymbol{\Delta 4}\\ \boldsymbol{\Delta 5}\\ \boldsymbol{\Delta 6}\\ \hline \boldsymbol{\Delta 7}\\ \boldsymbol{\Delta 8}\\ \boldsymbol{\Delta 9}\\ \hline 0\\0\\ \boldsymbol{\Delta 12} \end{bmatrix}$$

• MATRIZ DE RIGIDEZ GLOBAL DA ESTRUTURA

$$[K_G]^{Est} \cdot [\Delta_G] = [F_G] + [R_G]$$

$$\begin{array}{c} \\ 4\\5\\6\\7\\8\\9\\12\\1\\2\\3\\10\\11 \end{array} \begin{array}{c} 4 \;\; 5 \;\; 6 \;\; 7 \;\; 8 \;\; 9 \;\; 12 \;\; 1 \;\; 2 \;\; 3 \;\; 10 \;\; 11 \\ \left[\begin{array}{c|c} K_{LL} & K_{LF} \\ \hline K_{FL} & K_{FF} \end{array}\right] \end{array} * \begin{bmatrix} \Delta_{Li} \\ \hline \Delta_F \end{bmatrix} = \begin{bmatrix} F_{Li} \\ \hline F_F \end{bmatrix} + \begin{bmatrix} 0 \\ \hline R_F \end{bmatrix}$$

$$\begin{cases} [K_{LL}] \cdot [\Delta_{Li}] + [K_{LF}] \cdot [\Delta_F] = [F_L] & \Rightarrow \quad [\Delta_{Li}] \\ [K_{FL}] \cdot [\Delta_{Li}] + [K_{FF}] \cdot [\Delta_F] = [F_F] + [R_F] & \end{cases}$$

Na página seguinte estão representados o sistema $[K_G]^{Est} \cdot [\Delta_G] = [F_G] + [R_G]$ na forma inicial, com os graus de liberdade ordenados de forma crescente do 1 ao 12, e na forma rearranjada, separando graus de liberdade sem restrições de deslocamento (livres): 4, 5, 6, 7, 8, 9 e 12 e graus de liberdade com restrições de deslocamento (fixos): 1, 2, 3, 10 e 11.

K_G - Estrutura * Δ_G = F_G + R_G

	1	2	3	4	5	6	7	8	9	10	11	12	Δ_G	F_G	R_G
1	600 000	0	0	-600 000	0	0	0	0	0	0	0	0	0	0	R1
2	0	3 840	9 600	0	-3 840	9 600	0	0	0	0	0	0	0	-8,32	R2
3	0	9 600	32 000	0	-9 600	16 000	0	0	0	0	0	0	-0,002	-12,8	R3
4	-600 000	0	0	617 777,78	0	26 666,67	-17 777,78	0	26 666,67	0	0	0	Δ4	27,5926	0
5	0	-3 840	-9 600	0	1003 840	-9 600	0	-1000 000	0	0	0	0	Δ5	-87,4535	0
6	0	9 600	16 000	26 666,67	-9 600	85 333,33	-26 666,67	0	26 666,67	0	0	0	Δ6	3,4222	0
7	0	0	0	-17 777,78	0	-26 666,67	375 866,67	178 249,40	-24 281,53	-358 088,89	-178 249,40	2 385,14	Δ7	7,4074	0
8	0	0	0	0	-1000 000	0	178 249,40	1090 714,79	-4 770,28	-178 249,40	-90 714,79	-4 770,28	Δ8	-45,0880	0
9	0	0	0	26 666,67	0	26 666,67	-24 281,53	-4 770,28	77 184,73	-2 385,14	4 770,28	11 925,70	Δ9	29,0966	0
10	0	0	0	0	0	0	-358 088,89	-178 249,40	-2 385,14	358 088,89	178 249,40	-2 385,14	0	0	R10
11	0	0	0	0	0	0	-178 249,40	-90 714,79	4 770,28	178 249,40	90 714,79	4 770,28	0	-33,5410	R11
12	0	0	0	0	0	0	2 385,14	-4 770,28	11 925,70	-2 385,14	4 770,28	23 851,39	Δ12	-33,5410	0

