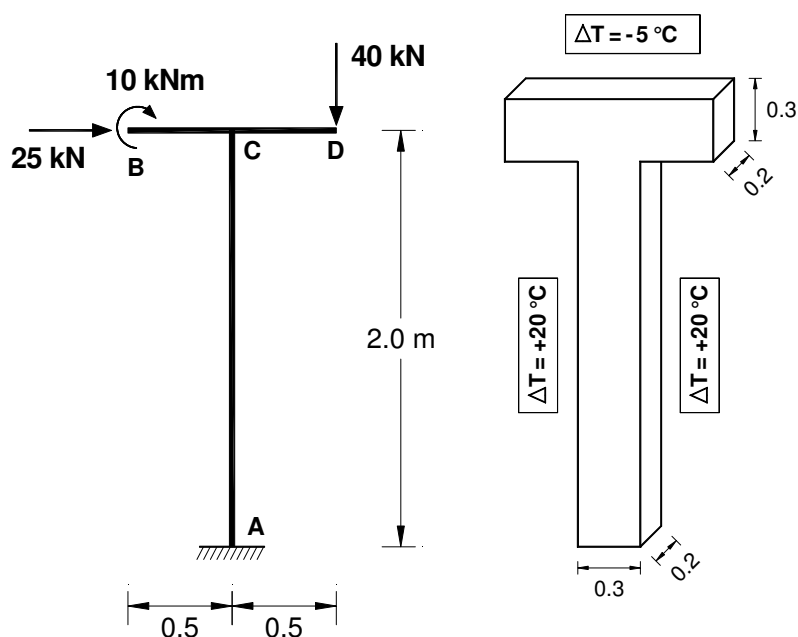


LICENCIATURA EM ENGENHARIA CIVIL

TEORIA DE ESTRUTURAS

TEOREMA DOS TRABALHOS VIRTUAIS



ESTRUTURA CONTÍNUA ISOSTÁTICA

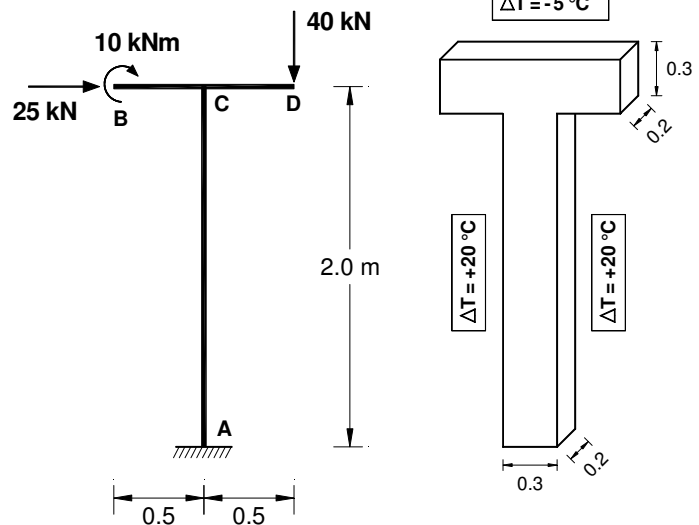
ISABEL ALVIM TELES

EXERCÍCIO PROPOSTO

Considere a estrutura representada na figura constituída por um material com as seguintes características:

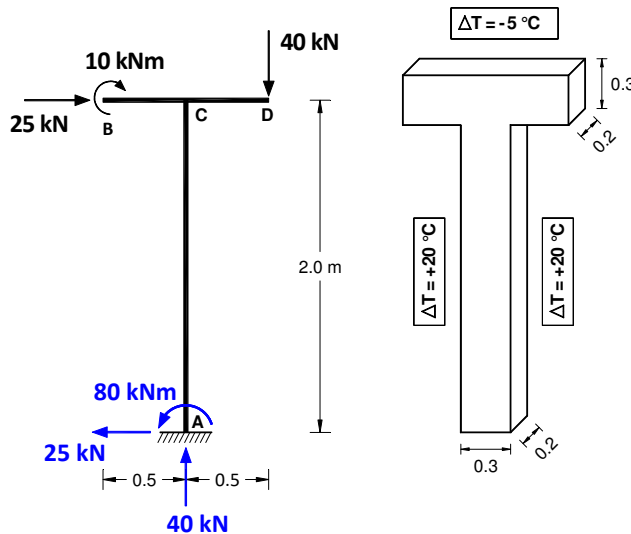
$$E = 25 \text{ GPa}; \nu = 0,2; \alpha = 1,4 \times 10^{-5} / ^\circ\text{C}$$

- Determine o deslocamento e rotação do ponto B;
- Determine o deslocamento e rotação do ponto D;
- Determine o deslocamento e rotação do ponto a meia altura do pilar AC.



