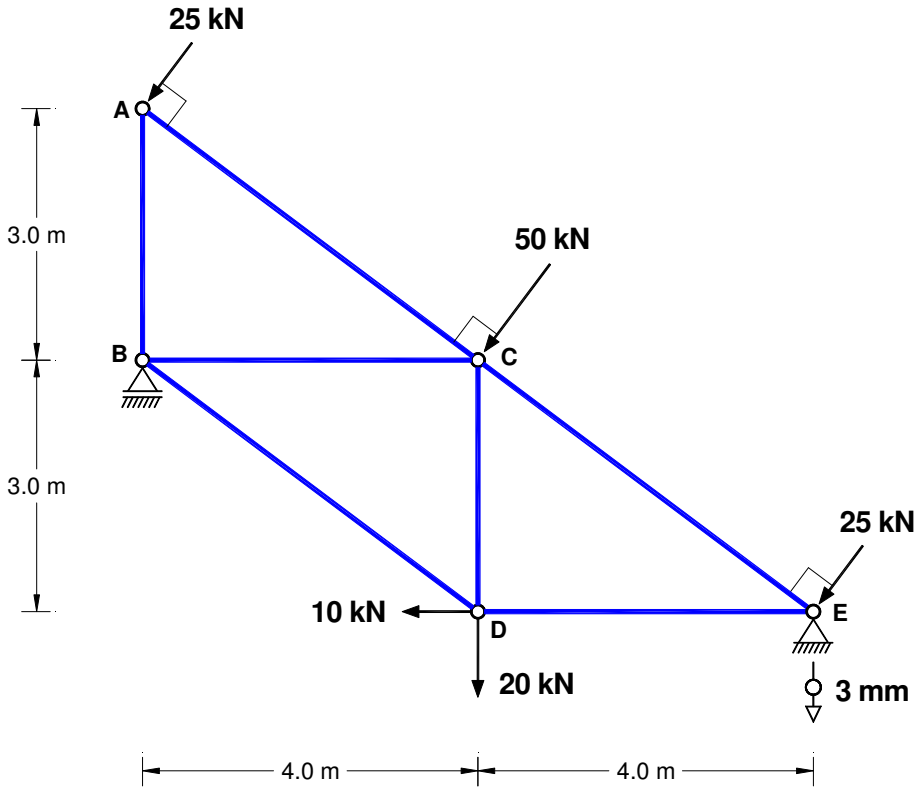
Instituto Superior de Engenharia do Porto

LICENCIATURA EM ENGENHARIA CIVIL

TEORIA DE ESTRUTURAS

TEOREMA DOS TRABALHOS VIRTUAIS



SISTEMA ARTICULADO PLANO (SAP) ISOSTÁTICO

FOLHA 3 – EX. 2

ISABEL ALVIM TELES

EXERCÍCIO

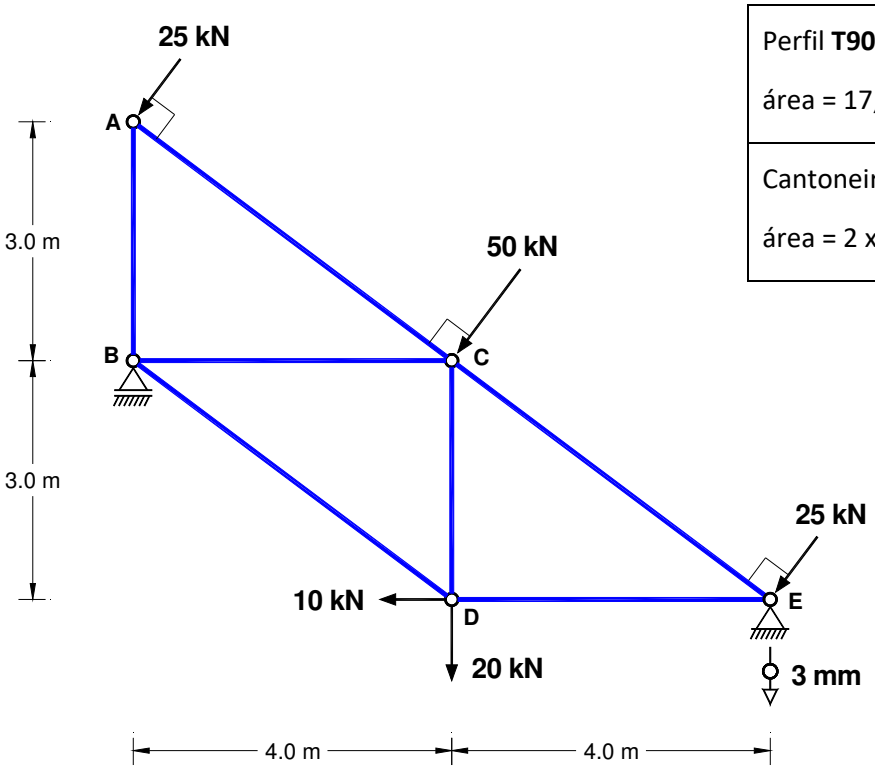
Considere a estrutura articulada plana representada na figura.

Todas as barras são em aço com E = 200 GPa e $\alpha = 1{,}2 \times 10^{-5}$ /°C.

As barras **AB**, **AC** e **CE** são constituídas por um perfil **T90**. As restantes barras são constituídas por duas cantoneiras **L90x7** soldadas.

Para além do assentamento vertical de 3 mm do apoio da direita e das forças nos nós indicadas, a estrutura está ainda submetida aos seguintes incrementos de temperatura:

Barras **AB**, **AC** e **CE**: ΔT = +30°C
Restantes barras: ΔT = +10ºC

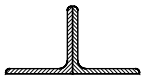


Perfil **T90**

área = 17,1 cm²

Cantoneiras **L90x7**

área = 2 x 12,2 cm²



a) Determine o deslocamento do nó **B**;

b) Determine o deslocamento do nó **D**;

c) Determine a rotação da barra **AB**;

d) Determine a rotação relativas das barras **BC** e **BD** (variação do ângulo **DBC**);

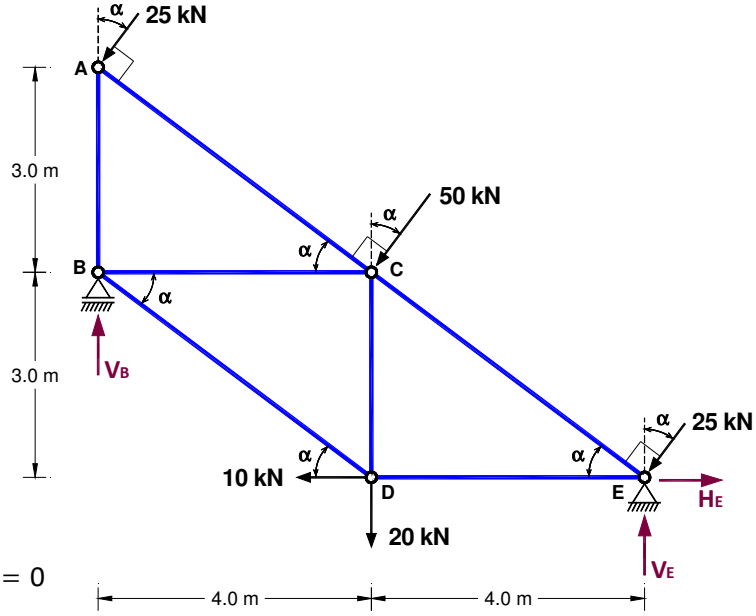
e) Determine a variação da distância entre o nó **A** e o nó **D**;

f) Determine a variação da distância entre o nó **C** e a barra **BD**.