K_G - Estrutura * Δ_G = F_G + R_G

	4	5	6	7	8	9	12	1	2	3	10	11	Δ_G	F_G	R_G
4	617 777,78	0	26 666,67	-17 777,78	0	26 666,67	0	-600 000	0	0	0	0	Δ4	27,5926	0
5	0	1003 840	-9 600	0	-1000 000	0	0	0	-3 840	-9 600	0	0	Δ5	-87,4535	0
6	26 666,67	-9 600	85 333,33	-26 666,67	0	26 666,67	0	0	9 600	16 000	0	0	Δ6	3,4222	0
7	-17 777,78	0	-26 666,67	375 866,67	178 249,40	-24 281,53	2 385,14	0	0	0	-358 088,89	-178 249,40	Δ7	7,4074	0
8	0	-1000 000	0	178 249,40	1090 714,79	-4 770,28	-4 770,28	0	0	0	-178 249,40	-90 714,79	Δ8	-45,0880	0
9	26 666,67	0	26 666,67	-24 281,53	-4 770,28	77 184,73	11 925,70	0	0	0	-2 385,14	4 770,28	Δ9	29,0966	0
12	0	0	0	2 385,14	-4 770,28	11 925,70	23 851,39	0	0	0	-2 385,14	4 770,28	Δ12	-33,5410	0
1	-600 000	0	0	0	0	0	0	600 000	0	0	0	0	0	0	R1
2	0	-3 840	9 600	0	0	0	0	0	3 840	9 600	0	0	0	-8,32	R2
3	0	-9 600	16 000	0	0	0	0	0	9 600	32 000	0	0	-0,002	-12,8	R3
10	0	0	0	-358 088,89	-178 249,40	-2 385,14	-2 385,14	0	0	0	358 088,89	178 249,40	0	0	R10
11	0	0	0	-178 249,40	-90 714,79	4 770,28	4 770,28	0	0	0	178 249,40	90 714,79	0	-33,5410	R11

$$[K_{LL}] \ast [\Delta_{Li}] + [K_{LF}] \ast [\Delta_F] = [F_L]$$

K_{LL}

	4	5	6	7	8	9	12
4	617 777.78	0	26 666.67	-17 777.78	0	26 666.67	0
5	0	1003 840	-9 600	0	-1000 000	0	0
6	26 666.67	-9 600	85 333.33	-26 666.67	0	26 666.67	0
7	-17 777.78	0	-26 666.67	375 866.67	178 249.40	-24 281.53	2 385.14
8	0	-1000 000	0	178 249.40	1090 714.79	-4 770.28	-4 770.28
9	26 666.67	0	26 666.67	-24 281.53	-4 770.28	77 184.73	11925.70
12	0	0	0	2 385.14	-4 770.28	11925.70	23 851.39

Δ_{Li}

4	Δ4
5	Δ5
6	Δ6
7	Δ7
8	Δ8
9	Δ9
12	Δ12

K_{LF}

	1	2	3	10	11
4	-600 000	0	0	0	0
5	0	-3 840	-9 600	0	0
6	0	9 600	16 000	0	0
7	0	0	0	-358 088.89	-178 249.40
8	0	0	0	-178 249.40	-90 714.79
9	0	0	0	-2 385.14	4 770.28
12	0	0	0	-2 385.14	4 770.28