RESOLUÇÃO

• **Cálculo das reacções**



$$E = 25 \text{ GPa} = 25 \times 10^6 \text{ kPa}$$

$$\nu = 0,2$$

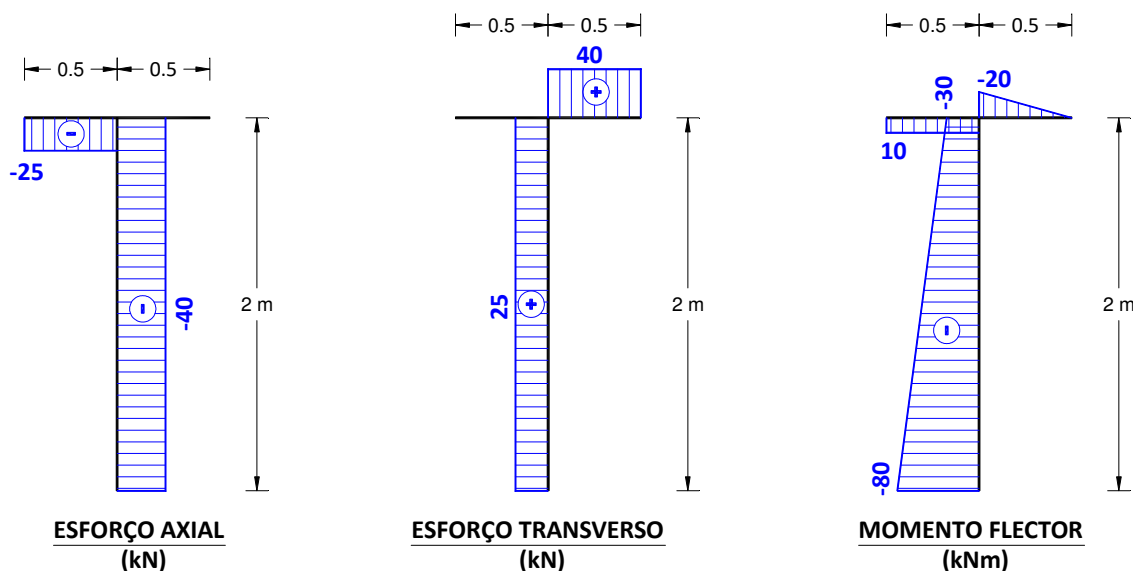
$$G = \frac{E}{2(1+\nu)} = \frac{25 \times 10^6}{2(1+0,2)} = 10,4167 \times 10^6 \text{ kPa}$$

$$\alpha = 1,4 \times 10^{-5} / ^\circ\text{C}$$

Pilar $\overline{AC}$ e vigas $\overline{BC}$ e $\overline{BD}$ ($b=0,20 \text{ m}$; $h=0,30 \text{ m}$)

$$\begin{cases} A = 0,20 \times 0,30 = 0,06 \text{ m}^2 \\ A' = \frac{5}{6} A = \frac{5}{6} \times 0,06 = 0,005 \text{ m}^2 \\ I = \frac{0,20 \times 0,30^3}{12} = 4,5 \times 10^{-4} \text{ m}^4 \end{cases}$$

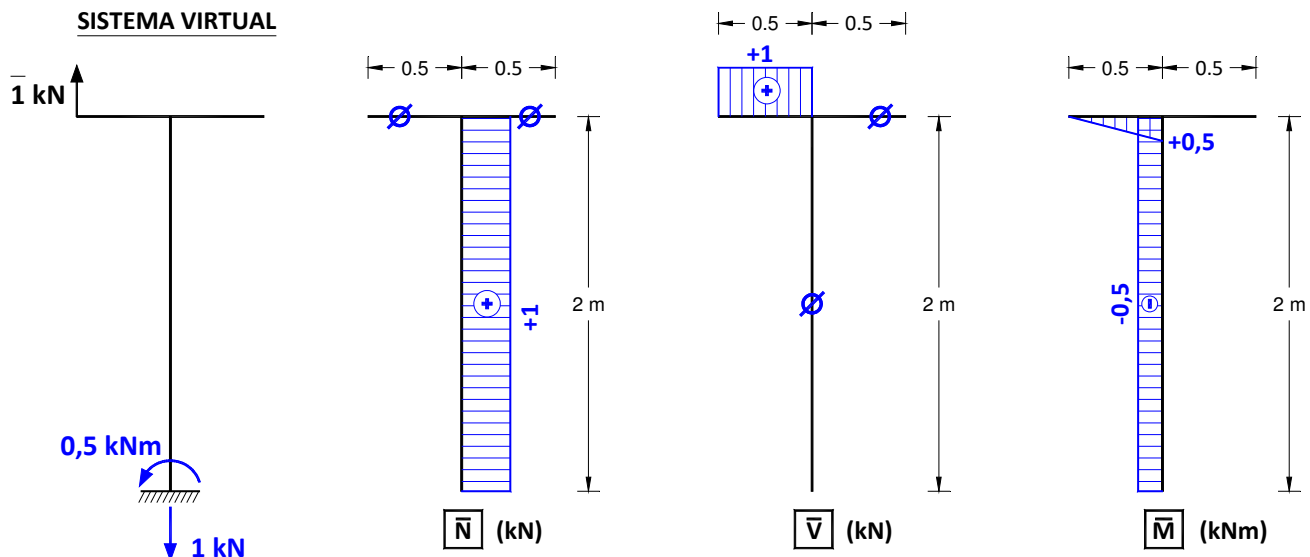
• **Diagramas de Esforços – sistema real**



a) DESLOCAMENTO DO PONTO B

a₁) Deslocamento vertical do ponto B

- Diagramas de Esforços – sistema virtual**



$$\bar{\mathbf{i}} \times \delta_B^v + \sum (\bar{\mathbf{R}} \times \text{assent. apoio}) = \int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{EA} dz + \int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{GA'} dz + \int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{EI} dz + \int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{EA} dz = \frac{1}{25 \times 10^6 \times 0,06} (-40 \times 2 \times 1) = -5,333 \times 10^{-5} \text{ m}$$