RESOLUÇÃO

- **Reacções nos apoios**

$$\alpha \Rightarrow \begin{cases} \cos\alpha = 0{,}8 \\ \operatorname{sen}\alpha = 0{,}6 \end{cases}$$



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_E = 0 \end{cases}$$

REFERENCIAL

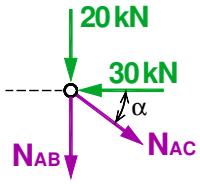
$$\begin{cases} H_E - 10 - 25\times0{,}6 - 50\times0{,}6 - 25\times0{,}6 = 0 \\ V_B + V_E - 20 - 25\times0{,}8 - 50\times0{,}8 - 25\times0{,}8 = 0 \\ 20\times4 + 50\times5 + 25\times10 - V_B\times8 = 0 \end{cases}$$

RESULTADO:

$$\begin{cases} H_E = 70.0 \text{ kN} \rightarrow \\ V_E = 27{,}5 \text{ kN} \uparrow \\ V_B = 72{,}5 \text{ kN} \uparrow \end{cases}$$

- **Sistema real - esforços instalados nas barras**

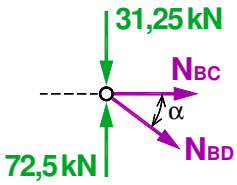
Nó A



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} 0{,}8\ N_{AC} - 15 = 0 \\ -N_{AB} - 0{,}6\ N_{AB} - 20 = 0 \end{cases}$$

$$\begin{cases} N_{AC} = 18{,}75 \text{ kN} \quad (T) \\ N_{AB} = -31{,}25 \text{ kN} \quad (C) \end{cases}$$

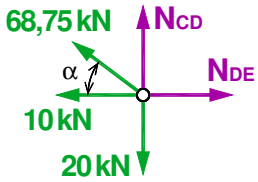
Nó B



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{BC} + 0{,}8\ N_{BD} = 0 \\ -0{,}6\ N_{BD} - 31{,}25 + 72{,}5 = 0 \end{cases}$$

$$\begin{cases} N_{BC} = -55 \text{ kN} \quad (C) \\ N_{BD} = 68{,}75 \text{ kN} \quad (T) \end{cases}$$

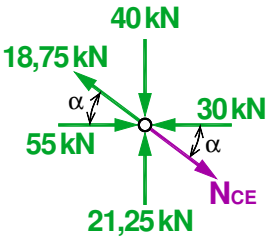
Nó D



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{DE} - 10 - 68{,}75 \times 0{,}8 = 0 \\ N_{CD} - 20 + 68{,}75 \times 0{,}6 = 0 \end{cases}$$

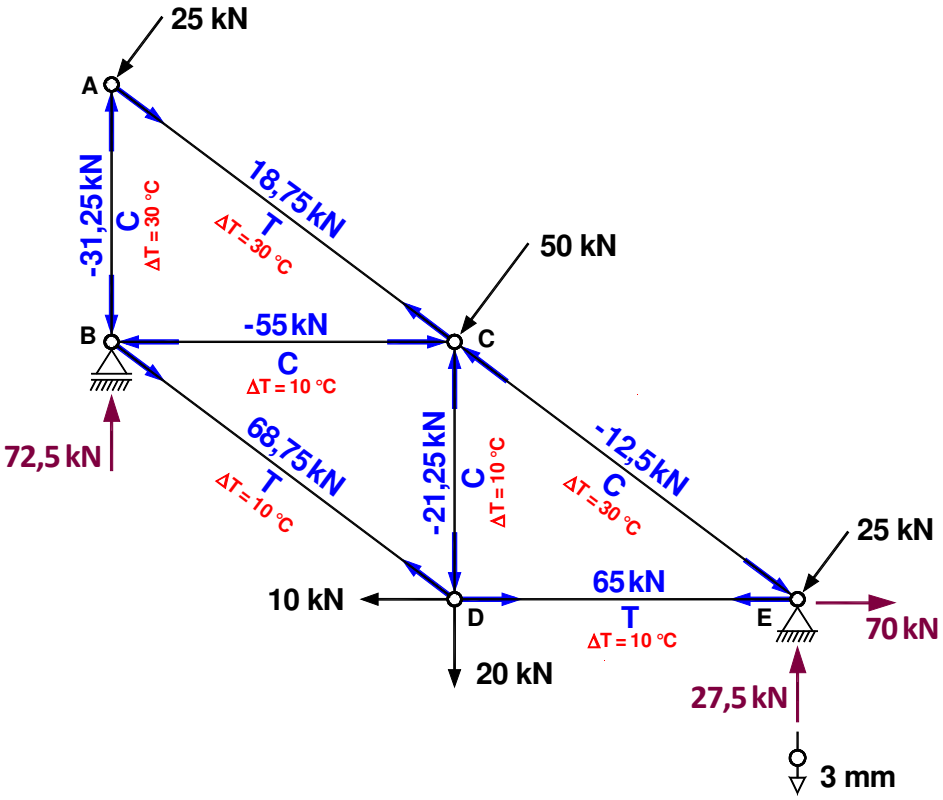
$$\begin{cases} N_{DE} = 65 \text{ kN} \quad (T) \\ N_{CD} = -21{,}25 \text{ kN} \quad (C) \end{cases}$$

Nó C



$$\sum F_x = 0 \quad \Rightarrow \quad 0{,}8\,N_{CE} - 30 + 55 - 18{,}75 \times 0{,}8 = 0$$

$$N_{CE} = -12{,}5 \text{ kN} \quad (C)$$

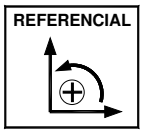


a) DESLOCAMENTO DO NÓ B

Como no **nó B** existe um apoio vertical, o seu deslocamento só pode ser horizontal.