Δ_F

1	0
2	0
3	-0.002
10	0
11	0

F_L

4	27.5926
5	-87.4535
6	3.4222
7	7.4074
8	-45.0880
9	29.0966
12	-33.5410

Δ4 =	2.0706E-04	m
Δ5 =	-2.3775E-02	m
Δ6 =	8.9026E-05	rad
Δ7 =	1.1614E-02	m
Δ8 =	-2.3761E-02	m
Δ9 =	3.8915E-03	rad
Δ12 =	-9.2656E-03	rad

$$[K_{FL}] \ast [\Delta_{Li}] + [K_{FF}] \ast [\Delta_F] = [F_F] + [R_F]$$

K_{FL}

	4	5	6	7	8	9	12
1	-600 000	0	0	0	0	0	0
2	0	-3 840	9 600	0	0	0	0
3	0	-9 600	16 000	0	0	0	0
10	0	0	0	-358 088.89	-178 249.40	-2 385.14	-2 385.14
11	0	0	0	-178 249.40	-90 714.79	4 770.28	4 770.28

Δ_{Li}

4	2.0706E-04
5	-2.3775E-02
6	8.9026E-05
7	1.1614E-02
8	-2.3761E-02
9	3.8915E-03
12	-9.2656E-03

K_{FF}

	1	2	3	10	11
1	600 000	0	0	0	0
2	0	3 840	9 600	0	0
3	0	9 600	32 000	0	0
10	0	0	0	358 088.89	178 249.40
11	0	0	0	178 249.40	90 714.79

Δ_F

1	0
2	0
3	-0.002
10	0
11	0

F_F

1	0
2	-8.32
3	-12.8
10	0
11	-33.54102

R_F

1	R 1
2	R 2
3	R 3
10	R 10
11	R 11

R1 =	-124.239	kN
R2 =	81.271	kN
R3 =	178.466	kNm
R8 =	89.239	kN
R10 =	93.131	kN

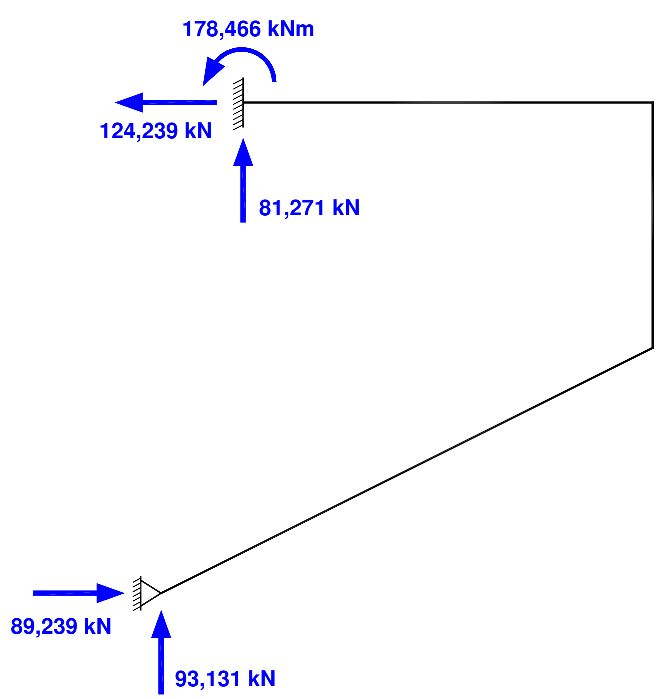
Deslocamentos dos nós da estrutura

$\Delta1 = 0$	
$\Delta2 = 0$	
$\Delta3 = -2{,}000 \times 10^{-3}$ rad	↻
$\Delta4 = 2{,}071 \times 10^{-4}$ m	→
$\Delta5 = -2{,}378 \times 10^{-2}$ m	↓
$\Delta6 = 8{,}903 \times 10^{-5}$ rad	↺
$\Delta7 = 1{,}161 \times 10^{-2}$ m	→
$\Delta8 = -2{,}376 \times 10^{-2}$ m	↓
$\Delta9 = 3{,}892 \times 10^{-3}$ rad	↺
$\Delta10 = 0$	
$\Delta11 = 0$	
$\Delta12 = -9{,}266 \times 10^{-3}$ rad	↻