$$\int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{GA'} dz = 0$$

$$\int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{EI} dz = \frac{1}{25 \times 10^6 \times 4,5 \times 10^{-4}} \left[\frac{0,5 \times 0,5}{2} \times 10 + \frac{-80 - 30}{2} \times 2 \times (-0,5) \right] = 0,005 \text{ m}$$

$$\int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz = 1 \times 2 \times 1,4 \times 10^{-5} \times 20 = 56 \times 10^{-5} \text{ m}$$

$$\int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz = \frac{0,5 \times 0,5}{2} \times 1,4 \times 10^{-5} \times \frac{20 + 5}{0,30} = 14,583 \times 10^{-5} \text{ m}$$

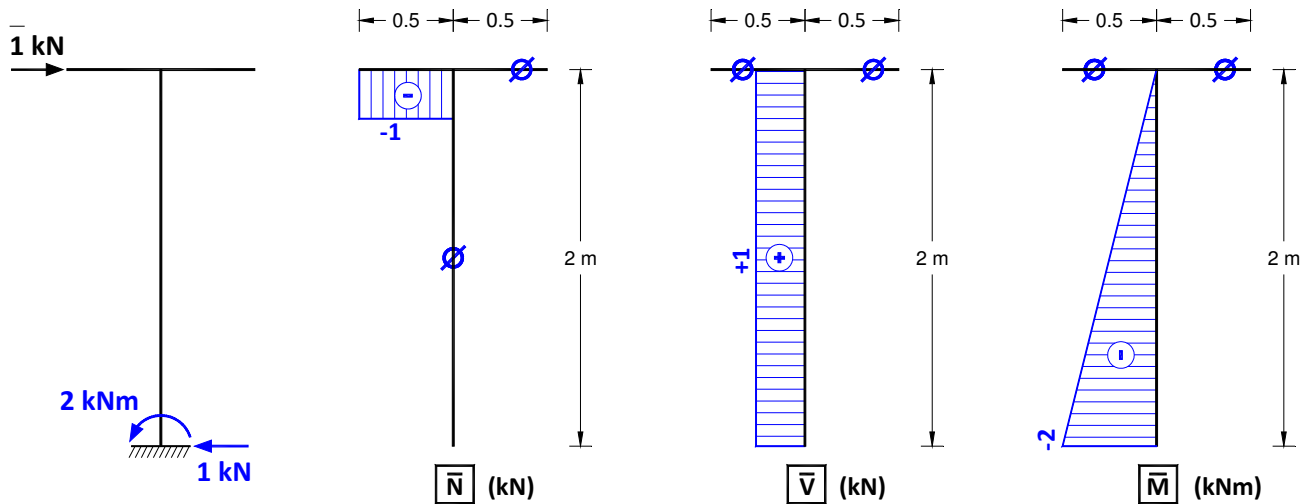
$$\delta_B^v = \int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{EA} dz + \int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{GA'} dz + \int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{EI} dz + \int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\delta_B^v = -5,333 \times 10^{-5} + 0,0 + 0,005 + 56 \times 10^{-5} + 14,583 \times 10^{-5} = 5,653 \times 10^{-3} \text{ m} = 5,653 \text{ mm} \quad \uparrow$$

a2) Deslocamento horizontal do ponto B

• **Diagramas de Esforços – sistema virtual**

SISTEMA VIRTUAL



$$\bar{\mathbf{i}} \times \delta_B^h + \sum (\bar{\mathbf{R}} \times \text{assent. apoio}) = \int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{E A} dz + \int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{G A'} dz + \int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{E I} dz + \int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{E A} dz = \frac{1}{25 \times 10^6 \times 0,06} (-1) \times 0,5 \times (-25) = 0,8333 \times 10^{-5} \text{ m}$$

$$\int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{G A'} dz = \frac{1}{10,4167 \times 10^6 \times 0,05} 1 \times 2 \times 25 = 9,6 \times 10^{-5} \text{ m}$$

$$\int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{E I} dz = \frac{1}{25 \times 10^6 \times 4,5 \times 10^{-4}} \left[\frac{-2 \times 2}{6} \times (-30 - 2 \times 80) \right] = 1,126 \times 10^{-2} \text{ m}$$

$$\int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz = -1 \times 0,5 \times 1,4 \times 10^{-5} \times \frac{-5 + 20}{2} = -5,25 \times 10^{-5} \text{ m}$$

$$\int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz = 0$$

$$\delta_B^h = \int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{E A} dz + \int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{G A'} dz + \int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{E I} dz + \int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