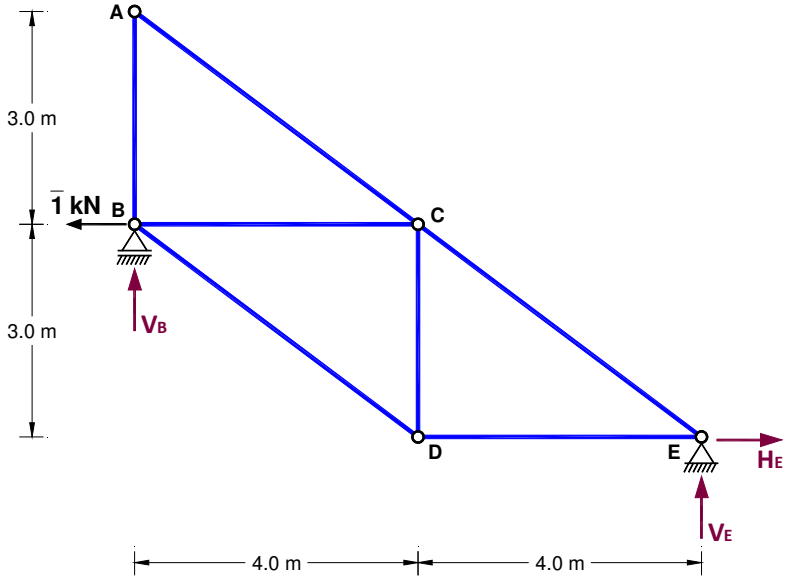
- **Sistema virtual**

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_E = 0 \end{cases}$$



$$\begin{cases} H_E - 1 = 0 \\ V_B + V_E = 0 \\ 1\times 3 - V_B \times 8 = 0 \end{cases}$$

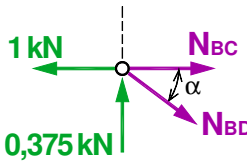
$$\begin{cases} H_E = 1 \text{ kN} \rightarrow \\ V_E = -0{,}375 \text{ kN} \downarrow \\ V_B = 0{,}375 \text{ kN} \uparrow \end{cases}$$



- **Sistema virtual - esforços instalados nas barras**

Nó A $\Rightarrow$ $\begin{cases} N_{AB} = 0 \\ N_{AC} = 0 \end{cases}$

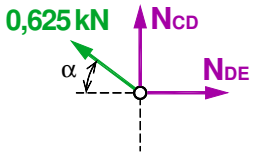
Nó B



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{BC} + 0{,}8\ N_{BD} - 1 = 0 \\ -0{,}6\ N_{BD} + 0{,}375 = 0 \end{cases}$$

$$\begin{cases} N_{BC} = 0{,}5 \text{ kN} \quad (T) \\ N_{BD} = 0{,}625 \text{ kN} \quad (T) \end{cases}$$

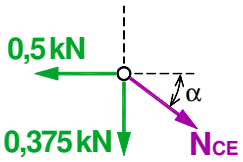
Nó D



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{DE} - 0{,}625\times 0{,}8 = 0 \\ N_{CD} + 0{,}625\times 0{,}6 = 0 \end{cases}$$

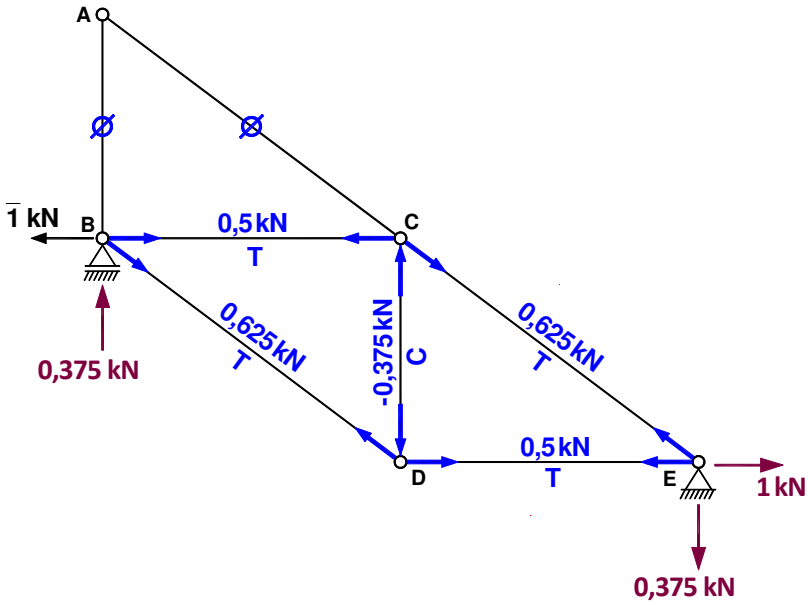
$$\begin{cases} N_{DE} = 0{,}5 \text{ kN} \quad (T) \\ N_{CD} = -0{,}375 \text{ kN} \quad (C) \end{cases}$$

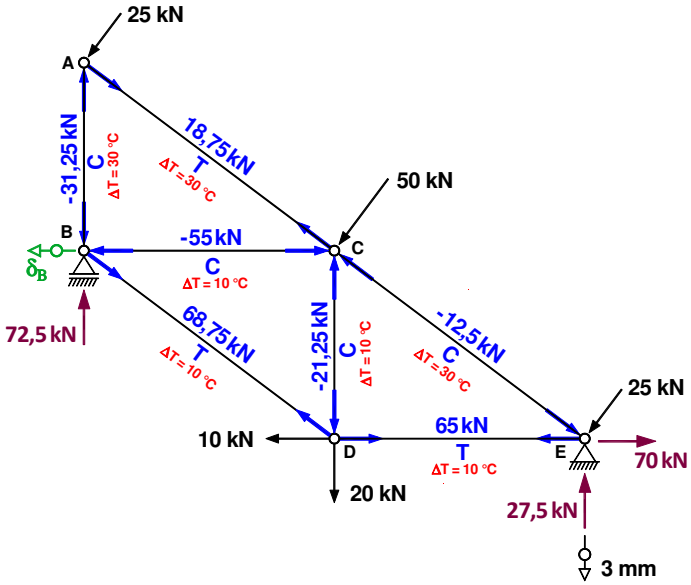
Nó C



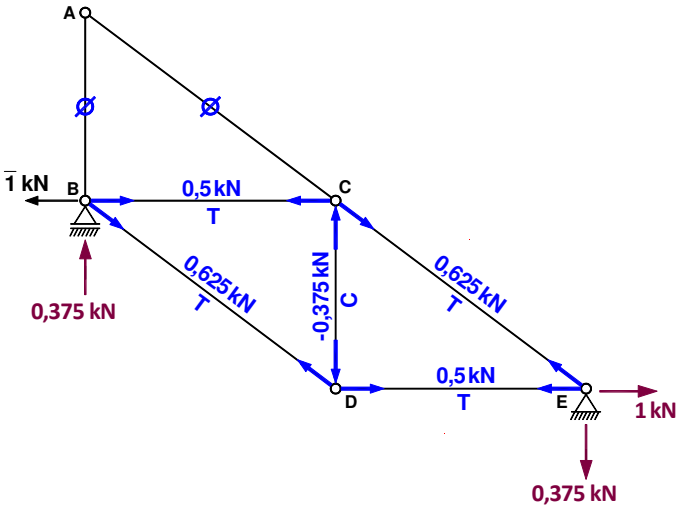
$\sum F_X = 0 \quad \Rightarrow \quad 0{,}8\,N_{CE} - 0{,}5 = 0$