ALÍNEA b)

Reacções nos apoios

R1 = H1 =	-124,239 kN	←
R2 = V1 =	81,271 kN	↑
R3 = M1 =	178,466 kNm	↺
R10 = H4 =	89,239 kN	→
R11 = V4 =	93,131 kN	↑



ALÍNEA c)

Esforços nas barras

$$[F_L] = [K_L] \,.\, [\Delta_L] - [F_{L\text{-}EN}]$$

$$[\Delta_L] = [T] \,.\, [\Delta_G]$$

$$[F_L] = [K_L] \,.\, [T] \,.\, [\Delta_G] - [F_{L\text{-}EN}]$$

BARRA 1 **Nós 1 - 2**

$\Delta L1 \Leftrightarrow \Delta G1 = 0$

$\Delta L2 \Leftrightarrow \Delta G2 = 0$

$\Delta L3 \Leftrightarrow \Delta G3 = -2{,}000 \times 10^{-3}$

$\Delta L4 \Leftrightarrow \Delta G4 = 2{,}071 \times 10^{-4}$

$\Delta L5 \Leftrightarrow \Delta G5 = -2{,}378 \times 10^{-2}$

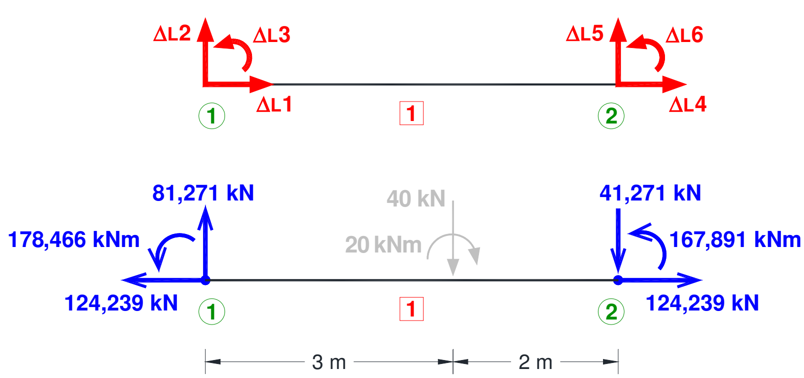
$\Delta L6 \Leftrightarrow \Delta G6 = 8{,}903 \times 10^{-5}$

F_L	=	***K_L***						*		**Δ *L***	-		***F_L-EN***
F_1		600 000	0	0	-600 000	0	0		1	0		1	0
F_2		0	3 840	9 600	0	-3 840	9 600		2	0		2	-8,32
F_3		0	9 600	32 000	0	-9 600	16 000		3	$-2{,}000 \times 10^{-3}$		3	-12,8
F_4		-600 000	0	0	600 000	0	0		4	$2{,}071 \times 10^{-4}$		4	0
F_5		0	-3840	-9 600	0	3840	-9 600		5	$-2{,}378 \times 10^{-2}$		5	-31,68
F_6		0	9 600	16 000	0	-9 600	32 000		6	$8{,}903 \times 10^{-5}$		6	31,2

Barra 1 - *Nós 1 - 2*

F_L - LOCAL

F_1	=	-124,239	kN
F_2		81,271	kN
F_3		178,466	Nm
F_4		124,239	kN
F_5		-41,271	kN
F_6		167,891	Nm