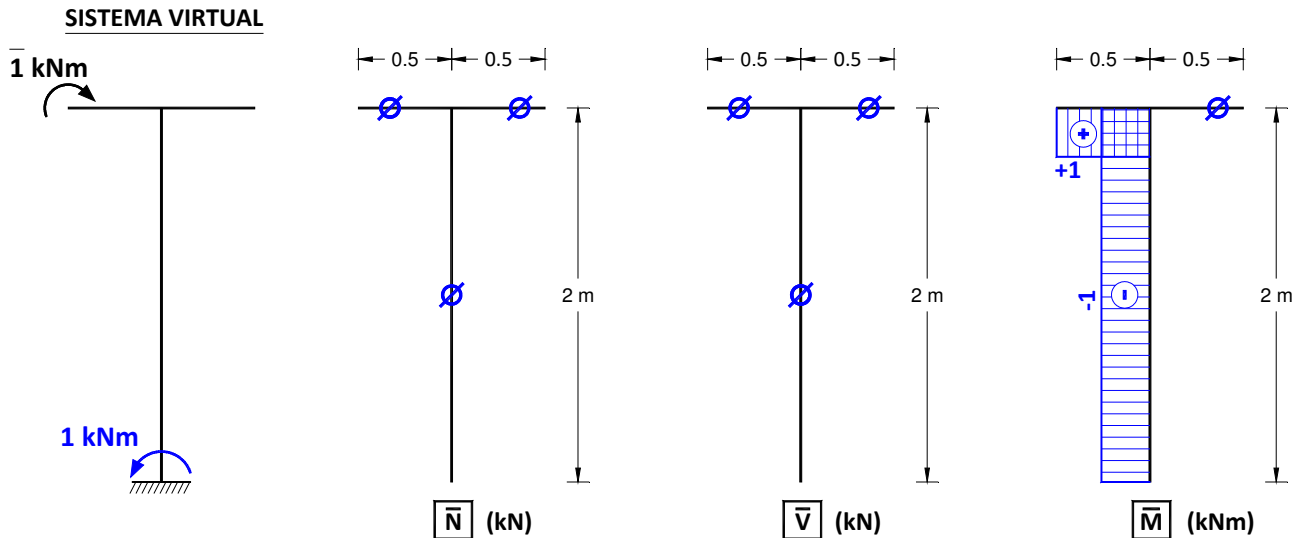
$$\delta_B^h = 0,8333 \times 10^{-5} + 9,6 \times 10^{-5} + 1,126 \times 10^{-2} - 5,25 \times 10^{-5} + 0 = 1,131 \times 10^{-2} \text{ m} = 1,131 \text{ cm} \rightarrow$$

• **Deslocamento do ponto B**

$$\delta_B = \sqrt{(\delta_B^v)^2 + (\delta_B^h)^2} = \sqrt{0,5653^2 + 1,131^2} = 1,264 \text{ cm} \nearrow$$

a₃) Rotação do ponto B

• **Diagramas de Esforços – sistema virtual**



$$1 \times \theta_B + \sum (\bar{R} \times \text{assent. apoio}) = \int_0^L \frac{\bar{N} \times N}{E A} dz + \int_0^L \frac{\bar{V} \times V}{G A'} dz + \int_0^L \frac{\bar{M} \times M}{E I} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\int_0^L \frac{\bar{N} \times N}{E A} dz = 0$$

$$\int_0^L \frac{\bar{V} \times V}{G A'} dz = 0$$

$$\int_0^L \frac{\bar{M} \times M}{E I} dz = \frac{1}{25 \times 10^6 \times 4,5 \times 10^{-4}} \left[\frac{-80 - 30}{2} \times 2 \times (-1) + 10 \times 0,5 \times 1 \right] = 1,0222 \times 10^{-2} \text{ rad}$$

$$\int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz = 0$$

$$\int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz = 1 \times 0,5 \times 1,4 \times 10^{-5} \times \frac{20 + 5}{0,30} = 58,333 \times 10^{-5} \text{ rad}$$

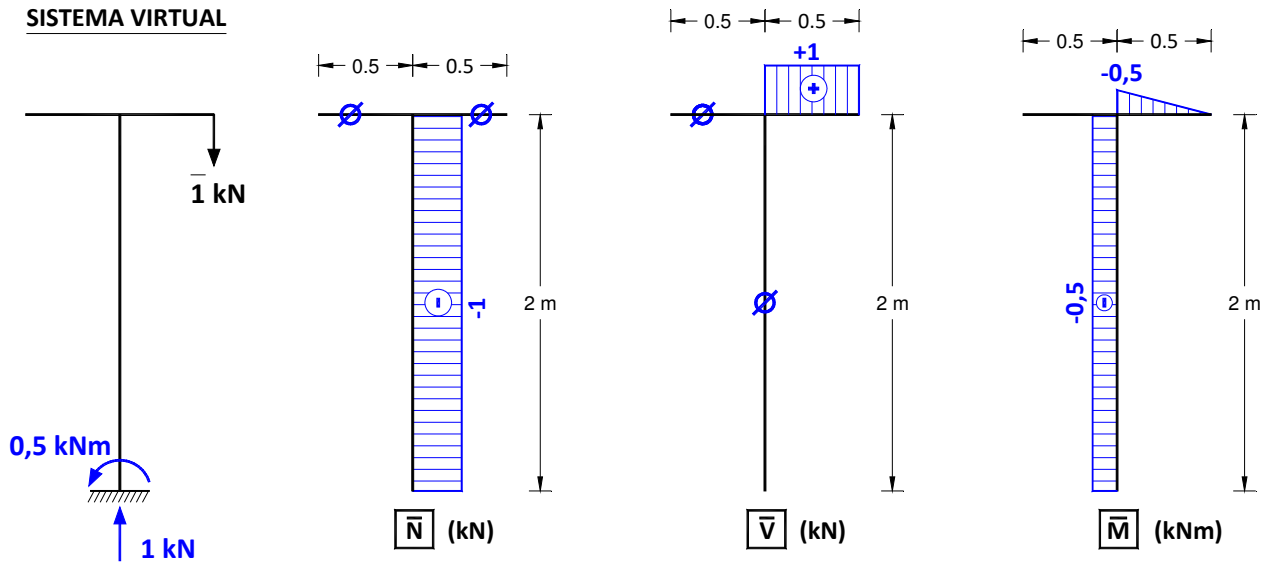
$$\theta_B = \int_0^L \frac{\bar{N} \times N}{E A} dz + \int_0^L \frac{\bar{V} \times V}{G A'} dz + \int_0^L \frac{\bar{M} \times M}{E I} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\theta_B = 1,0222 \times 10^{-2} + 58,333 \times 10^{-5} = 1,081 \times 10^{-2} \text{ rad} \quad \curvearrowright$$

