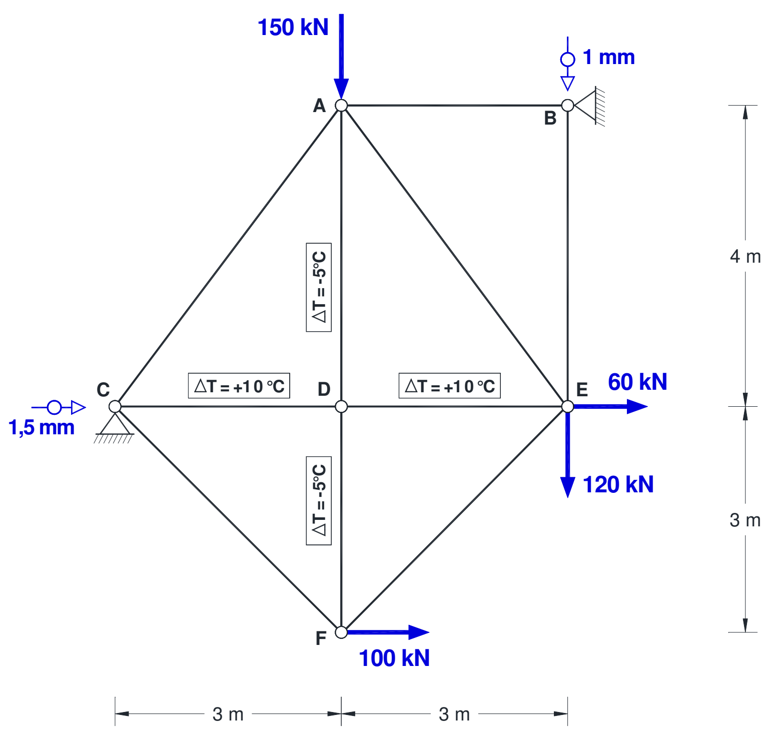
LICENCIATURA EM ENGENHARIA CIVIL

TEORIA DE ESTRUTURAS

MÉTODO DAS FORÇAS



SISTEMA ARTICULADO PLANO (SAP) HIPERESTÁTICO

ISABEL ALVIM TELES

EXERCÍCIO PROPOSTO

Considere a estrutura articulada plana representada na figura.

Todas as barras são constituídas por perfis cuja secção transversal apresenta uma área de 10 cm^2.

Para além dos deslocamentos dos apoios e das forças nos nós indicadas, algumas barras estão submetidas a variações uniformes de temperatura:

barras **AD** e **DF**: $\Delta T = -5$ °C

barras **CD** e **DE**: $\Delta T = +10$ °C

Características do material constituinte das barras: E = 200 GPa

$\alpha = 1,5 \times 10^{-5}$ /°C

Resolva as alíneas seguintes aplicando o Método das Forças.

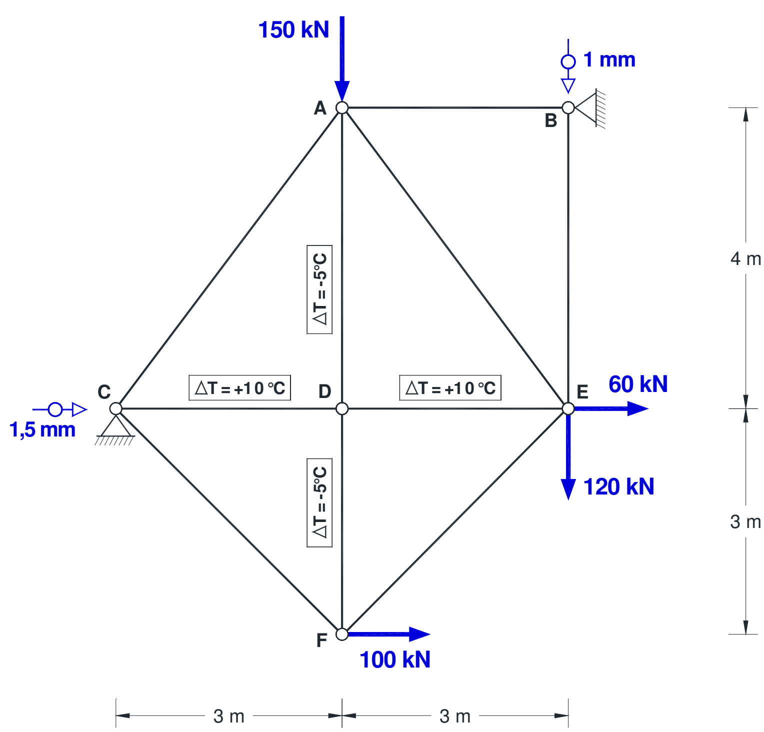
a) Determine as reações nos apoios e os esforços instalados em todas as barras;

b) Determine o deslocamento do nó **E**;

c) Determine a rotação da barra **BE**;

d) Determine o deslocamento vertical do nó **A**;

e) Determine qual deveria ser o assentamento vertical do apoio **C** para que a barra **AB** se mantivesse horizontal após deformação;



RESOLUÇÃO

Alínea a)

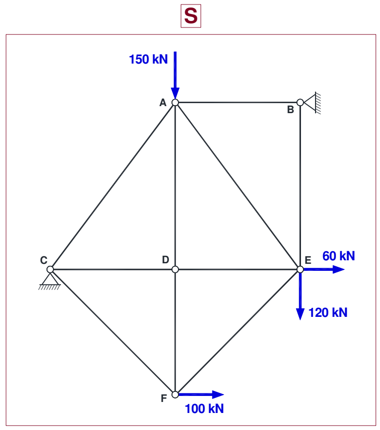
A estrutura é 1 vez hiperestática por condições externas e 1 vez hiperestática por condições internas. Logo a estrutura é hiperestática de grau 2.

O sistema base (**S_0**) adotado na resolução será a estrutura isostática que se obtém substituindo o apoio duplo em **C** por um apoio simples (suprimiu-se a incógnita correspondente à reação horizontal em **C**) e eliminando-se a barra **EF**.

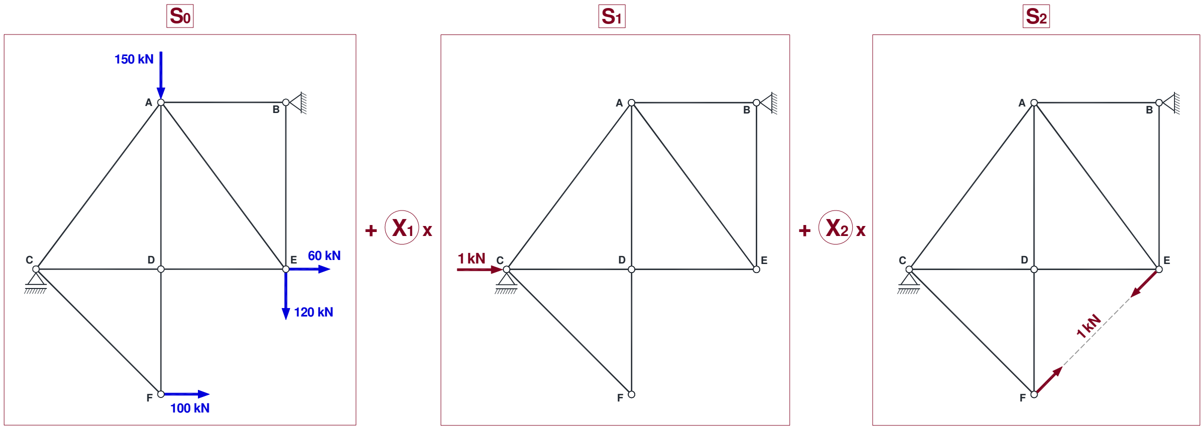
A incógnita hiperestática **X_1** corresponderá à reação horizontal vertical do apoio do nó **C** e a incógnita hiperestática **X_2** corresponderá ao esforço axial da barra **EF**.