$N_{CE} = 0.625 \text{ kN} \quad (T)$





S - Sistema real



$\bar{S}$ - Sistema virtual

$$\bar{1} \times \delta_B + \Sigma\, \bar{R} \times \text{assent. apoio} = \Sigma\, \bar{N} \frac{N\,L}{E\,A} + \Sigma\, \bar{N} \cdot \alpha\, \Delta T\, L$$

BARRAS	L (m)	E (kPa)	A (m^2)	N (kN)	$\bar{N}$ (kN)	ΔT (°C)	α.ΔT.L (α=1,2x10^{-5}/°C)	$\bar{N}$.α .ΔT. L	$\bar{N}\frac{NL}{EA}$
AB	3	200 x 10^6	17,1 x 10^{-4}	-31,25	0	30	108 x 10^{-5}	0	0
AC	5	200 x 10^6	17,1 x 10^{-4}	18,75	0	30	180 x 10^{-5}	0	0
CE	5	200 x 10^6	17,1 x 10^{-4}	-12,50	0,625	30	180 x 10^{-5}	112,5 x 10^{-5}	-11,422 x 10^{-5}
BC	4	200 x 10^6	24,4 x 10^{-4}	-55,00	0,50	10	48 x 10^{-5}	24 x 10^{-5}	-22,541 x 10^{-5}
BD	5	200 x 10^6	24,4 x 10^{-4}	68,75	0,625	10	60 x 10^{-5}	37,5 x 10^{-5}	44,025 x 10^{-5}
CD	3	200 x 10^6	24,4 x 10^{-4}	-21,25	-0,375	10	36 x 10^{-5}	-13,5 x 10^{-5}	4,899 x 10^{-5}
DE	4	200 x 10^6	24,4 x 10^{-4}	65,00	0,50	10	48 x 10^{-5}	24 x 10^{-5}	26,639 x 10^{-5}
Σ								184,5 x 10^{-5}	41,6 x 10^{-5}

$$\bar{1} \times \delta_B + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{N L}{E A} + \Sigma \bar{N} . \alpha \Delta T L$$

$\delta_B + (-0,375) \times (-0,003) = 41,6 \times 10^{-5} + 184,5 \times 10^{-5}$

$\delta_B = 1,14 \times 10^{-3}$ m = 1,14 mm ◄⊖

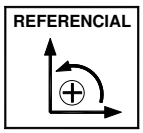
b) DESLOCAMENTO DO NÓ D

Vamos calcular separadamente o deslocamento horizontal (δ_D^h) e o deslocamento vertical (δ_D^v) do **nó D**.

b1) DESLOCAMENTO HORIZONTAL DO NÓ D (δ_D^h)

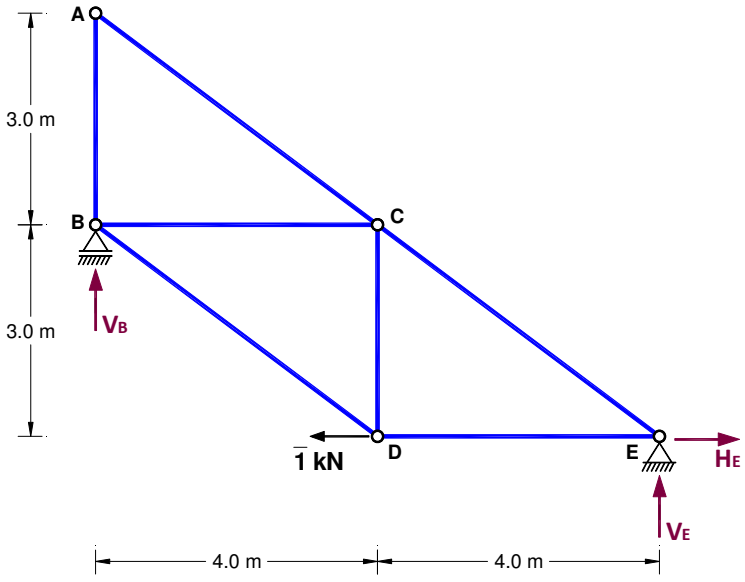
- **Sistema virtual**

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_E = 0 \end{cases}$$

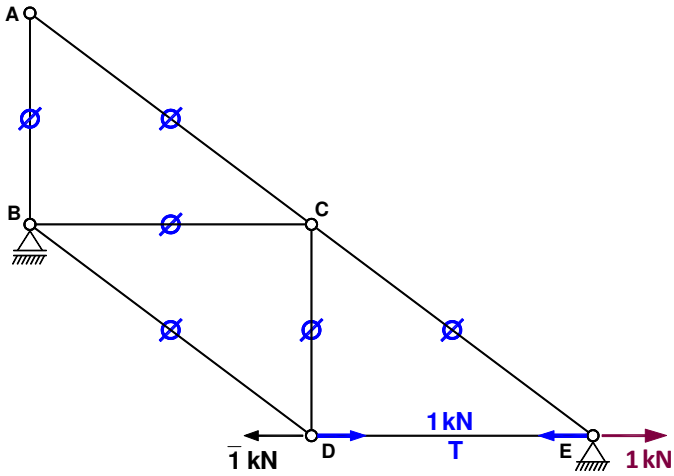


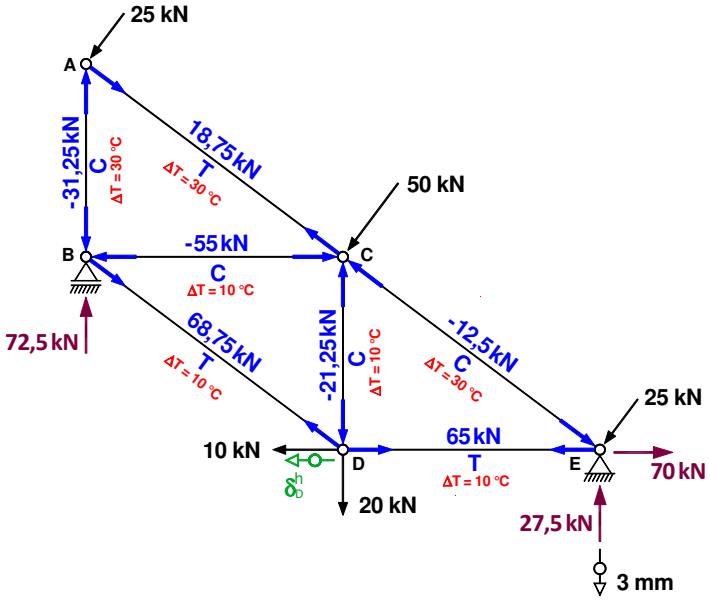
$$\begin{cases} H_E - 1 = 0 \\ V_B + V_E = 0 \\ V_B \times 8 = 0 \end{cases}$$

$$\begin{cases} H_E = 1 \text{ kN} \rightarrow \\ V_E = 0 \\ V_B = 0 \end{cases}$$