b) DESLOCAMENTO DO PONTO D

b₁) Deslocamento vertical do ponto D

- Diagramas de Esforços – sistema virtual**



$$\bar{\mathbf{i}} \times \delta_D^v + \sum (\bar{\mathbf{R}} \times \text{assent. apoio}) = \int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{EA} dz + \int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{GA'} dz + \int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{EI} dz + \int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{EA} dz = \frac{1}{25 \times 10^6 \times 0,06} (-1) \times 2 \times (-40) = 5,333 \times 10^{-5} \text{ m}$$

$$\int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{GA'} dz = \frac{1}{10,4167 \times 10^6 \times 0,05} 1 \times 0,5 \times 40 = 3,840 \times 10^{-5} \text{ m}$$

$$\int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{EI} dz = \frac{1}{25 \times 10^6 \times 4,5 \times 10^{-4}} \left[\frac{-80 - 30}{2} \times 2 \times (-0,5) + \frac{-0,5 \times 0,5 \times (-20)}{3} \right] = 5,037 \times 10^{-3} \text{ m}$$

$$\int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz = -1 \times 2 \times 1,4 \times 10^{-5} \times 20 = -56 \times 10^{-5} \text{ m}$$

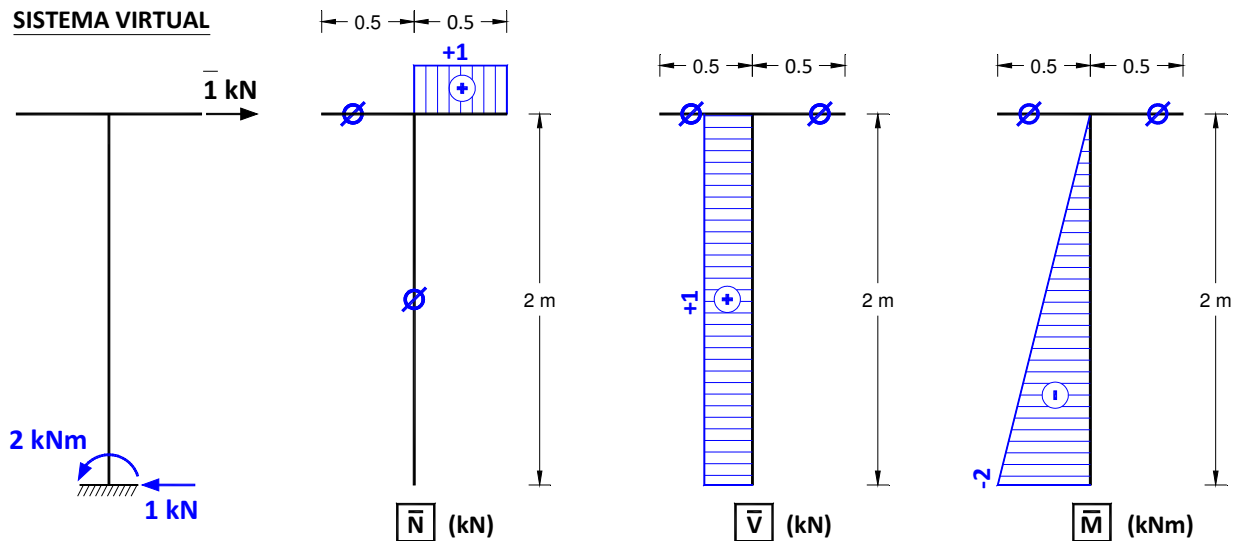
$$\int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz = \frac{-0,5 \times 0,5}{2} \times 1,4 \times 10^{-5} \times \frac{20 + 5}{0,30} = -14,583 \times 10^{-5} \text{ m}$$

$$\delta_D^v = \int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{EA} dz + \int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{GA'} dz + \int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{EI} dz + \int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\delta_D^v = 5,333 \times 10^{-5} + 3,840 \times 10^{-5} + 5,037 \times 10^{-3} - 56 \times 10^{-5} - 14,583 \times 10^{-5} = 4,42 \times 10^{-3} \text{ m} = 4,42 \text{ mm} \downarrow$$

b₂) Deslocamento horizontal do ponto D

• **Diagramas de Esforços – sistema virtual**



$$\bar{\mathbf{i}} \times \delta_D^h + \sum (\bar{\mathbf{R}} \times \text{assent. apoio}) = \int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{EA} dz + \int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{GA'} dz + \int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{EI} dz + \int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{EA} dz = 0$$