BARRA 2 Nós 2 - 3

$\Delta L1 \Leftrightarrow -\Delta G5 = 2{,}378 \times 10^{-2}$

$\Delta L2 \Leftrightarrow -\Delta G4 = 2{,}071 \times 10^{-4}$

$\Delta L3 \Leftrightarrow \Delta G6 = 8{,}903 \times 10^{-5}$

$\Delta L4 \Leftrightarrow -\Delta G8 = 2{,}376 \times 10^{-2}$

$\Delta L5 \Leftrightarrow -\Delta G7 = 1{,}161 \times 10^{-2}$

$\Delta L6 \Leftrightarrow \Delta G9 = 3{,}891 \times 10^{-3}$

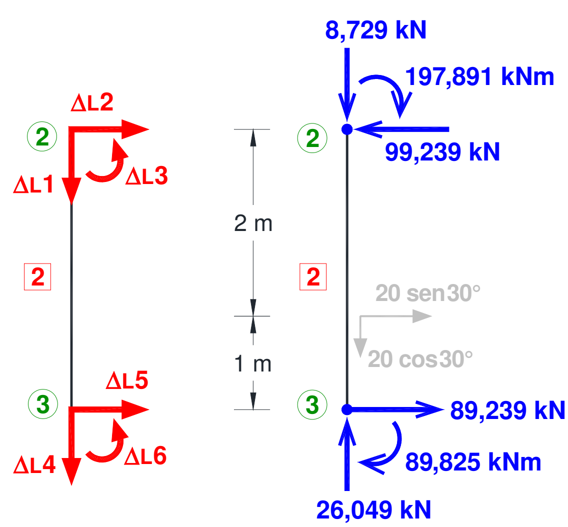
$F_L = K_L * \Delta_L - F_{L-EN}$

F_L	K_L							Δ_L		F_{L-EN}
F_1	1000 000	0	0	-1000 000	0	0	1	2,378E-02	1	5,7735
F_2	0	17 777,78	26 666,67	0	-17 777,78	26 666,67	2	2,071E-04	2	2,5926
F_3	0	26 666,67	53 333,33	0	-26 666,67	26 666,67	3	8,903E-05	3	2,2222
F_4	-1000 000	0	0	1000 000	0	0	4	2,376E-02	4	11,5470
F_5	0	-17 777,78	-26 666,67	0	17 777,78	-26 666,67	5	1,161E-02	5	7,4074
F_6	0	26 666,67	26 666,67	0	-26 666,67	53 333,33	6	3,891E-03	6	-4,4444

Barra 2 - *Nós 2 - 3*

F_L - LOCAL

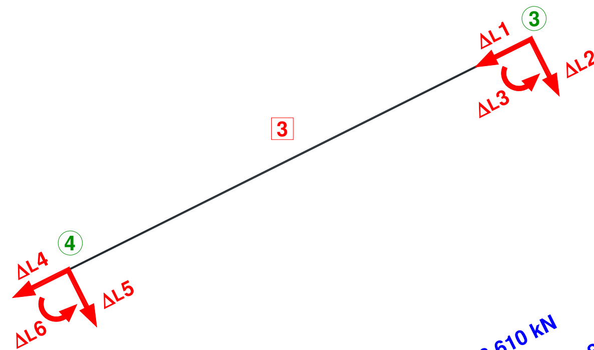
F_1	=	8,729	kN
F_2		-99,239	kN
F_3		-197,891	Nm
F_4		-26,049	kN
F_5		89,239	kN
F_6		-89,825	Nm