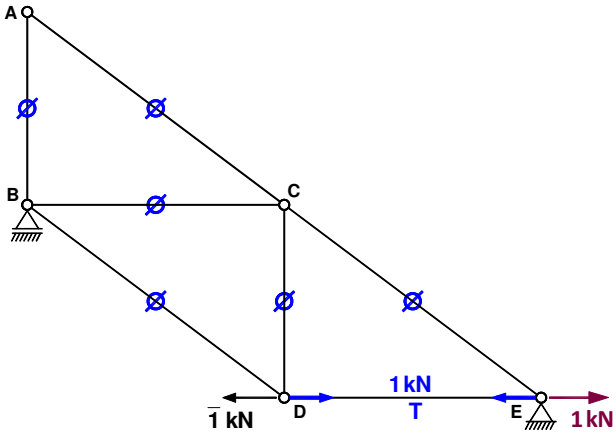


- **Sistema virtual - esforços instalados nas barras**





S - Sistema real



$\overline{\mathbf{S}}$ - Sistema virtual

$$\bar{1} \times \delta_D^h + \Sigma\, \bar{R} \times \text{assent. apoio} = \Sigma\, \bar{N}\, \frac{N\, L}{E\, A} + \Sigma\, \bar{N} \,.\, \alpha\, \Delta T\, L$$

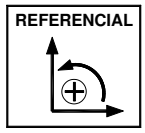
$$\delta_D^h = 1x\, \frac{65 \times 4}{200x10^6 \times\ 24{,}4x10^{-4}} + 1 \times 1{,}2x10^{-5} \times 10 \times 4$$

$$\delta_D^h = 1{,}013 \times 10^{-3}\ \text{m} = 1{,}013\ \text{mm} \leftarrow$$

b2) DESLOCAMENTO VERTICAL DO NÓ D (δ_D^v)

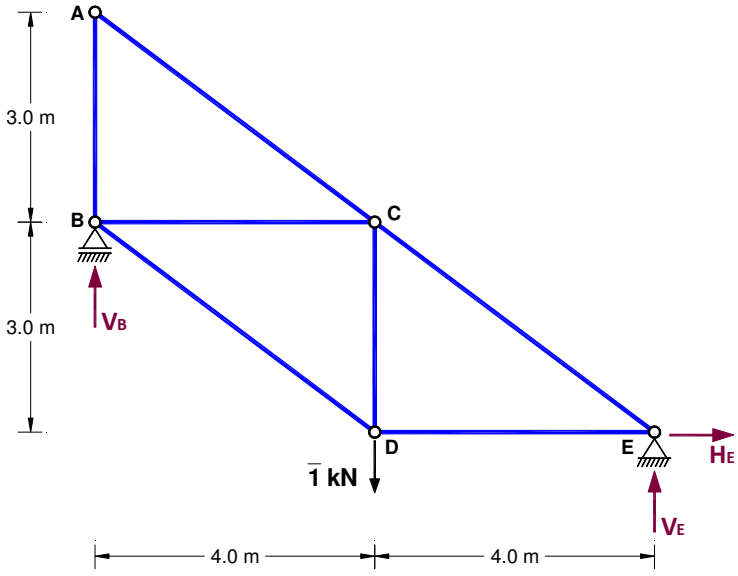
- **Sistema virtual**

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_E = 0 \end{cases}$$



$$\begin{cases} H_E = 0 \\ V_B + V_E = 1 \\ 1\times 4 - V_B \times 8 = 0 \end{cases}$$

$$\begin{cases} H_E = 0 \\ V_E = 0{,}5 \text{ kN} \uparrow \\ V_B = 0{,}5 \text{ kN} \uparrow \end{cases}$$

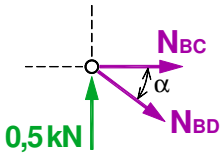


- **Sistema virtual - esforços instalados nas barras**

Nó A $\Rightarrow$

$$\begin{cases} N_{AB} = 0 \\ N_{AC} = 0 \end{cases}$$

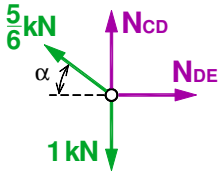
Nó B



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{BC} + 0{,}8\ N_{BD} = 0 \\ -0{,}6\ N_{BD} + 0{,}5 = 0 \end{cases}$$

$$\begin{cases} N_{BC} = -\frac{2}{3} \text{ kN} \quad (C) \\ N_{BD} = \frac{5}{6} \text{ kN} \quad (T) \end{cases}$$

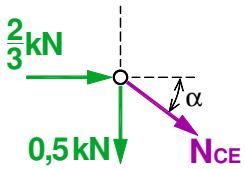
Nó D



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{DE} - \frac{5}{6}\times 0{,}8 = 0 \\ N_{CD} + \frac{5}{6}\times 0{,}6 - 1 = 0 \end{cases}$$

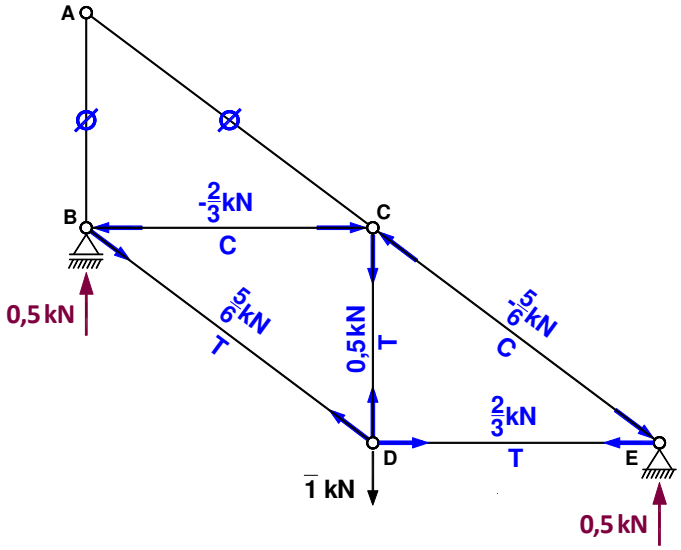
$$\begin{cases} N_{DE} = \frac{2}{3} \text{ kN} \quad (T) \\ N_{CD} = 0{,}5 \text{ kN} \quad (T) \end{cases}$$