$$\mathbf{S = S_0 + X_1 \times S_1 + X_2 \times S_2}$$

S

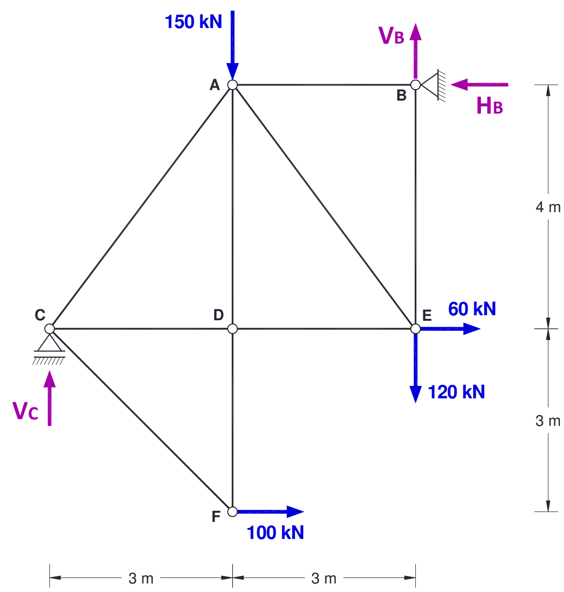


S0 S1 S2

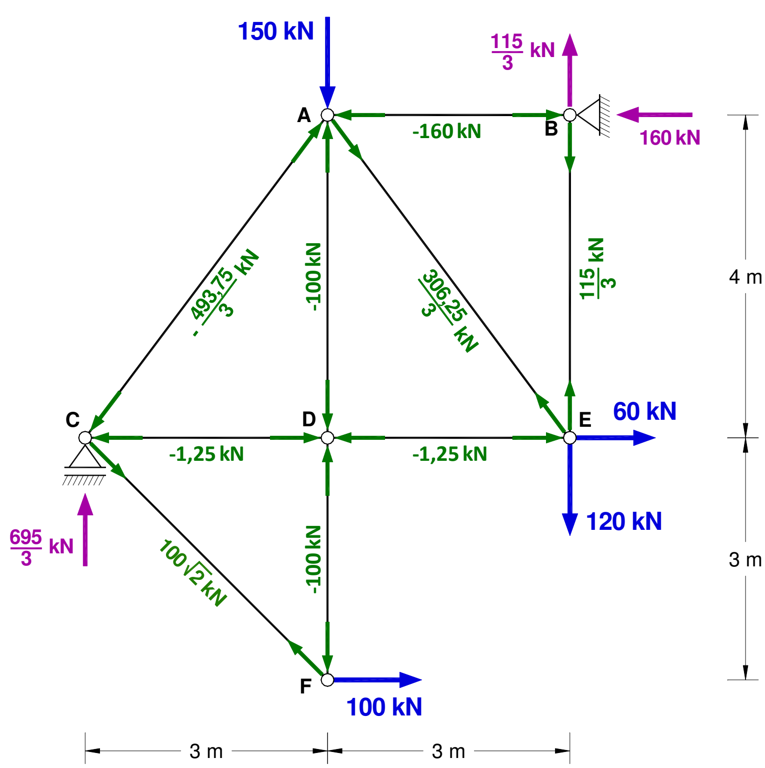


• Cálculo da estrutura S_0

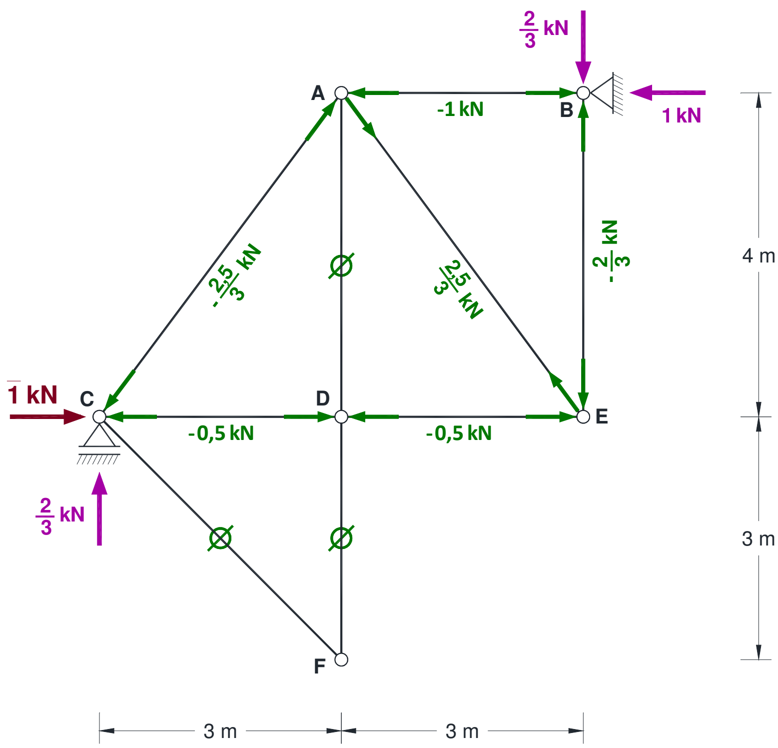
$$\begin{cases} \sum F_X = 0 \Rightarrow H_B = 100 + 60 \\ \sum F_Y = 0 \Rightarrow V_B + V_C = 150 + 120 \\ \sum M_B = 0 \Rightarrow 100\times7 + 60\times4 + 150\times3 = 6\,V_C \end{cases}$$



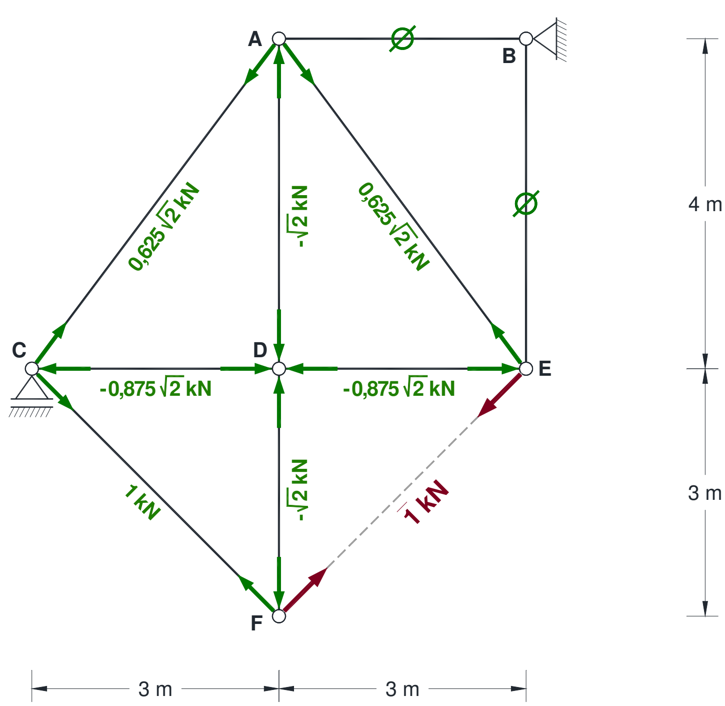
$$\begin{cases} H_B = 160 \text{ kN} \leftarrow \\ V_B = \dfrac{115}{3} \text{ kN} \uparrow \\ V_C = \dfrac{695}{3} \text{ kN} \uparrow \end{cases}$$



- **Cálculo da estrutura S_1**



- **Cálculo da estrutura S_2**



MÉTODO DAS FORÇAS	$\begin{cases} \sum F_{ext}^{S1} \times \Delta_R = \sum N_1 \frac{NL}{EA} + \sum N_1\, \alpha\, \Delta T\, L \\ \sum F_{ext}^{S2} \times \Delta_R = \sum N_2 \frac{NL}{EA} + \sum N_2\, \alpha\, \Delta T\, L \end{cases}$

Considerando:

