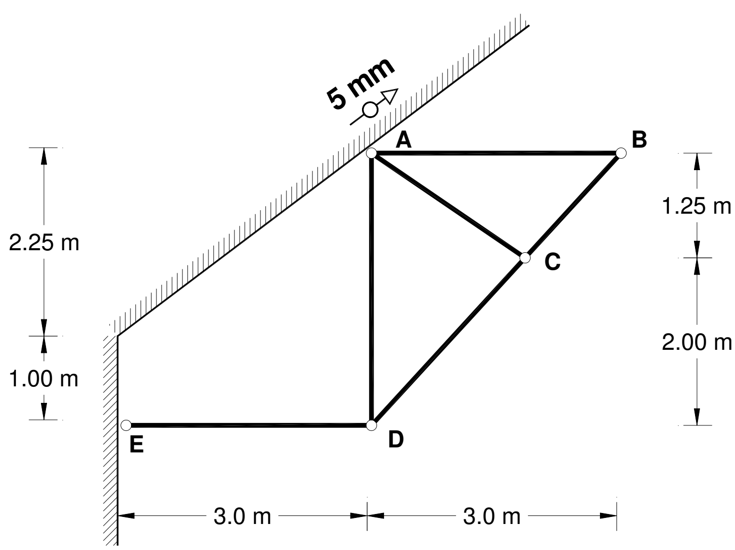
LICENCIATURA EM ENGENHARIA CIVIL

TEORIA DE ESTRUTURAS

TEOREMA DOS TRABALHOS VIRTUAIS



SISTEMA ARTICULADO PLANO (SAP) ISOSTÁTICO

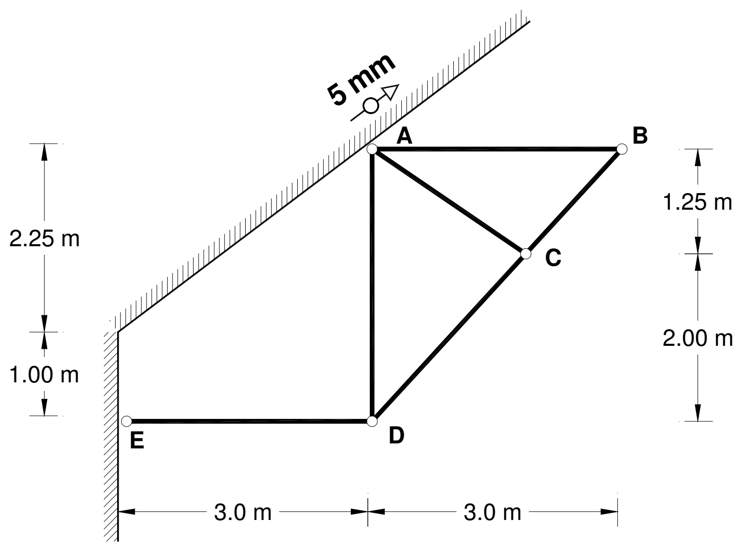
ISABEL ALVIM TELES

EXERCÍCIO PROPOSTO

Uma estrutura articulada plana foi fixada à parede e tecto de uma câmara frigorífica.

Devido a um erro, a fixação do nó **A** sofreu um desvio de 5 mm em relação ao pretendido (ver figura).

Dentro da câmara frigorífica as barras ficam submetidas a uma variação de temperatura de -25 °C.



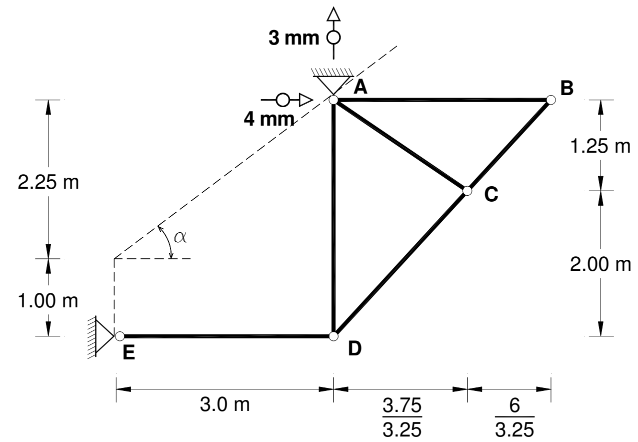
Aço constituinte das barras:	E = 210 GPa	$\alpha = 1{,}5\text{x}10^{-5}$ /°C
Barra ED Perfil circular: Ø = 80mm	**Restantes barras** Perfil tubular: Ø = 80 mm esp = 8 mm	

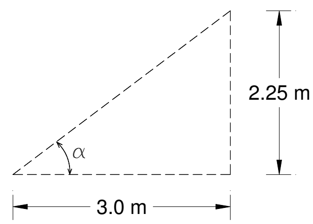
a) Determine deslocamento do nó **B**;

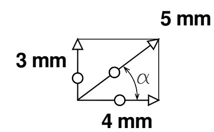
b) Determine deslocamento do nó **D**;

c) Determine a força vertical a aplicar em **D** para que a barra **ED** fique horizontal.