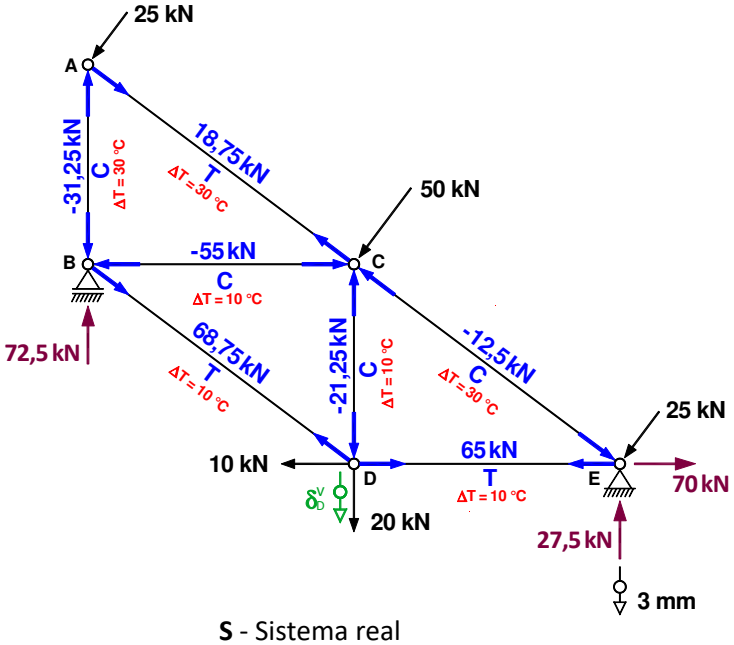
Nó C



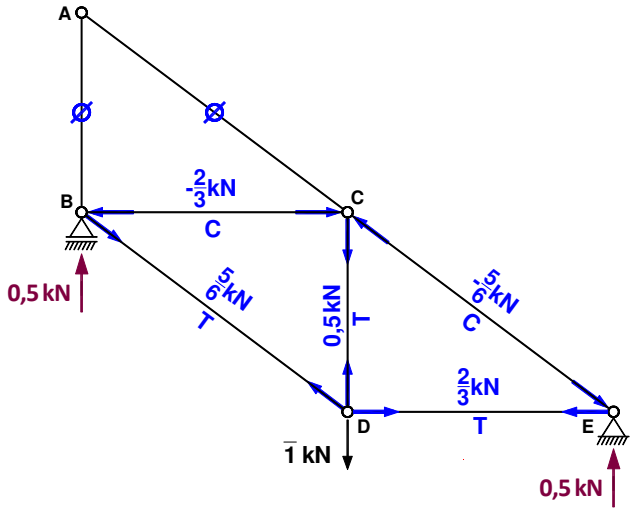
$$\sum F_y = 0 \quad \Rightarrow \quad -0{,}6\,N_{CE} - 0{,}5 = 0$$

$$N_{CE} = -\frac{5}{6}\ \text{kN} \quad (C)$$





S - Sistema real



$\bar{S}$ - Sistema virtual

$$\bar{1} \times \delta_D^v + \Sigma\, \bar{R} \times \text{assent. apoio} = \Sigma\, \bar{N}\, \frac{N\,L}{E\,A} + \Sigma\, \bar{N} \,.\, \alpha\, \Delta T\, L$$

BARRAS	L (m)	E (kPa)	A (m^2)	N (kN)	$\bar{N}$ (kN)	ΔT (°C)	α.ΔT.L (α=1,2x10^{-5}/°C)	$\bar{N}$.α .ΔT. L	$\bar{N}\frac{NL}{EA}$
AB	3	200×10^6	$17{,}1 \times 10^{-4}$	-31,25	0	30	108×10^{-5}	0	0
AC	5	200×10^6	$17{,}1 \times 10^{-4}$	18,75	0	30	180×10^{-5}	0	0
CE	5	200×10^6	$17{,}1 \times 10^{-4}$	-12,50	$-\frac{5}{6}$	30	180×10^{-5}	-150×10^{-5}	$15{,}229 \times 10^{-5}$
BC	4	200×10^6	$24{,}4 \times 10^{-4}$	-55,00	$-\frac{2}{3}$	10	48×10^{-5}	-32×10^{-5}	$30{,}055 \times 10^{-5}$
BD	5	200×10^6	$24{,}4 \times 10^{-4}$	68,75	$\frac{5}{6}$	10	60×10^{-5}	50×10^{-5}	$58{,}700 \times 10^{-5}$
CD	3	200×10^6	$24{,}4 \times 10^{-4}$	-21,25	0,5	10	36×10^{-5}	18×10^{-5}	$-6{,}532 \times 10^{-5}$
DE	4	200×10^6	$24{,}4 \times 10^{-4}$	65,00	$\frac{2}{3}$	10	48×10^{-5}	32×10^{-5}	$35{,}519 \times 10^{-5}$
Σ								-82×10^{-5}	$132{,}97 \times 10^{-5}$

$$\bar{1} \times \delta_D^v + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{N L}{E A} + \Sigma \bar{N} . \alpha \Delta T L$$

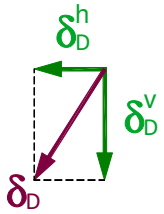
$\delta_D^v + 0{,}5 \times (-0{,}003) = -82 \times 10^{-5} + 132{,}97 \times 10^{-5}$

$\delta_D^v = 2{,}010 \times 10^{-3}$ m = 2,010 mm ↓

b3) DESLOCAMENTO TOTAL DO NÓ D (δ_D)