$N = N_0 + \mathbf{X_1} N_1 + \mathbf{X_2} N_2$

$$\Rightarrow \begin{cases} \sum F_{ext}^{S1} \times \Delta_R = \sum N_1 \frac{N_0 L}{EA} + \sum N_1 \frac{N_1 L}{EA}.\mathbf{X_1} + \sum N_1 \frac{N_2 L}{EA}.\mathbf{X_2} + \sum N_1\, \alpha\, \Delta T\, L \\ \sum F_{ext}^{S2} \times \Delta_R = \sum N_2 \frac{N_0 L}{EA} + \sum N_2 \frac{N_1 L}{EA}.\mathbf{X_1} + \sum N_2 \frac{N_2 L}{EA}.\mathbf{X_2} + \sum N_2\, \alpha\, \Delta T\, L \end{cases}$$

$$\begin{cases} \delta_{10} + \delta_{11}.\, \mathbf{X_1} + \delta_{12}.\, \mathbf{X_2} = 0 \\ \delta_{20} + \delta_{21}.\, \mathbf{X_1} + \delta_{22}.\, \mathbf{X_2} = 0 \end{cases}$$

sendo:

$\delta_{10} = \sum N_1 \frac{N_0 L}{EA} + \sum N_1\, \alpha\, \Delta T\, L - \sum F_{ext}^{S1} \times \Delta_R$	$\delta_{11} = \sum N_1 \frac{N_1 L}{EA}$	$\delta_{12} = \sum N_1 \frac{N_2 L}{EA}$
$\delta_{20} = \sum N_2 \frac{N_0 L}{EA} + \sum N_2\, \alpha\, \Delta T\, L - \sum F_{ext}^{S2} \times \Delta_R$	$\delta_{21} = \sum N_2 \frac{N_1 L}{EA}$	$\delta_{22} = \sum N_2 \frac{N_2 L}{EA}$

E A = 200 x 10^6 x 10 x 10^{-4} = 2 x 10^5 kPa x m^2 α = 1,5 x 10^{-5} /°C

BARRAS	L (m)	N_0 (kN)	N_1 (kN)	N_2 (kN)	$N_1 \frac{N_0 L}{EA}$	$N_2 \frac{N_0 L}{EA}$	$N_1 \frac{N_1 L}{EA}$	$N_1 \frac{N_2 L}{EA}$	$N_2 \frac{N_2 L}{EA}$	$N_1.\alpha.\Delta T.L$	$N_2.\alpha.\Delta T.L$
AB	3	-160	-1	0	2,400x10^{-3}	0	1,500x10^{-5}	0	0	-	-
AC	5	$-\frac{493,75}{3}$	$-\frac{2,5}{3}$	$0,625\sqrt{2}$	3,429x10^{-3}	-3,63 x10^{-3}	1,736x10^{-5}	-1,841x10^{-5}	1,953x10^{-5}	-	-
AD	4	-100	0	$-\sqrt{2}$	0	2,828x10^{-3}	0	0	4,000x10^{-5}	0	4,243x10^{-4}
AE	5	$\frac{306,25}{3}$	$\frac{2,5}{3}$	$0,625\sqrt{2}$	2,127x10^{-3}	2,256x10^{-3}	1,736x10^{-5}	1,841x10^{-5}	1,953x10^{-5}	-	-
BE	4	$\frac{115}{3}$	$-\frac{2}{3}$	0	-5.111x10^{-4}	0	8,889x10^{-6}	0	0	-	-
CD	3	-1.25	-0,5	$-0,875\sqrt{2}$	9.375x10^{-6}	2,320x10^{-5}	3,750x10^{-6}	9,281x10^{-6}	2,297x10^{-5}	-2,25x10^{-4}	-5,568x10^{-4}
DE	3	-1.25	-0,5	$-0,875\sqrt{2}$	9.375x10^{-6}	2,320x10^{-5}	3,750x10^{-6}	9,281x10^{-6}	2,297x10^{-5}	-2,25x10^{-4}	-5,568x10^{-4}
CF	$3\sqrt{2}$	$100\sqrt{2}$	0	1	0	3,000x10^{-3}	0	0	2,121x10^{-5}	-	-
DF	3	-100	0	$-\sqrt{2}$	0	2,121x10^{-3}	0	0	3,000x10^{-5}	0	3,182x10^{-4}
EF	$3\sqrt{2}$	-	-	1	0	0	0	0	2,121x10^{-5}	-	-
Σ					7,463x10^{-3}	6,615x10^{-3}	6,611x10^{-5}	1,856x10^{-5}	1,974x10^{-4}	-4,50x10^{-4}	-3,712x10^{-4}

$$\delta_{10} = \sum N_1 \frac{N_0 L}{EA} + \sum N_1 \alpha \Delta T L - \sum F_{ext}^{S1} \times \Delta_R =$$

$$= 7{,}463 \times 10^{-3} - 4{,}5 \times 10^{-4} - \left(\frac{2}{3} \times 0{,}001 + 1 \times 0{,}0015\right) = 4{,}846 \times 10^{-3}$$

$$\delta_{20} = \sum N_2 \frac{N_0 L}{EA} + \sum N_2 \alpha \Delta T L - \sum F_{ext}^{S2} \times \Delta_R =$$

$$= 6{,}615 \times 10^{-3} - 3{,}712 \times 10^{-4} - 0 = 6{,}244 \times 10^{-3}$$

$$\delta_{11} = \sum N_1 \frac{N_1 L}{EA} = 6{,}611 \times 10^{-5}$$

$$\delta_{12} = \delta_{21} = \sum N_1 \frac{N_2 L}{EA} = \sum N_2 \frac{N_1 L}{EA} = 1{,}856 \times 10^{-5}$$

$$\delta_{22} = \sum N_2 \frac{N_2 L}{EA} = 1{,}974 \times 10^{-4}$$

$$\begin{cases} \delta_{10} + \delta_{11} \cdot \mathbf{X_1} + \delta_{12} \cdot \mathbf{X_2} = 0 \\ \delta_{20} + \delta_{21} \cdot \mathbf{X_1} + \delta_{22} \cdot \mathbf{X_2} = 0 \end{cases} \Rightarrow \begin{cases} 4{,}846 \times 10^{-3} + 6{,}611 \times 10^{-5} \times \mathbf{X_1} + 1{,}856 \times 10^{-5} \times \mathbf{X_2} = 0 \\ 6{,}244 \times 10^{-3} + 1{,}856 \times 10^{-5} \times \mathbf{X_1} + 1{,}974 \times 10^{-4} \times \mathbf{X_2} = 0 \end{cases}$$

$$\begin{cases} \mathbf{X_1} = -66{,}17 \\ \mathbf{X_2} = -25{,}41 \end{cases}$$

$$\begin{cases} H_B = (H_B)^{S0} + \mathbf{X_1} \cdot (H_B)^{S1} + \mathbf{X_2} \cdot (H_B)^{S2} \\ V_B = (V_B)^{S0} + \mathbf{X_1} \cdot (V_B)^{S1} + \mathbf{X_2} \cdot (V_B)^{S2} \\ H_C = (H_C)^{S0} + \mathbf{X_1} \cdot (H_C)^{S1} + \mathbf{X_2} \cdot (H_C)^{S2} \\ V_C = (V_C)^{S0} + \mathbf{X_1} \cdot (V_C)^{S1} + \mathbf{X_2} \cdot (V_C)^{S2} \end{cases} \Rightarrow \begin{cases} H_B = -160 + (-66{,}17) \times (-1) + (-25{,}41) \times 0 \\ V_B = \frac{115}{3} + (-66{,}17) \times (-\frac{2}{3}) + (-25{,}41) \times 0 \\ H_C = 0 + (-66{,}17) \times 1 + (-25{,}41) \times 0 \\ V_C = \frac{695}{3} + (-66{,}17) \times \frac{2}{3} + (-25{,}41) \times 0 \end{cases}$$

$$\begin{cases} H_B = -93{,}83 \text{ kN} \leftarrow \\ V_B = 82{,}45 \text{ kN} \uparrow \\ H_C = -88{,}17 \text{ kN} \leftarrow \\ V_C = 187{,}55 \text{ kN} \uparrow \end{cases}$$