BARRA 3 Nós 3 - 4

$F_L = K_L * T * \Delta_G - F_{L\text{-}EN}$

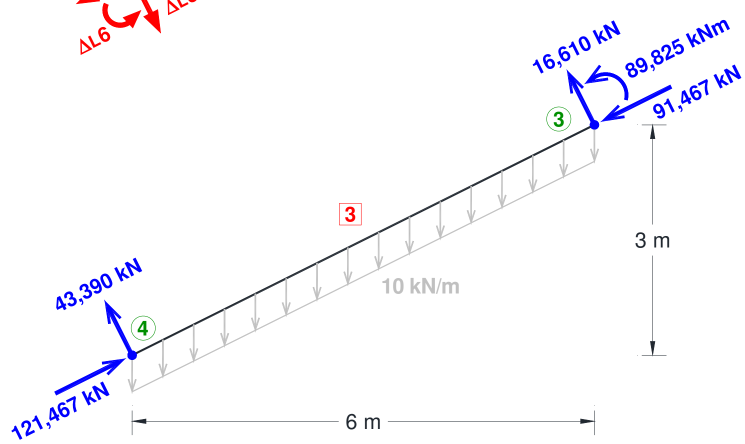
F_L	K_L						T							Δ_G		$F_{L\text{-}EN}$
F_1	447 213,60	0	0	-447 213,60	0	0	-0,8944	-0,4472	0	0	0	0	7	1,161E-02	1	15
F_2	0	1 590,09	5 333,33	0	-1 590,09	5 333,33	0,4472	-0,8944	0	0	0	0	8	-2,376E-02	2	30
F_3	0	5 333,33	23 851,39	0	-5 333,33	11 925,70	0	0	1	0	0	0	9	3,891E-03	3	33,5410
F_4	-447 213,60	0	0	447 213,60	0	0	0	0	0	-0,8944	-0,4472	0	10	0	4	15
F_5	0	-1 590,09	-5 333,33	0	1 590,09	-5 333,33	0	0	0	0,4472	-0,8944	0	11	0	5	30
F_6	0	5 333,33	11 925,70	0	-5 333,33	23 851,39	0	0	0	0	0	1	12	-9,266E-03	6	-33,5410