RESOLUÇÃO







$$\alpha = \text{arctg}\ \frac{2{,}25}{3} \Rightarrow \begin{cases} \cos\alpha = 0{,}8 \\ \text{sen}\ \alpha = 0{,}6 \end{cases}$$

$$\Rightarrow \begin{cases} \text{componente horizontal do assentamento de apoio} = 0{,}8 \times 5 = 4\ \text{mm} \\ \text{componente vertical do assentamento de apoio} = 0{,}6 \times 5 = 3\ \text{mm} \end{cases}$$

$E = 210 \times 10^6$ kPa

$$\text{Barra ED} \Rightarrow \text{Área} = \frac{\pi \times 0{,}08^2}{4}\ m^2 \Rightarrow EA = 1055575{,}13\ \text{kN}$$

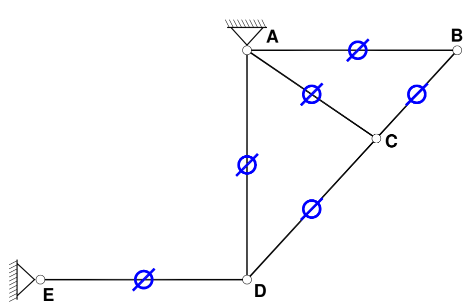
$$\text{Restantes barras} \Rightarrow \text{Área} = \frac{\pi \times (0{,}08^2 - 0{,}064^2)}{4}\ m^2 \Rightarrow EA = 380007{,}047\ \text{kN}$$

$\Delta T = -25^\circ C$ (em todas as barras) $\qquad \alpha = 1{,}5 \times 10^{-5}$ /°C

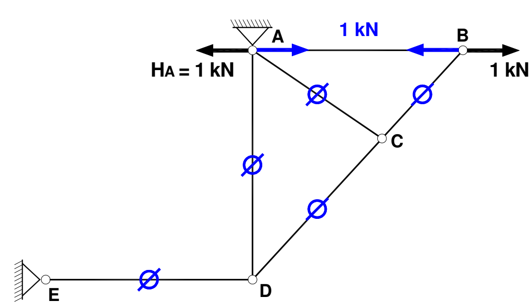
a) DESLOCAMENTO DO NÓ B

- **Deslocamento horizontal do nó B**

S - Sistema real



$\overline{S}$ - Sistema virtual



$$\overline{1} \times \delta_h^B + \Sigma \overline{R} \times \text{assent. apoio} = \Sigma \overline{N} \frac{N L}{E A} + \Sigma \overline{N}\, \alpha\, \Delta T\, L$$

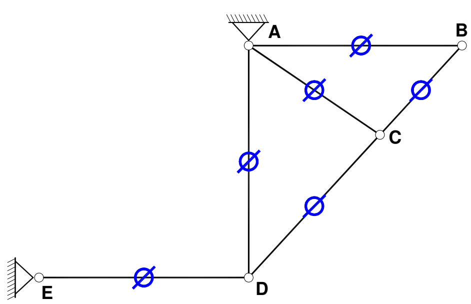
BARRAS	L (m)	E A (kPa x m²)	N (kN)	$\bar{N}$ (kN)	α.ΔT.L	$\bar{N}$.α.ΔT.L	$\bar{N}\frac{NL}{EA}$
AB	3	$3{,}8001 \times 10^5$	0	1	$-1{,}1250 \times 10^{-3}$	$-1{,}125 \times 10^{-3}$	0
AC	2,22953	$3{,}8001 \times 10^5$	0	0	$-8{,}3607 \times 10^{-4}$	0	0
AD	3,25	$3{,}8001 \times 10^5$	0	0	$-1{,}2188 \times 10^{-3}$	0	0
DC	2,72182	$3{,}8001 \times 10^5$	0	0	$-1{,}0207 \times 10^{-3}$	0	0
CB	1,70114	$3{,}8001 \times 10^5$	0	0	$-6{,}3793 \times 10^{-4}$	0	0
ED	3	$10{,}5558 \times 10^5$	0	0	$-1{,}1250 \times 10^{-3}$	0	0
						$\Sigma = -1{,}125 \times 10^{-3}$	$\Sigma = 0$

$$\delta_h^B = \Sigma \bar{N} \frac{N L}{E A} + \Sigma \bar{N}\, \alpha\, \Delta T\, L - \Sigma \bar{R} \times \text{assent. apoio}$$