• Esforços nas barras

$$N_{barra} = (N_{barra})^{S0} + \mathbf{X_1}.(N_{barra})^{S1} + \mathbf{X_2}.(N_{barra})^{S2}$$

$$N_{AB} = -160 + (-66{,}17)\times(-1) + (-25{,}41)\times 0 = -93{,}83\text{ kN} \quad \text{(compressão)}$$

$$N_{AC} = -\frac{493{,}75}{3} + (-66{,}17)\times(-\frac{2{,}5}{3}) + (-25{,}41)\times 0{,}625\sqrt{2} = -131{,}90\text{ kN} \quad \text{(compressão)}$$

$$N_{AD} = -100 + (-66{,}17)\times 0 + (-25{,}41)\times(-\sqrt{2}) = -64{,}06\text{ kN} \quad \text{(compressão)}$$

$$N_{AE} = \frac{306{,}25}{3} + (-66{,}17)\times\frac{2{,}5}{3} + (-25{,}41)\times 0{,}625\sqrt{2} = 24{,}48\text{ kN} \quad \text{(tracção)}$$

$$N_{BE} = \frac{115}{3} + (-66{,}17)\times(-\frac{2}{3}) + (-25{,}41)\times 0 = 82{,}45\text{ kN} \quad \text{(tracção)}$$

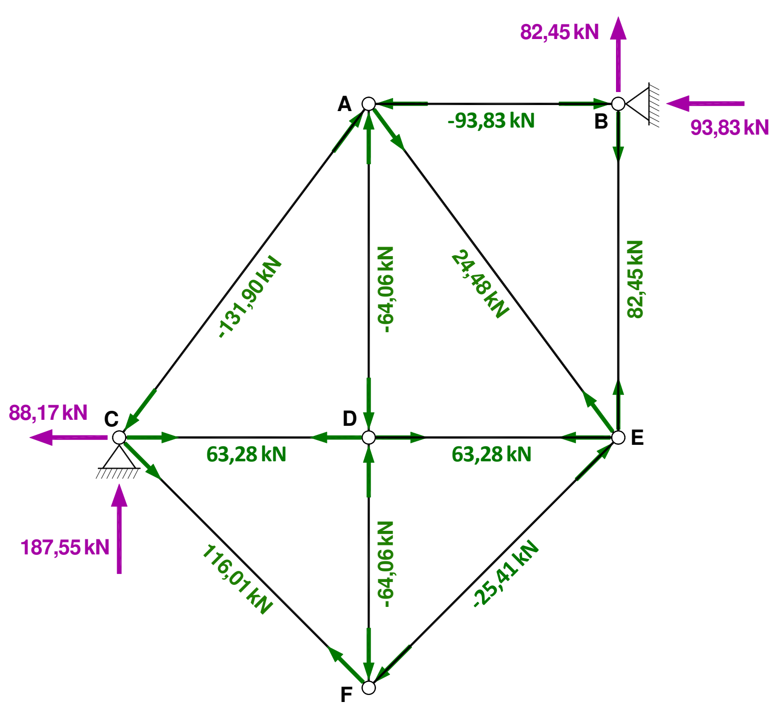
$$N_{CD} = -1{,}25 + (-66{,}17)\times(-0{,}5) + (-25{,}41)\times(-0{,}875\sqrt{2}) = 63{,}28\text{ kN} \quad \text{(tracção)}$$

$$N_{DE} = -1{,}25 + (-66{,}17)\times(-0{,}5) + (-25{,}41)\times(-0{,}875\sqrt{2}) = 63{,}28\text{ kN} \quad \text{(tracção)}$$

$$N_{CF} = 100\sqrt{2} + (-66{,}17)\times 0 + (-25{,}41)\times 1 = 116{,}01\text{ kN} \quad \text{(tracção)}$$

$$N_{DF} = -100 + (-66{,}17)\times 0 + (-25{,}41)\times(-\sqrt{2}) = -64{,}06\text{ kN} \quad \text{(compressão)}$$

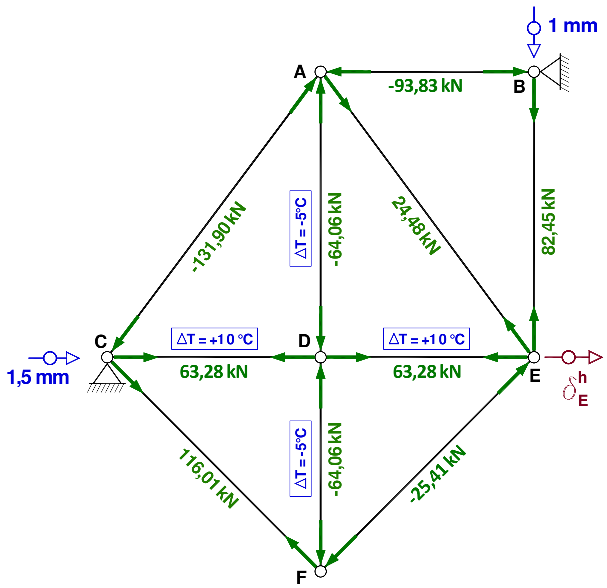
$$N_{EF} = 0 + (-66{,}17)\times 0 + (-25{,}41)\times 1 = -25{,}41\text{ kN} \quad \text{(compressão)}$$



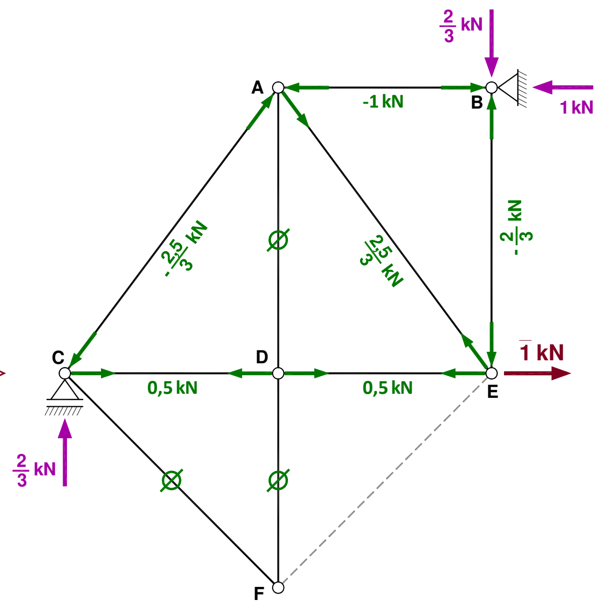
Alínea b)

• Deslocamento horizontal de E: δ_E^h

SISTEMA REAL



SISTEMA VIRTUAL



$E\,A = 200 \times 10^{6} \times 10 \times 10^{-4} = 2 \times 10^{5}$ kPa x m^2

$\alpha = 1{,}5 \times 10^{-5}$ /°C

BARRAS	L (m)	N (kN)	$\overline{N}$ (kN)	$\overline{N}\frac{NL}{EA}$	$\overline{N}.\alpha.\Delta T.L$
AB	3	-93,83	-1	2,400x10^{-3}	-
AC	5	-131,90	$-\frac{2,5}{3}$	3,429x10^{-3}	-
AD	4	-64,06	0	0	0
AE	5	24,48	$\frac{2,5}{3}$	2,127x10^{-3}	-
BE	4	82,45	$-\frac{2}{3}$	-5.111x10^{-4}	-
CD	3	63,28	-0,5	9.375x10^{-6}	2,25x10^{-4}
DE	3	63,28	-0,5	9.375x10^{-6}	2,25x10^{-4}
CF	$3\sqrt{2}$	116,01	0	0	-
DF	3	-64,06	0	0	0
EF	$3\sqrt{2}$	-25,41	-	0	-
Σ				4,515x10^{-3}	4,50x10^{-4}