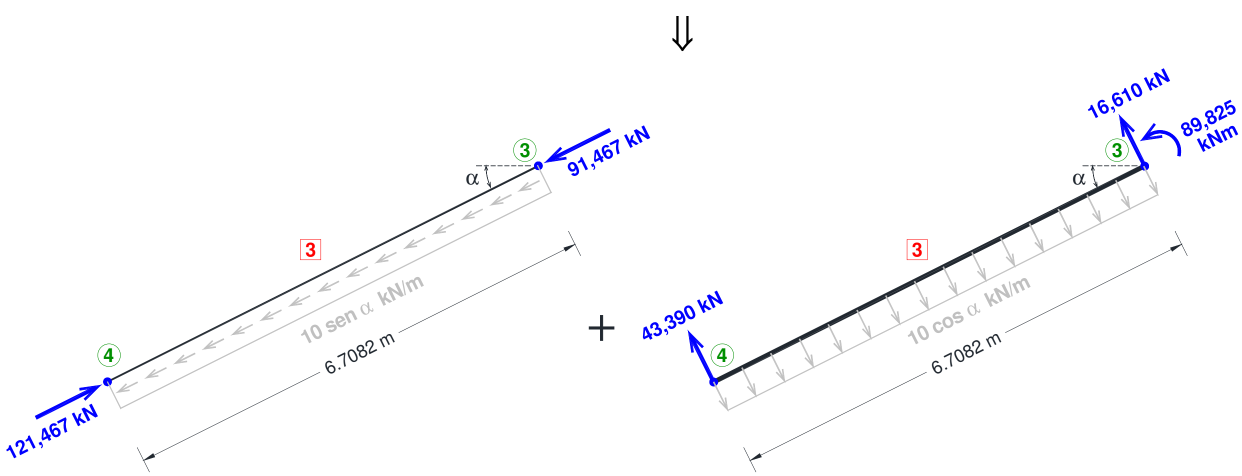


Barra 3 - Nós 3 - 4

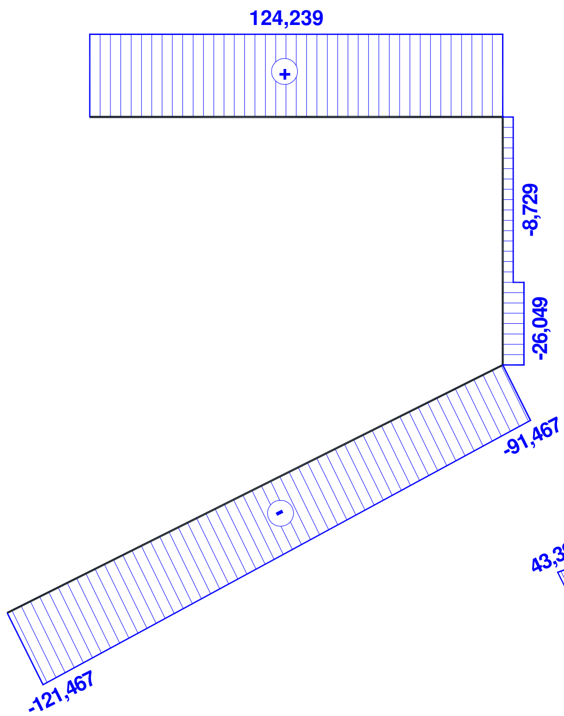
F_L - LOCAL

F_1	=	91,467	kN
F_2		-16,610	kN
F_3		89,825	Nm
F_4		-121,467	kN
F_5		-43,390	kN
F_6		0	Nm

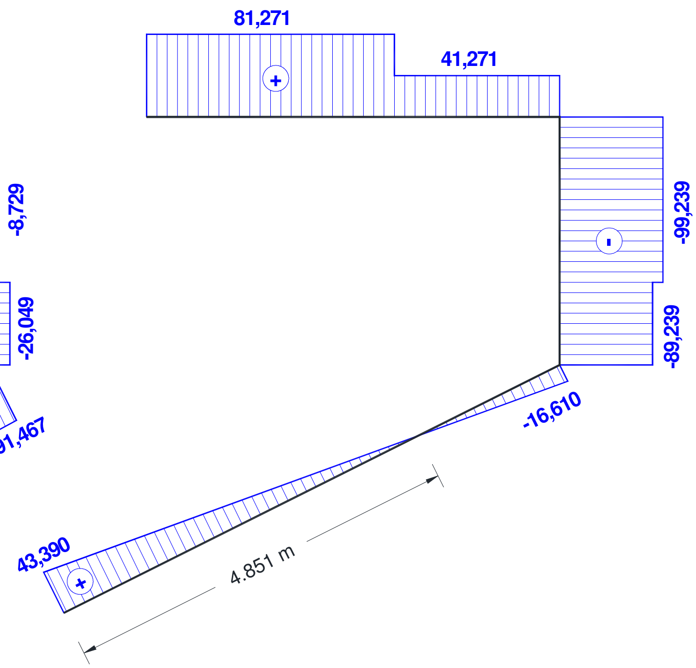




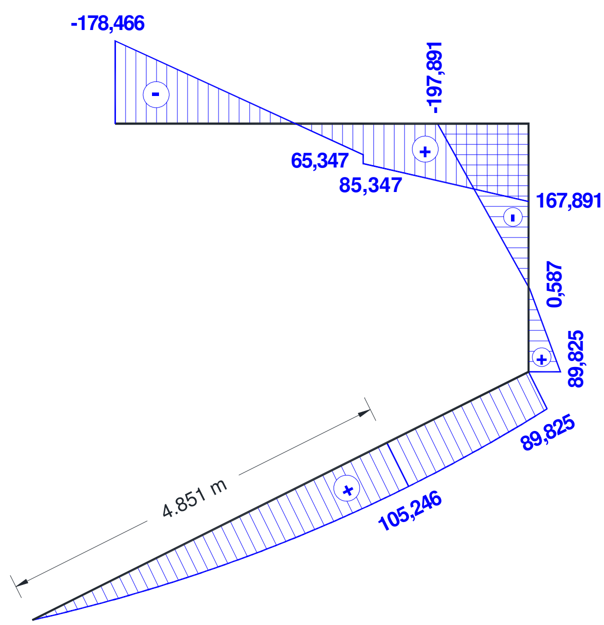
Diagramas de esforços



ESFORÇO AXIAL (kN)



ESFORÇO TRANSVERSO (kN)



MOMENTOS FLECTORES (kNm)