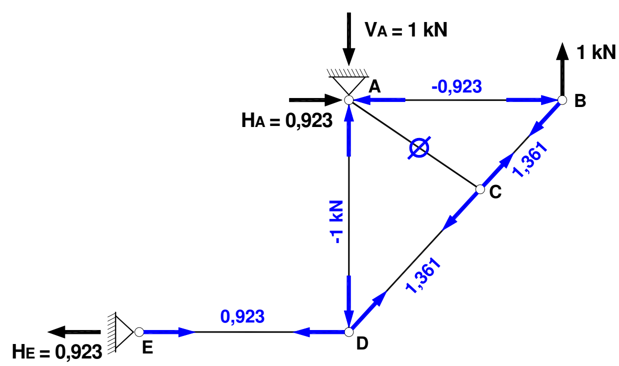
$$\delta_h^B = 0{,}0 - 1{,}125 \times 10^{-3} - 0{,}004 \times (-1) = 2{,}875 \times 10^{-3}\ \text{m} = 2{,}875\ \text{mm} \ \rightarrow$$

- **Deslocamento vertical do nó B**

S - Sistema real



$\bar{\mathbf{S}}$ - Sistema virtual



$$\bar{1} \times \delta_v^B + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{N L}{E A} + \Sigma \bar{N}\, \alpha\, \Delta T\, L$$

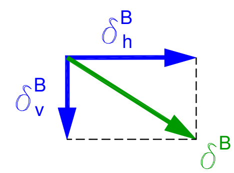
BARRAS	L (m)	E A (kPa x m²)	N (kN)	$\bar{N}$ (kN)	α.ΔT.L	$\bar{N}$.α.ΔT.L	$\bar{N}\frac{NL}{EA}$
AB	3	$3{,}8001 \times 10^5$	0	-0,92308	$-1{,}1250 \times 10^{-3}$	$1{,}0385 \times 10^{-3}$	0
AC	2,22953	$3{,}8001 \times 10^5$	0	0	$-8{,}3607 \times 10^{-4}$	0	0
AD	3,25	$3{,}8001 \times 10^5$	0	-1	$-1{,}2188 \times 10^{-3}$	$1{,}2188 \times 10^{-3}$	0
DC	2,72182	$3{,}8001 \times 10^5$	0	1,36091	$-1{,}0207 \times 10^{-3}$	$-1{,}3891 \times 10^{-3}$	0
CB	1,70114	$3{,}8001 \times 10^5$	0	1,36091	$-6{,}3793 \times 10^{-4}$	$-8{,}6816 \times 10^{-4}$	0
ED	3	$10{,}5558 \times 10^5$	0	0,92308	$-1{,}1250 \times 10^{-3}$	$-1{,}0385 \times 10^{-3}$	0
						$\Sigma = -1{,}0385 \times 10^{-3}$	$\Sigma = 0$

$$\delta_v^B = \Sigma \bar{N} \frac{NL}{EA} + \Sigma \bar{N}\, \alpha\, \Delta T\, L - \Sigma \bar{R} \text{ x assent. apoio}$$

$$\delta_v^B = 0{,}0 - 1{,}0385 \times 10^{-3} - 0{,}004 \times 0{,}92308 - (-1) \times 0{,}003 = -1{,}731 \times 10^{-3} \text{ m} = -1{,}731 \text{ mm} \downarrow$$

- **Deslocamento do nó B**

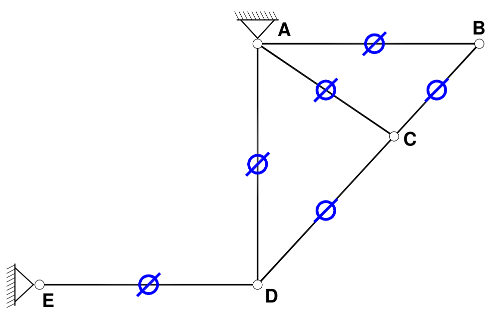
$$\delta^B = \sqrt{(\delta_h^B)^2 + (\delta_v^B)^2} = \sqrt{2{,}875^2 + 1{,}731^2} = 3{,}36 \text{ mm}$$