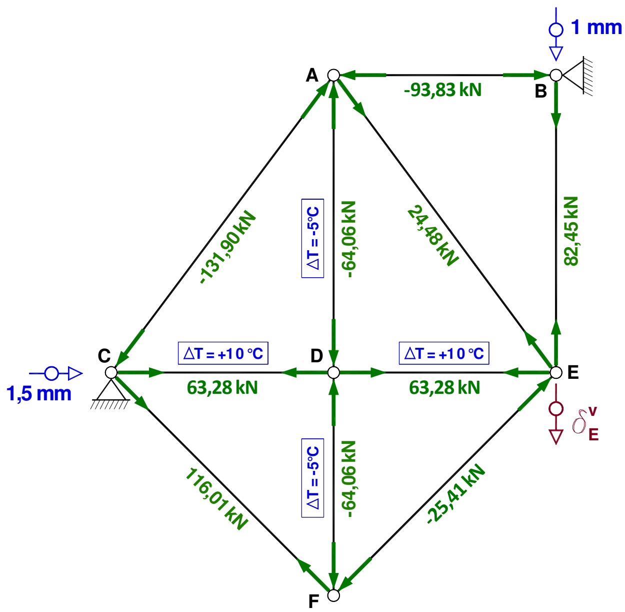
$$\overline{1} \times \delta_E^h + \Sigma\, \overline{R} \times \text{assent. apoio} = \Sigma\, \overline{N}\, \frac{N\,L}{E\,A} + \Sigma\, \overline{N}\, \alpha\, \Delta T\, L$$

$$\delta_E^h + \frac{2}{3} \times 0{,}001 = 4{,}515 \times 10^{-3} + 4{,}5 \times 10^{-4}$$

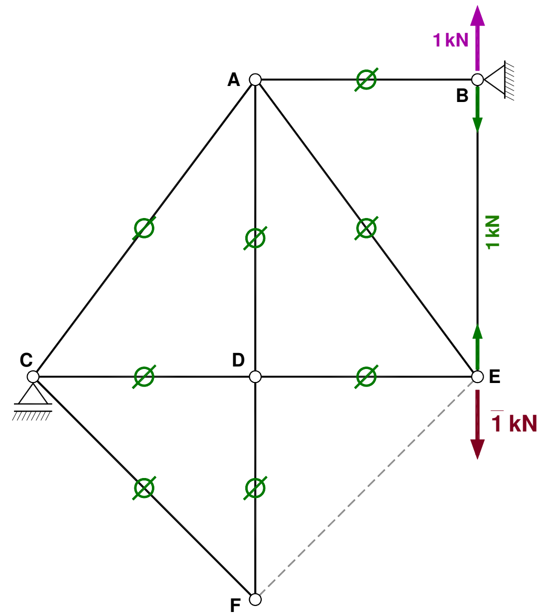
$$\delta_E^h = 4{,}40 \times 10^{-3} \text{ m} = 4{,}30 \text{ mm} \rightarrow$$

• Deslocamento vertical de E: δ_E^v

SISTEMA REAL



SISTEMA VIRTUAL



$E\,A = 200 \times 10^6 \times 10 \times 10^{-4} = 2 \times 10^5$ kPa x m²

$\alpha = 1{,}5 \times 10^{-5}$ /°C

BARRAS	L (m)	N (kN)	$\bar{N}$ (kN)	$\bar{N}\frac{NL}{EA}$	$\bar{N}.\alpha.\Delta T.L$
AB	3	-93,83	0	0	-
AC	5	-131,90	0	0	-
AD	4	-64,06	0	0	0
AE	5	24,48	0	0	-
BE	4	82,45	1	1.649x10^{-3}	-
CD	3	63,28	0	0	0
DE	3	63,28	0	0	0
CF	$3\sqrt{2}$	116,01	0	0	-
DF	3	-64,06	0	0	0
EF	$3\sqrt{2}$	-25,41	-	0	-
Σ				1,649x10^{-3}	0

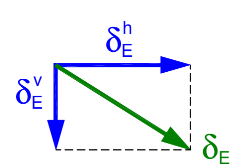
$$\bar{1} \times \delta_E^v + \Sigma\,\bar{R} \times \text{assent. apoio} = \Sigma\,\bar{N}\frac{NL}{EA} + \Sigma\,\bar{N}\,\alpha\,\Delta T\,L$$

$$\delta_E^v + 1\times(-0{,}001) = 1{,}649\times10^{-3} + 0$$

$$\delta_E^v = 2{,}65 \times 10^{-3}\ \text{m} = 2{,}65\ \text{mm}\ \downarrow$$

• Deslocamento do nó E

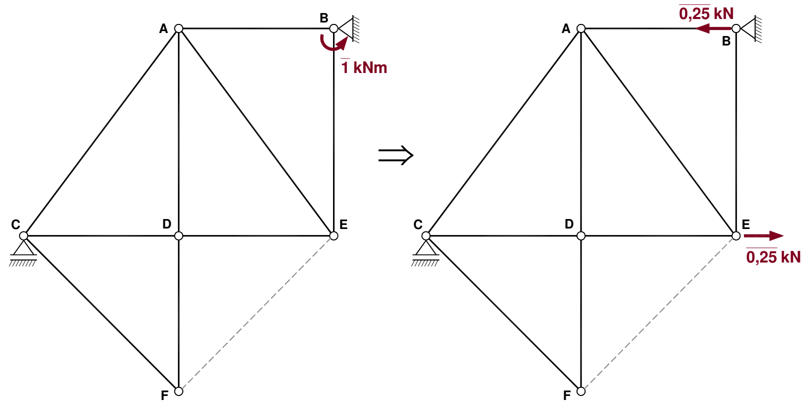
$$\delta_E = \sqrt{(\delta_E^h)^2 + (\delta_E^v)^2} = \sqrt{4{,}30^2 + 2{,}65^2} = 5{,}05\ \text{mm}\ \searrow$$



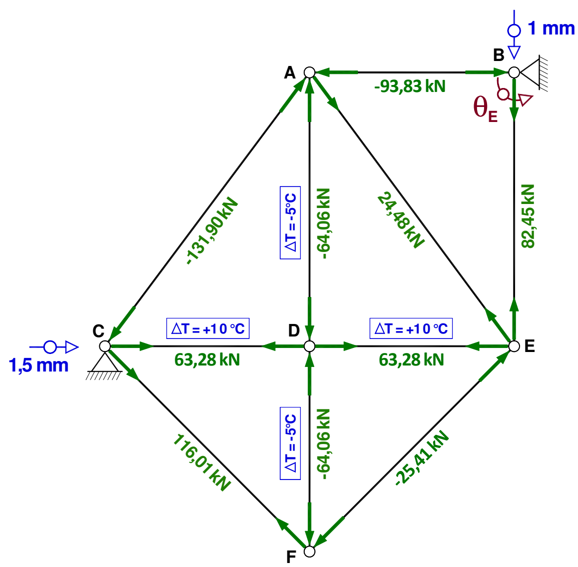
Alínea c)

• **Rotação da barra BE:** θ_{BE}

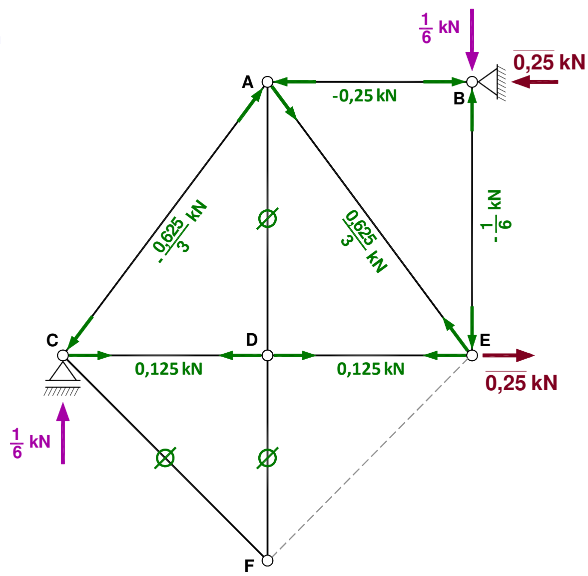
Solicitação do sistema virtual:



SISTEMA REAL



SISTEMA VIRTUAL



BARRAS	L (m)	N (kN)	$\bar{N}$ (kN)	$\bar{N}\frac{NL}{EA}$	$\bar{N}.\alpha.\Delta T.L$
AB	3	-93,83	-0,25	$3{,}52\times10^{-4}$	
AC	5	-131,90	$-\frac{0{,}625}{3}$	$6{,}870\times10^{-4}$	
AD	4	-64,06	0	0	0
AE	5	24,48	$\frac{0{,}625}{3}$	$1{,}275\times10^{-4}$	
BE	4	82,45	$-\frac{1}{6}$	$-2{,}748\times10^{-4}$	
CD	3	63,28	0,125	$1{,}187\times10^{-4}$	$5{,}625\times10^{-5}$
DE	3	63,28	0,125	$1{,}187\times10^{-4}$	$5{,}625\times10^{-5}$
CF	$3\sqrt{2}$	116,01	0	0	-
DF	3	-64,06	0	0	0
EF	$3\sqrt{2}$	-25,41	-	0	-
Σ				$1{,}129\times10^{-3}$	$1{,}125\times10^{-4}$