$$\int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{GA'} dz = \frac{1}{10,4167 \times 10^6 \times 0,05} 1 \times 2 \times 25 = 9,6 \times 10^{-5} \text{ m}$$

$$\int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{EI} dz = \frac{1}{25 \times 10^6 \times 4,5 \times 10^{-4}} \left[\frac{-2 \times 2}{6} \times (-30 - 2 \times 80) \right] = 1,126 \times 10^{-2} \text{ m}$$

$$\int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz = 1 \times 0,5 \times 1,4 \times 10^{-5} \times \frac{-5 + 20}{2} = 5,25 \times 10^{-5} \text{ m}$$

$$\int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz = 0$$

$$\delta_D^h = \int_0^L \frac{\bar{\mathbf{N}} \times \mathbf{N}}{EA} dz + \int_0^L \frac{\bar{\mathbf{V}} \times \mathbf{V}}{GA'} dz + \int_0^L \frac{\bar{\mathbf{M}} \times \mathbf{M}}{EI} dz + \int_0^L \bar{\mathbf{N}} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{\mathbf{M}} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

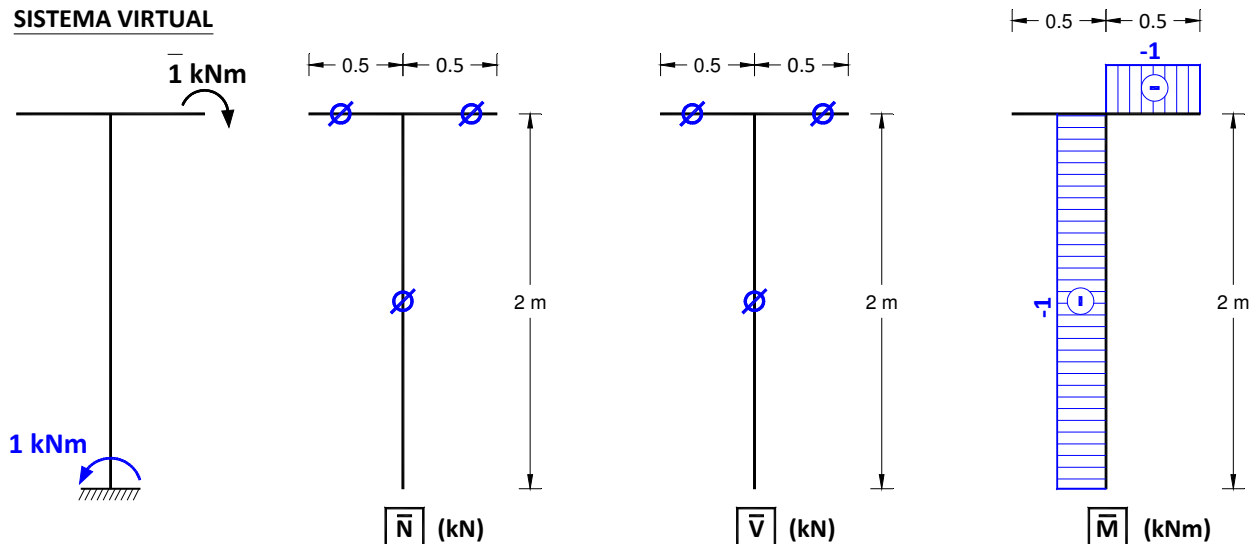
$$\delta_D^h = 0 + 9,6 \times 10^{-5} + 1,126 \times 10^{-2} + 5,25 \times 10^{-5} + 0 = 1,14 \times 10^{-2} \text{ m} = 1,14 \text{ cm} \rightarrow$$

• **Deslocamento do ponto D**

$$\delta_D = \sqrt{(\delta_D^v)^2 + (\delta_D^h)^2} = \sqrt{0,442^2 + 1,14^2} = 1,22 \text{ cm} \searrow$$

b₃) Rotação do ponto D

• **Diagramas de Esforços – sistema virtual**



$$1 \times \theta_D + \Sigma(\bar{R} \times \text{assent. apoio}) = \int_0^L \frac{\bar{N} \times N}{EA} dz + \int_0^L \frac{\bar{V} \times V}{GA'} dz + \int_0^L \frac{\bar{M} \times M}{EI} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\int_0^L \frac{\bar{N} \times N}{EA} dz = 0$$

$$\int_0^L \frac{\bar{V} \times V}{GA'} dz = 0$$

$$\int_0^L \frac{\bar{M} \times M}{EI} dz = \frac{1}{25 \times 10^6 \times 4,5 \times 10^{-4}} \left[\frac{-80 - 30}{2} \times 2 \times (-1) + \frac{-20 \times 0,5}{2} \times (-1) \right] = 1,0222 \times 10^{-2} \text{ rad}$$

$$\int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz = 0$$

$$\int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz = -1 \times 0,5 \times 1,4 \times 10^{-5} \times \frac{20 + 5}{0,30} = -58,333 \times 10^{-5} \text{ rad}$$

$$\theta_D = \int_0^L \frac{\bar{N} \times N}{EA} dz + \int_0^L \frac{\bar{V} \times V}{GA'} dz + \int_0^L \frac{\bar{M} \times M}{EI} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