$\delta_D^h = 1{,}013 \times 10^{-3}$ m = 1,013 mm ←

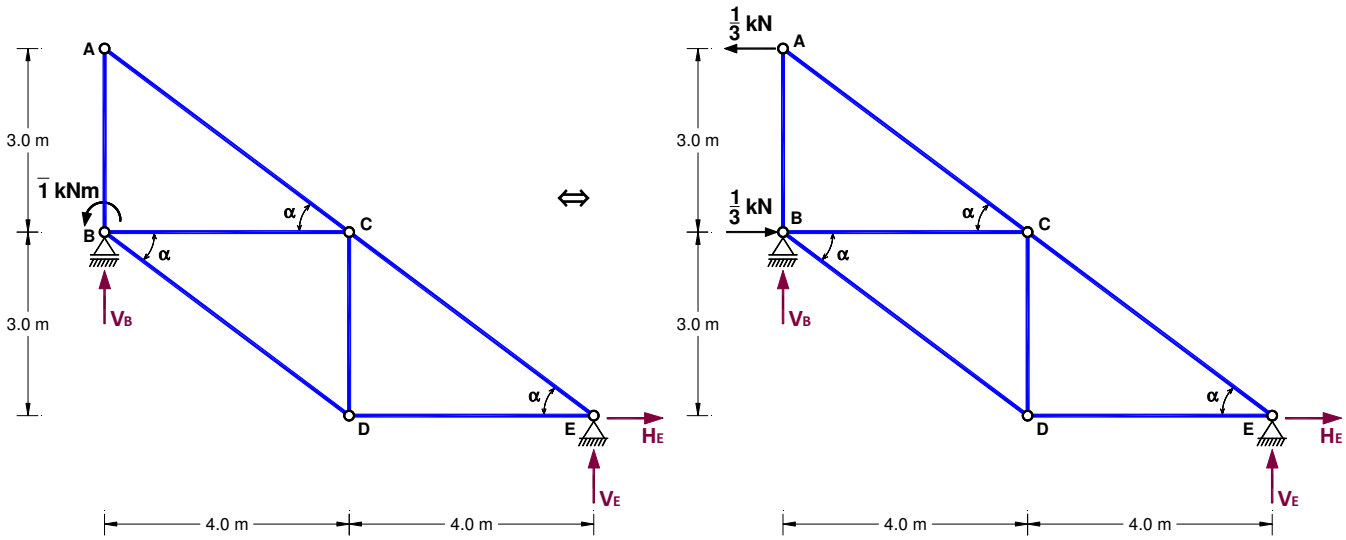
$\delta_D^v = 2{,}010 \times 10^{-3}$ m = 2,010 mm ↓



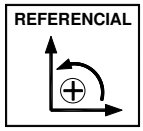
$$\delta_D = \sqrt{(\delta_D^h)^2 + (\delta_D^v)^2} = \sqrt{1{,}013^2 + 2{,}010^2} = 2{,}25 \text{ mm} \swarrow$$

c) ROTAÇÃO DA BARRA $\overline{AB}$

- **Sistema virtual**

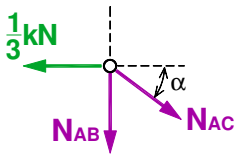


$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_E = 0 \end{cases} \Rightarrow \begin{cases} H_E = 0 \\ V_B + V_E = 1 \\ 1 - V_B \times 8 = 0 \end{cases} \Rightarrow \begin{cases} H_E = 0 \\ V_E = -0{,}125 \text{ kN} \downarrow \\ V_B = 0{,}125 \text{ kN} \uparrow \end{cases}$$



- **Sistema virtual - esforços instalados nas barras**

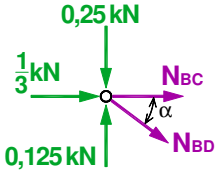
Nó A



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} 0{,}8\ N_{AC} - \frac{1}{3} = 0 \\ -0{,}6\ N_{AC} - N_{AB} = 0 \end{cases}$$

$$\begin{cases} N_{AC} = \frac{5}{12} \text{ kN} \quad (T) \\ N_{AB} = -0{,}25 \text{ kN} \quad (C) \end{cases}$$

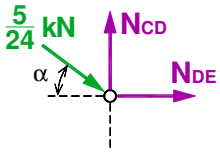
Nó B



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{BC} + 0{,}8\ N_{BD} + \frac{1}{3} = 0 \\ -0{,}6\ N_{BD} - 0{,}25 + 0{,}125 = 0 \end{cases}$$

$$\begin{cases} N_{BC} = -\frac{1}{6} \text{ kN} \quad (C) \\ N_{BD} = -\frac{5}{24} \text{ kN} \quad (C) \end{cases}$$

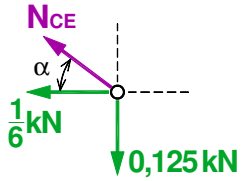
Nó D



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{DE} + \frac{5}{24}\times 0{,}8 = 0 \\ N_{CD} - \frac{5}{24}\times 0{,}6 = 0 \end{cases}$$

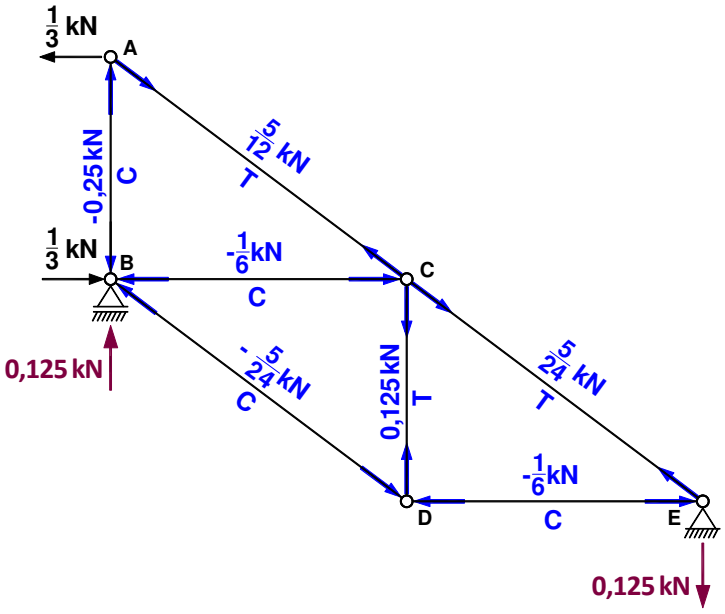
$$\begin{cases} N_{DE} = -\frac{1}{6}\ \text{kN} \quad (C) \\ N_{CD} = 0{,}125\ \text{kN} \quad (T) \end{cases}$$