E A = $200 \times 10^6 \times 10 \times 10^{-4} = 2 \times 10^5$ kPa x m²

$\alpha = 1{,}5 \times 10^{-5}$ /°C

$$\bar{1} \times \theta_{BE} + \sum \bar{R} \times \text{assent. apoio} = \sum \bar{N}\frac{NL}{EA} + \sum \bar{N}\,\alpha\,\Delta T\,L$$

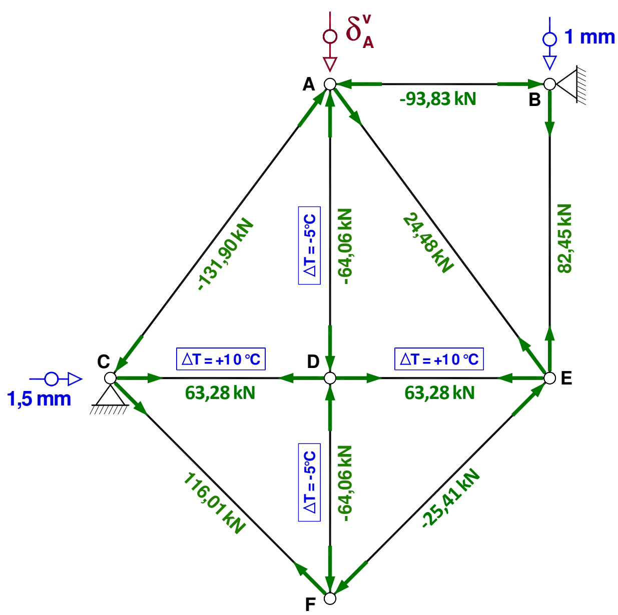
$$\theta_{DE} + \frac{1}{6}\times 0{,}001 = 1{,}129\times10^{-3} + 1{,}125\times10^{-4}$$

$$\theta_{DE} = 1{,}075 \times 10^{-3} \text{ rad} \circlearrowleft$$

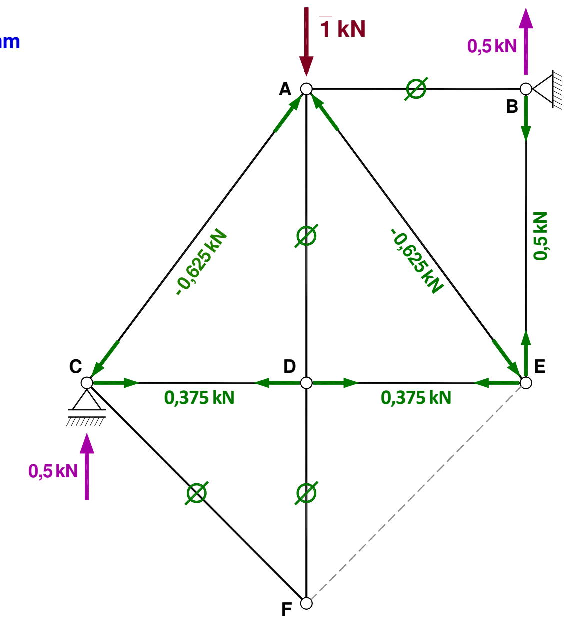
Alínea d)

• **Deslocamento vertical de A:** δ_A^v

SISTEMA REAL



SISTEMA VIRTUAL



E A = 200 x 10^6 x 10 x 10^{-4} = 2 x 10^5 kPa x m^2

α = 1,5 x 10^{-5} /°C

BARRAS	L (m)	N (kN)	$\bar{N}$ (kN)	$\bar{N}\frac{NL}{EA}$	$\bar{N}.\alpha.\Delta T.L$
AB	3	-93,83	0	0	
AC	5	-131,90	-0,625	2,061x10^{-3}	
AD	4	-64,06	0	0	0
AE	5	24,48	-0,625	-3,825x10^{-4}	
BE	4	82,45	0,5	8,245 x10^{-4}	
CD	3	63,28	0,375	3,560x10^{-4}	1,688x10^{-4}
DE	3	63,28	0,375	3,560x10^{-4}	1,688x10^{-4}
CF	$3\sqrt{2}$	116,01	0	0	-
DF	3	-64,06	0	0	0
EF	$3\sqrt{2}$	-25,41	-	0	-
Σ				3,215 x10^{-3}	3,375x10^{-4}

$$\bar{1} \times \delta_A^v + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{NL}{EA} + \Sigma \bar{N}\, \alpha\, \Delta T\, L$$

$$\delta_A^v + 0{,}5 \times (-0{,}001) = 3{,}215 \times 10^{-3} + 3{,}375 \times 10^{-4}$$

$$\delta_A^v = 4{,}05 \times 10^{-3} \text{ m} = 4{,}05 \text{ mm} \downarrow$$

Alínea e)

O nó **C** tem um deslocamento vertical de 1 mm (↓).

Para que a barra **AB** se mantenha horizontal, o nó **A** terá que ter também um deslocamento vertical de 1mm (↓). Como o nó **A** tem um deslocamento vertical de 4,05 mm (↓), o assentamento de apoio do nó **C** tem que produzir um levantamento(↑) de 3,05 mm no nó **A**.

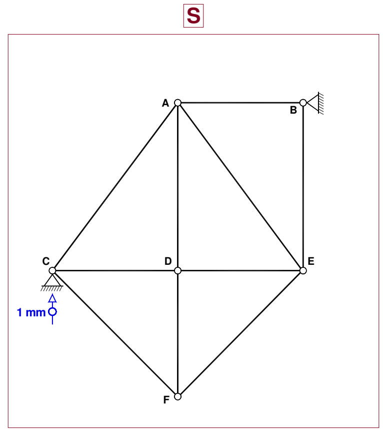
Se soubermos qual o deslocamento vertical do nó **A** devido a um assentamento unitário do nó **C**, poderemos depois determinar qual o assentamento que produzirá um levantamento de 3,05 mm.

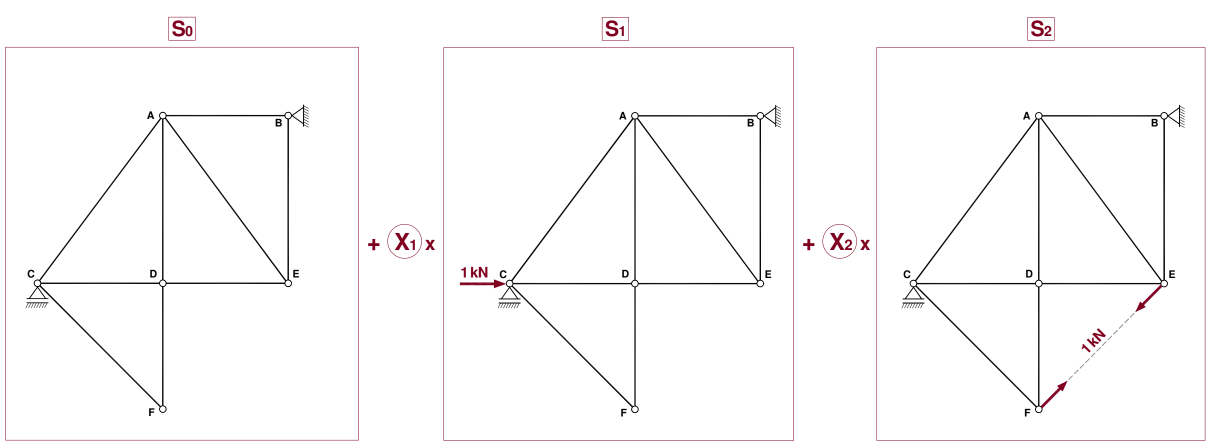
Vamos então determinar qual o deslocamento vertical do nó **A** devido a um assentamento vertical de **1mm** (↑) do apoio **C**.