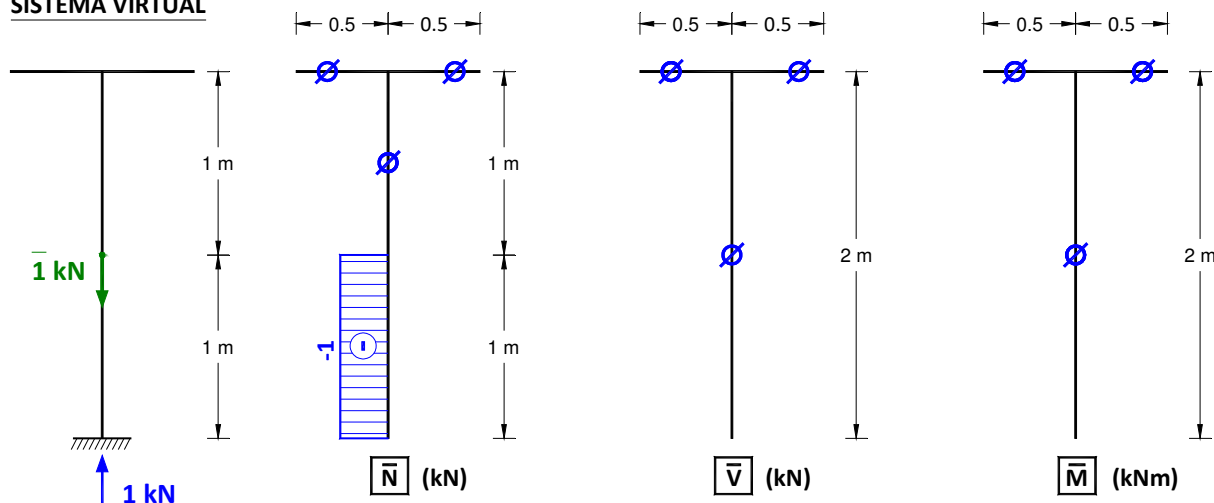
$$\theta_D = 1,0222 \times 10^{-2} - 58,333 \times 10^{-5} = 9,639 \times 10^{-3} \text{ rad} \quad \curvearrowright$$

c) DESLOCAMENTO E ROTAÇÃO A MEIA ALTURA DE AC

c₁) Deslocamento vertical

• **Diagramas de Esforços – sistema virtual**

SISTEMA VIRTUAL



$$\bar{1} \times \delta^v + \Sigma (\bar{R} \times \text{assent. apoio}) = \int_0^L \frac{\bar{N} \times N}{E A} dz + \int_0^L \frac{\bar{V} \times V}{G A'} dz + \int_0^L \frac{\bar{M} \times M}{E I} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\int_0^L \frac{\bar{N} \times N}{E A} dz = \frac{1}{25 \times 10^6 \times 0,06} (-1) \times 1 \times (-40) = 2,6667 \times 10^{-5} \text{ m}$$

$$\int_0^L \frac{\bar{V} \times V}{G A'} dz = 0$$

$$\int_0^L \frac{\bar{M} \times M}{E I} dz = 0$$

$$\int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz = -1 \times 1 \times 1,4 \times 10^{-5} \times 20 = -28 \times 10^{-5} \text{ m}$$

$$\int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz = 0$$

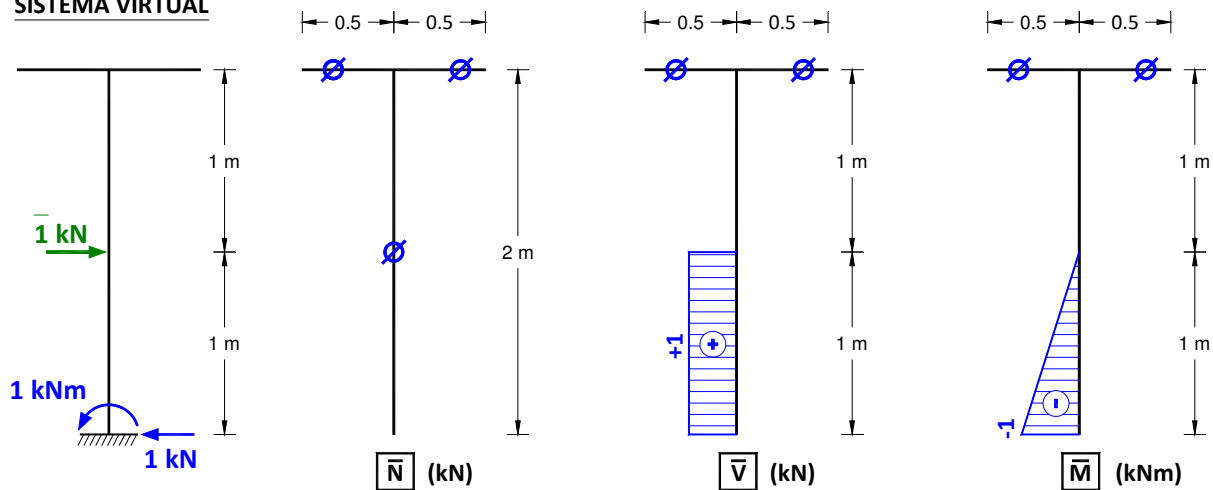
$$\delta^v = \int_0^L \frac{\bar{N} \times N}{E A} dz + \int_0^L \frac{\bar{V} \times V}{G A'} dz + \int_0^L \frac{\bar{M} \times M}{E I} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\delta^v = 2,6667 \times 10^{-5} - 28 \times 10^{-5} = -25,33 \times 10^{-5} \text{ m} = -0,253 \text{ mm} \uparrow$$