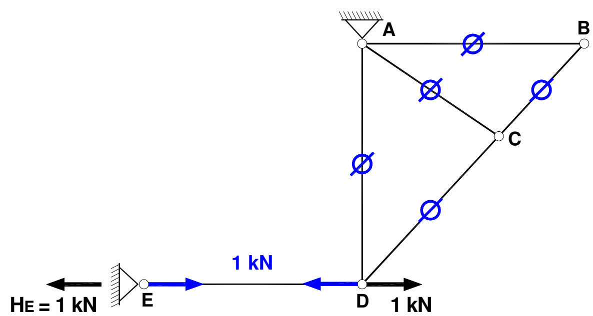
b) DESLOCAMENTO DO NÓ D

- **Deslocamento horizontal do nó D**

S - Sistema real



$\bar{S}$ - Sistema virtual



$$\bar{1} \times \delta_h^D + \Sigma \bar{R} \text{ x assent. apoio} = \Sigma \bar{N} \frac{NL}{EA} + \Sigma \bar{N}\, \alpha\, \Delta T\, L$$

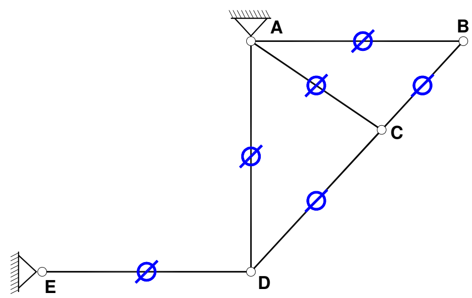
BARRAS	L (m)	E A (kPa x m^2)	N (kN)	$\bar{N}$ (kN)	α.ΔT.L	$\bar{N}$.α .ΔT. L	$\bar{N}\frac{NL}{EA}$
AB	3	3,8001 x 10^5	0	0	-1,1250 x 10^{-3}	0	0
AC	2,22953	3,8001 x 10^5	0	0	-8,3607 x 10^{-4}	0	0
AD	3,25	3,8001 x 10^5	0	0	-1,2188 x 10^{-3}	0	0
DC	2,72182	3,8001 x 10^5	0	0	-1,0207 x 10^{-3}	0	0
CB	1,70114	3,8001 x 10^5	0	0	-6,3793 x 10^{-4}	0	0
ED	3	10,5558 x 10^5	0	1	-1,1250 x 10^{-3}	-1,125 x 10^{-3}	0
						Σ= -1,125 x 10^{-3}	Σ = 0

$$\delta_h^D = \Sigma \bar{N} \frac{NL}{EA} + \Sigma \bar{N}\, \alpha\, \Delta T\, L - \Sigma \bar{R} \text{ x assent. apoio}$$

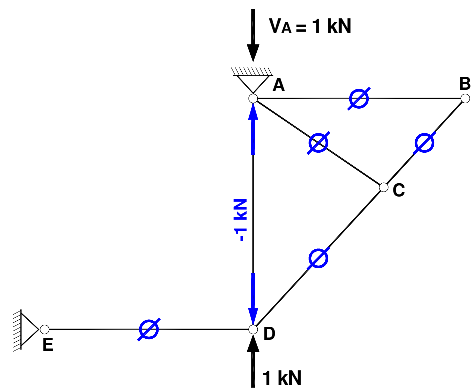
$$\delta_h^D = 0{,}0 - 1{,}125 \times 10^{-3} - 0{,}0 = -1{,}125 \times 10^{-3} \text{ m} = -1{,}125 \text{ mm} \leftarrow$$

- **Deslocamento vertical do nó D**

S - Sistema real



$\overline{S}$ - Sistema virtual



$$\overline{1} \times \delta_v^D + \Sigma \overline{R} \times \text{assent. apoio} = \Sigma \overline{N} \frac{N L}{E A} + \Sigma \overline{N} \alpha \Delta T L$$

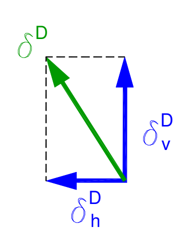
BARRAS	L (m)	E A (kPa x m²)	N (kN)	$\overline{N}$ (kN)	α.ΔT.L	$\overline{N}$.α.ΔT.L	$\overline{N}\frac{NL}{EA}$
AB	3	$3{,}8001 \times 10^5$	0	0	$-1{,}1250 \times 10^{-3}$	0	0
AC	2,22953	$3{,}8001 \times 10^5$	0	0	$-8{,}3607 \times 10^{-4}$	0	0
AD	3,25	$3{,}8001 \times 10^5$	0	-1	$-1{,}2188 \times 10^{-3}$	$1{,}2188 \times 10^{-3}$	0
DC	2,72182	$3{,}8001 \times 10^5$	0	0	$-1{,}0207 \times 10^{-3}$	0	0
CB	1,70114	$3{,}8001 \times 10^5$	0	0	$-6{,}3793 \times 10^{-4}$	0	0
ED	3	$10{,}5558 \times 10^5$	0	0	$-1{,}1250 \times 10^{-3}$	$1{,}2188 \times 10^{-3}$	0
						$\Sigma = 1{,}2188 \times 10^{-3}$	$\Sigma = 0$