Para proceder a este cálculo, é necessário previamente calcular as reacções e esforços instalados na estrutura devido ao assentamento vertical de **1mm** (↑) do apoio **C**.

$$\mathbf{S} = \mathbf{S_0} + \mathbf{X_1} \times \mathbf{S_1} + \mathbf{X_2} \times \mathbf{S_2}$$

S



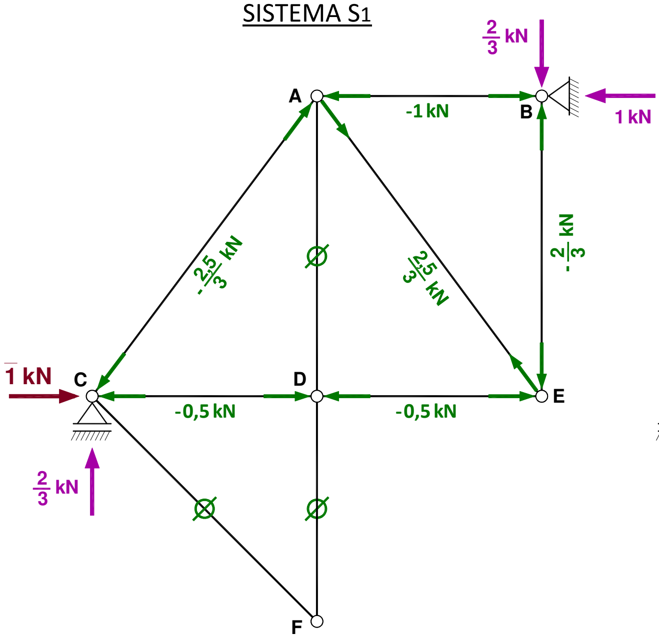


$$\begin{cases} \delta_{10} + \delta_{11} \cdot \mathbf{X_1} + \delta_{12} \cdot \mathbf{X_2} = 0 \\ \delta_{20} + \delta_{21} \cdot \mathbf{X_1} + \delta_{22} \cdot \mathbf{X_2} = 0 \end{cases}$$

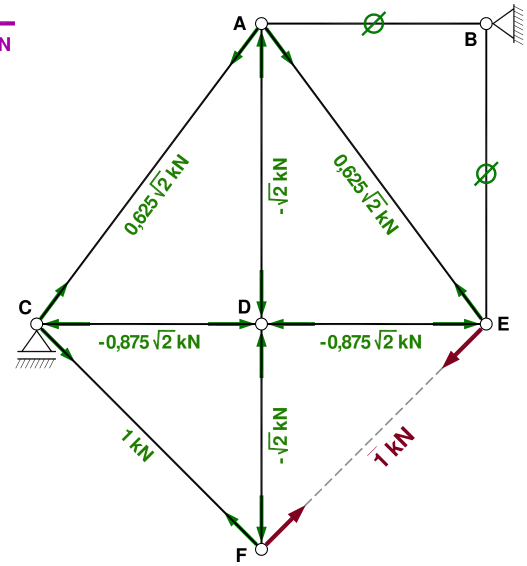
sendo:

$\delta_{10} = \Sigma\, N_1 \frac{N_0\, L}{E\, A} - \Sigma F_{ext}^{S1} \times \Delta_R$	$\delta_{11} = \Sigma\, N_1 \frac{N_1\, L}{E\, A}$	$\delta_{12} = \Sigma\, N_1 \frac{N_2\, L}{E\, A}$
$\delta_{20} = \Sigma\, N_2 \frac{N_0\, L}{E\, A} - \Sigma F_{ext}^{S2} \times \Delta_R$	$\delta_{21} = \Sigma\, N_2 \frac{N_1\, L}{E\, A}$	$\delta_{22} = \Sigma\, N_2 \frac{N_2\, L}{E\, A}$

SISTEMA S1



SISTEMA S2



BARRAS	L (m)	N_0 (kN)	N_1 (kN)	N_2 (kN)	$N_1 \frac{N_0 L}{EA}$	$N_2 \frac{N_0 L}{EA}$	$N_1 \frac{N_1 L}{EA}$	$N_1 \frac{N_2 L}{EA}$	$N_2 \frac{N_2 L}{EA}$
AB	3	0	-1	0	0	0	1,500x10^{-5}	0	0
AC	5	0	$-\frac{2,5}{3}$	$0,625\sqrt{2}$	0	0	1,736x10^{-5}	-1,841x10^{-5}	1,953x10^{-5}
AD	4	0	0	$-\sqrt{2}$	0	0	0	0	4,000x10^{-5}
AE	5	0	$\frac{2,5}{3}$	$0,625\sqrt{2}$	0	0	1,736x10^{-5}	1,841x10^{-5}	1,953x10^{-5}
BE	4	0	$-\frac{2}{3}$	0	0	0	8,889x10^{-6}	0	0
CD	3	0	-0,5	$-0,875\sqrt{2}$	0	0	3,750x10^{-6}	9,281x10^{-6}	2,297x10^{-5}
DE	3	0	-0,5	$-0,875\sqrt{2}$	0	0	3,750x10^{-6}	9,281x10^{-6}	2,297x10^{-5}
CF	$3\sqrt{2}$	0	0	1	0	0	0	0	2,121x10^{-5}
DF	3	0	0	$-\sqrt{2}$	0	0	0	0	3,000x10^{-5}
EF	$3\sqrt{2}$	-	-	1	0	0	0	0	2,121x10^{-5}
Σ					0	0	6,611x10^{-5}	1,856x10^{-5}	1,974x10^{-4}

$$\delta_{10} = \sum N_1 \frac{N_0 L}{EA} - \sum F_{ext}^{S1} \times \Delta_R = 0 - \left(\frac{2}{3} \times 0{,}001\right) = -\frac{2}{3} \times 10^{-3}$$

$$\delta_{20} = \sum N_2 \frac{N_0 L}{EA} - \sum F_{ext}^{S2} \times \Delta_R = 0 - 0 = 0$$

$$\delta_{11} = \sum N_1 \frac{N_1 L}{EA} = 6{,}611 \times 10^{-5}$$

$$\delta_{12} = \delta_{21} = \sum N_1 \frac{N_2 L}{EA} = \sum N_2 \frac{N_1 L}{EA} = 1{,}856 \times 10^{-5}$$

$$\delta_{22} = \sum N_2 \frac{N_2 L}{EA} = 1{,}974 \times 10^{-4}$$

$$\begin{cases} \delta_{10} + \delta_{11} \cdot \mathbf{X_1} + \delta_{12} \cdot \mathbf{X_2} = 0 \\ \delta_{20} + \delta_{21} \cdot \mathbf{X_1} + \delta_{22} \cdot \mathbf{X_2} = 0 \end{cases} \Rightarrow \begin{cases} -\frac{2}{3} \times 10^{-3} + 6{,}611 \times 10^{-5} \times \mathbf{X_1} + 1{,}856 \times 10^{-5} \times \mathbf{X_2} = 0 \\ 0 + 1{,}856 \times 10^{-5} \times \mathbf{X_1} + 1{,}974 \times 10^{-4} \times \mathbf{X_2} = 0 \end{cases}$$