c2) Deslocamento horizontal

• **Diagramas de Esforços – sistema virtual**

SISTEMA VIRTUAL



$$\bar{1} \times \delta^h + \sum (\bar{R} \times \text{assent. apoio}) = \int_0^L \frac{\bar{N} \times N}{EA} dz + \int_0^L \frac{\bar{V} \times V}{GA'} dz + \int_0^L \frac{\bar{M} \times M}{EI} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\int_0^L \frac{\bar{N} \times N}{EA} dz = 0$$

$$\int_0^L \frac{\bar{V} \times V}{GA'} dz = \frac{1}{10,4167 \times 10^6 \times 0,05} 1 \times 1 \times 25 = 4,80 \times 10^{-5} \text{ m}$$

$$\int_0^L \frac{\bar{M} \times M}{EI} dz = \frac{1}{25 \times 10^6 \times 4,5 \times 10^{-4}} \left[\frac{-1 \times 1}{6} \times (-55 - 2 \times 80) \right] = 3,185 \times 10^{-3} \text{ m}$$

$$\int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz = 0$$

$$\int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz = 0$$

$$\delta = \int_0^L \frac{\bar{N} \times N}{EA} dz + \int_0^L \frac{\bar{V} \times V}{GA'} dz + \int_0^L \frac{\bar{M} \times M}{EI} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\delta^h = 4,80 \times 10^{-5} + 3,185 \times 10^{-3} = 3,233 \times 10^{-3} \text{ m} = 3,233 \text{ mm} \rightarrow$$

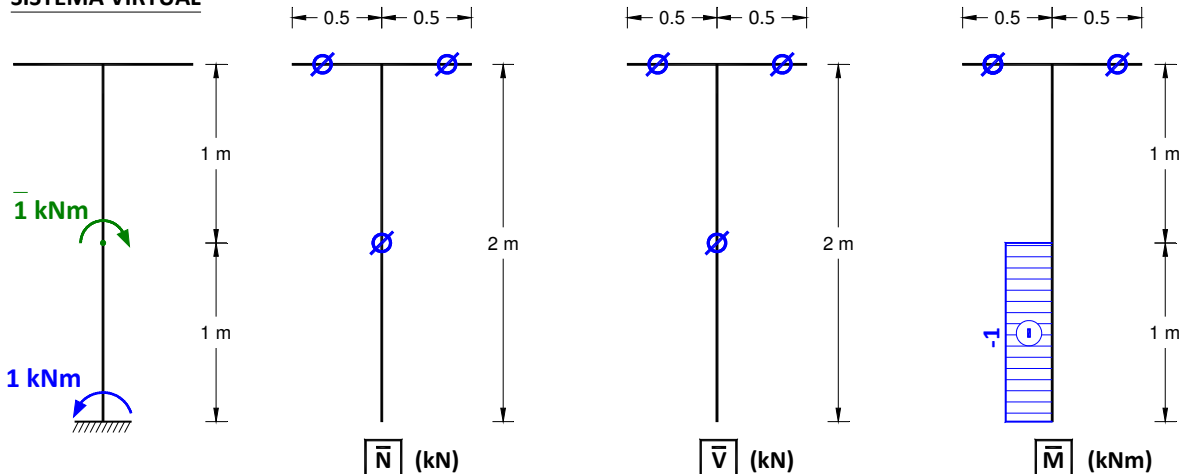
• **Deslocamento do ponto a meia altura da barra AC**

$$\delta = \sqrt{(\delta^v)^2 + (\delta^h)^2} = \sqrt{(-0,253)^2 + 3,233^2} = 3,24 \text{ mm} \nearrow$$

c3) Rotação do ponto a meia altura da barra AC

• **Diagramas de Esforços – sistema virtual**

SISTEMA VIRTUAL



$$\bar{1} \times \theta + \sum (\bar{R} \times \text{assent. apoio}) = \int_0^L \frac{\bar{N} \times N}{E A} dz + \int_0^L \frac{\bar{V} \times V}{G A'} dz + \int_0^L \frac{\bar{M} \times M}{E I} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\int_0^L \frac{\bar{N} \times N}{E A} dz = 0$$

$$\int_0^L \frac{\bar{V} \times V}{G A'} dz = 0$$

$$\int_0^L \frac{\bar{M} \times M}{E I} dz = \frac{1}{25 \times 10^6 \times 4,5 \times 10^{-4}} \left[\frac{-80 - 55}{2} \times 1 \times (-1) \right] = 6 \times 10^{-3} \text{ rad}$$

$$\int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz = 0$$

$$\int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz = 0$$

$$\theta = \int_0^L \frac{\bar{N} \times N}{E A} dz + \int_0^L \frac{\bar{V} \times V}{G A'} dz + \int_0^L \frac{\bar{M} \times M}{E I} dz + \int_0^L \bar{N} \cdot \alpha \cdot T_m \cdot dz + \int_0^L \bar{M} \cdot \alpha \cdot \frac{T_{\text{inf}} - T_{\text{sup}}}{h} dz$$

$$\theta = 6 \times 10^{-3} \text{ rad} \quad \curvearrowright$$