$$\delta_v^D = \Sigma \overline{N} \frac{N L}{E A} + \Sigma \overline{N} \alpha \Delta T L - \Sigma \overline{R} \times \text{assent. apoio}$$

$$\delta_v^D = 0{,}0 + 1{,}2188 \times 10^{-3} - (-1) \times 0{,}003 = 4{,}219 \times 10^{-3}\ \text{m} = 4{,}219\ \text{mm} \uparrow$$

- **Deslocamento do nó D**

$$\delta^D = \sqrt{(\delta_h^D)^2 + (\delta_v^D)^2} = \sqrt{1{,}125^2 + 4{,}219^2} = 4{,}37\ \text{mm}$$

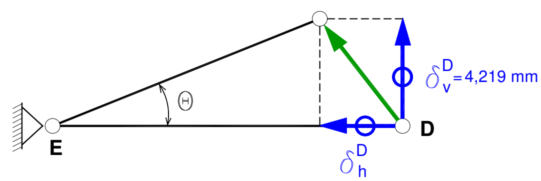


c) DETERMINAÇÃO DA FORÇA VERTICAL A APLICAR EM D PARA QUE A BARRA ED FIQUE HORIZONTAL

Na alínea anterior já foi calculado o deslocamento vertical do nó **D** (ver figura).

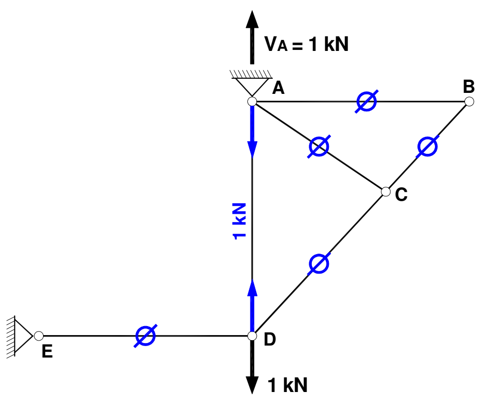
Para que a barra **ED** fique horizontal a força vertical **F** a aplicar em **D** terá de anular o deslocamento δ_V^D.

Ou seja, a força vertical **F** deverá provocar na estrutura um deslocamento de 4,219 mm (↓).

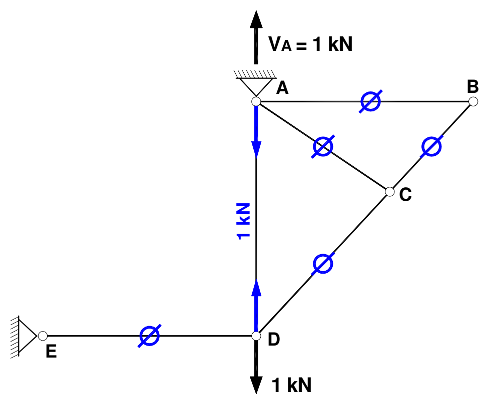


Vamos começar por calcular o deslocamento introduzido no nó **D** por uma força vertical **F=1KN**.

S - Sistema real



$\overline{\mathbf{S}}$ - Sistema virtual



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AB	3	3,8001 x 10^5	0	0	0
AC	2,22953	3,8001 x 10^5	0	0	0
AD	3,25	3,8001 x 10^5	0	1	8,5525 x 10^{-6}
DC	2,72182	3,8001 x 10^5	0	0	0
CB	1,70114	3,8001 x 10^5	0	0	0
ED	3	10,5558 x 10^5	0	0	0
					Σ = 8,5525 x 10^{-6}

$$\overline{1} \times \delta_V^D = \Sigma \overline{N} \frac{NL}{EA} \quad \Rightarrow \quad \delta_V^D = \Sigma \overline{N} \frac{NL}{EA} = 8,5525 \times 10^{-6}\ \text{m} = 0,00855\ \text{mm}\ (\downarrow)$$

Se $\quad F = 1\text{kN}\ (\downarrow) \quad \Rightarrow \quad \delta_V^D = 0,00855\ \text{mm}\ (\downarrow)$,

então para anular o deslocamento de 4,219 mm será necessário aplicar uma força:

$$F = \frac{1 \times 4,219}{0,00855} = 493,45\text{kN}\ (\downarrow)$$