$$\begin{cases} \mathbf{X_1} = 10{,}328 \\ \mathbf{X_2} = -0{,}974 \end{cases}$$

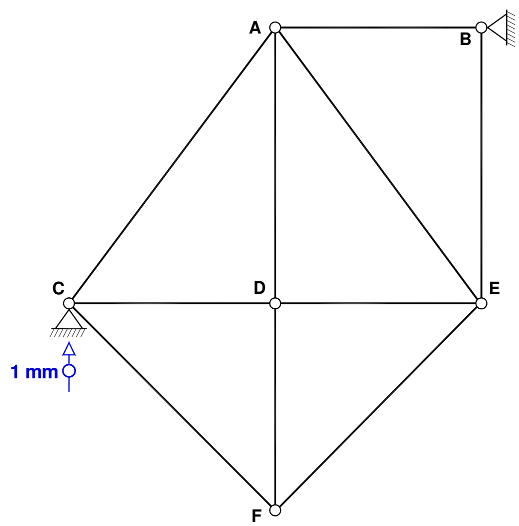
$$\begin{cases} H_B = (H_B)^{S0} + \mathbf{X_1} \cdot (H_B)^{S1} + \mathbf{X_2} \cdot (H_B)^{S2} \\ V_B = (V_B)^{S0} + \mathbf{X_1} \cdot (V_B)^{S1} + \mathbf{X_2} \cdot (V_B)^{S2} \\ H_C = (H_C)^{S0} + \mathbf{X_1} \cdot (H_C)^{S1} + \mathbf{X_2} \cdot (H_C)^{S2} \\ V_C = (V_C)^{S0} + \mathbf{X_1} \cdot (V_C)^{S1} + \mathbf{X_2} \cdot (V_C)^{S2} \end{cases} \Rightarrow \begin{cases} H_B = 0 + 10{,}328 \times (-1) + (-0{,}974) \times 0 \\ V_B = 0 + 10{,}328 \times (-\frac{2}{3}) + (-0{,}974) \times 0 \\ H_C = 0 + 10{,}328 \times 1 + (-0{,}974) \times 0 \\ V_C = 0 + 10{,}328 \times \frac{2}{3} + (-0{,}974) \times 0 \end{cases} \Rightarrow \begin{cases} H_B = -10{,}328 \text{ kN} \leftarrow \\ V_B = 6{,}885 \text{ kN} \uparrow \\ H_C = 10{,}328 \text{ kN} \rightarrow \\ V_C = 6{,}885 \text{ kN} \uparrow \end{cases}$$

- **Esforços nas barras**

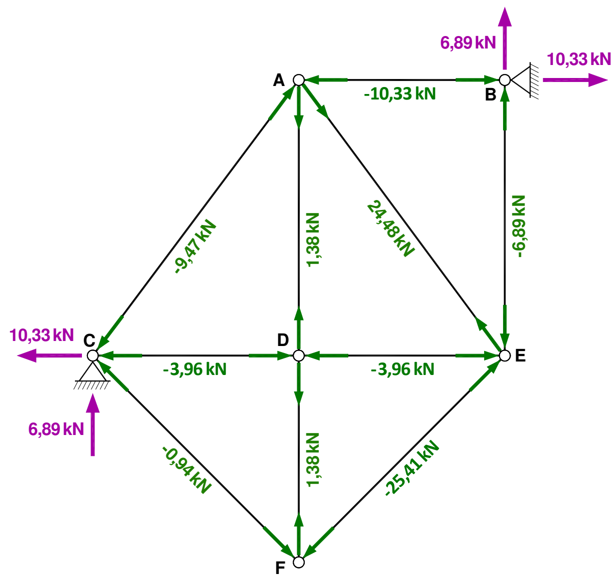
$$N_{barra} = (N_{barra})^{S0} + \mathbf{X_1} \cdot (N_{barra})^{S1} + \mathbf{X_2} \cdot (N_{barra})^{S2}$$

BARRAS	N_0 (kN)	N_1 (kN)	N_2 (kN)	N (kN)
AB	0	-1	0	-10,33
AC	0	$-\frac{2,5}{3}$	$0{,}625\sqrt{2}$	-9,47
AD	0	0	$-\sqrt{2}$	1,38
AE	0	$\frac{2,5}{3}$	$0{,}625\sqrt{2}$	7,75
BE	0	$-\frac{2}{3}$	0	-6,89
CD	0	-0,5	$-0{,}875\sqrt{2}$	-3,96
DE	0	-0,5	$-0{,}875\sqrt{2}$	-3,96
CF	0	0	1	-0,97
DF	0	0	$-\sqrt{2}$	1,38
EF	-	-	1	-0,97

SOLICITAÇÃO

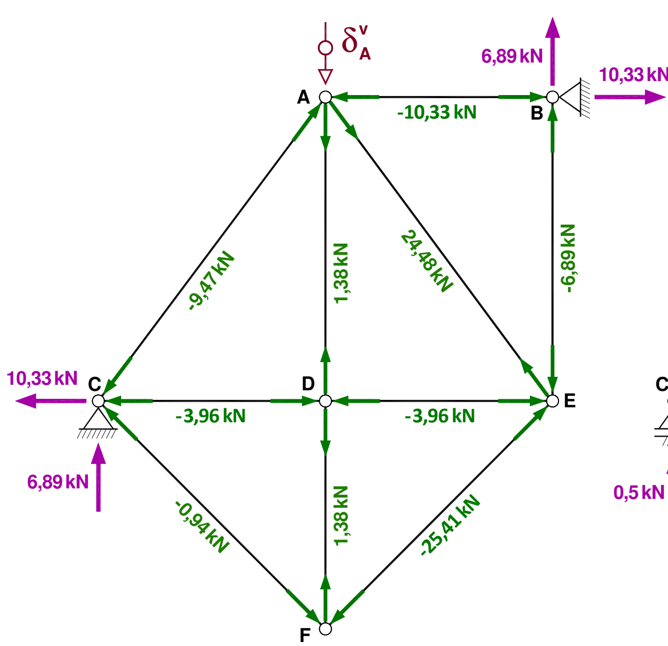


ESFORÇOS E REAÇÕES

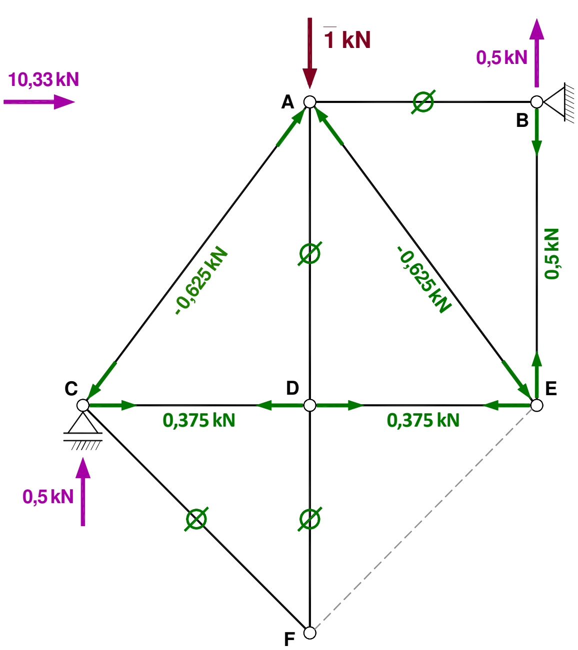


- **Determinação do deslocamento vertical de A devido a um deslocamento de 1mm (↑) do apoio C**

SISTEMA REAL



SISTEMA VIRTUAL



E A = 200 x 10^6 x 10 x 10^{-4} = 2 x 10^5 kPa x m^2

BARRAS	L (m)	N (kN)	$\overline{N}$ (kN)	$\overline{N}\frac{NL}{EA}$
AB	3	-10,33	0	0
AC	5	-9,47	-0,625	1,480x10^{-4}
AD	4	1,38	0	0
AE	5	7,75	-0,625	-1,211x10^{-4}
BE	4	-6,89	0,5	-6,890x10^{-5}
CD	3	-3,96	0,375	-2,228x10^{-5}
DE	3	-3,96	0,375	-2,228x10^{-5}
CF	$3\sqrt{2}$	-0,97	0	0
DF	3	1,38	0	0
EF	$3\sqrt{2}$	-0,97	-	0
Σ				-8,658x10^{-5}

$$\overline{1} \times \delta_A^v + \sum \overline{R} \times \text{assent. apoio} = \sum \overline{N}\frac{NL}{EA}$$

$$\delta_A^v + 0{,}5 \times 0{,}001 = -8{,}658 \times 10^{-5}$$

$$\delta_A^v = -5{,}87 \times 10^{-4} \text{ m} = -0{,}587 \text{ mm} \uparrow$$

Assentamento de apoio de 1mm (↑) $\Rightarrow$ $\delta_A^v = -0{,}587$ mm ↑

Qual o assentamento do apoio **ΔC**? $\Rightarrow$ $\delta_A^v = -3{,}05$ mm ↑

$$\Delta C = \frac{3{,}05 \times 1}{0{,}587} = 5{,}20 \text{ mm} \uparrow$$