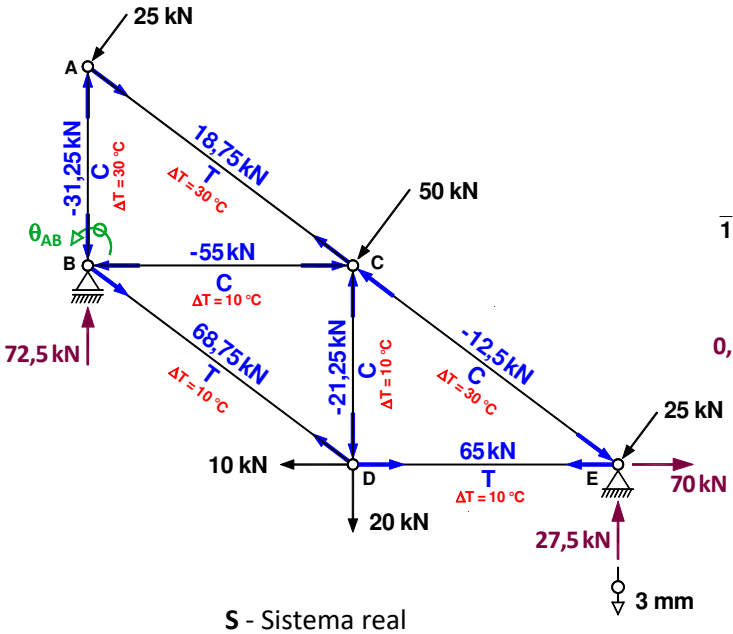
Nó E



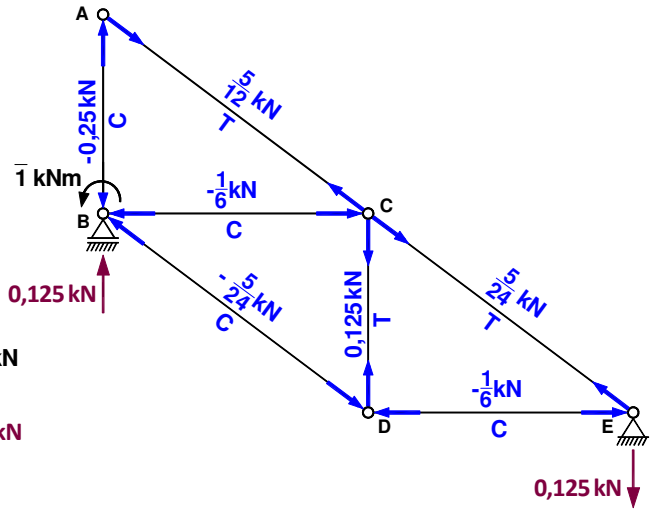
$$\sum F_y = 0 \quad \Rightarrow \quad 0{,}6\,N_{CE} - 0{,}125 = 0$$

$$N_{CE} = \frac{5}{24}\ \text{kN} \quad (T)$$





S - Sistema real



$\overline{\mathbf{S}}$ - Sistema virtual

$$\bar{1} \times \theta_{AB} + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{N L}{E A} + \Sigma \bar{N} . \alpha \Delta T L$$

BARRAS	L (m)	E (kPa)	A (m^2)	N (kN)	$\bar{N}$ (kN)	ΔT (ºC)	α.ΔT.L (α=1,2x10^{-5}/ºC)	$\bar{N}$.α .ΔT. L	$\bar{N} \frac{N L}{E A}$
AB	3	200×10^6	$17{,}1 \times 10^{-4}$	-31,25	-0,25	30	108×10^{-5}	-27×10^{-5}	$6{,}853 \times 10^{-5}$
AC	5	200×10^6	$17{,}1 \times 10^{-4}$	18,75	$\frac{5}{12}$	30	180×10^{-5}	75×10^{-5}	$11{,}422 \times 10^{-5}$
CE	5	200×10^6	$17{,}1 \times 10^{-4}$	-12,50	$\frac{5}{24}$	30	180×10^{-5}	$37{,}5 \times 10^{-5}$	$-3{,}807 \times 10^{-5}$
BC	4	200×10^6	$24{,}4 \times 10^{-4}$	-55,00	$-\frac{1}{6}$	10	48×10^{-5}	-8×10^{-5}	$7{,}514 \times 10^{-5}$
BD	5	200×10^6	$24{,}4 \times 10^{-4}$	68,75	$-\frac{5}{24}$	10	60×10^{-5}	$-12{,}5 \times 10^{-5}$	$-14{,}675 \times 10^{-5}$
CD	3	200×10^6	$24{,}4 \times 10^{-4}$	-21,25	0,125	10	36×10^{-5}	$4{,}5 \times 10^{-5}$	$-1{,}633 \times 10^{-5}$
DE	4	200×10^6	$24{,}4 \times 10^{-4}$	65,00	$-\frac{1}{6}$	10	48×10^{-5}	-8×10^{-5}	$-8{,}880 \times 10^{-5}$
Σ								$61{,}5 \times 10^{-5}$	$-3{,}206 \times 10^{-5}$

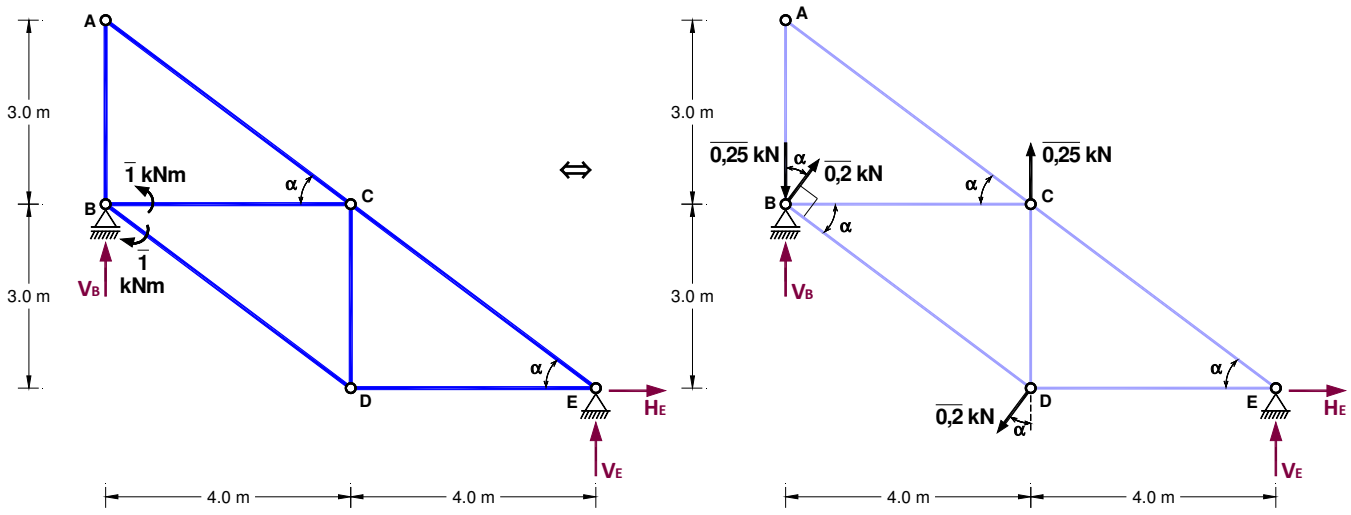
$$\bar{1} \times \theta_{AB} + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{N L}{E A} + \Sigma \bar{N} . \alpha \Delta T L$$

$$\theta_{AB} + (-0{,}125) \times (-0{,}003) = 61{,}5 \times 10^{-5} - 3{,}206 \times 10^{-5}$$

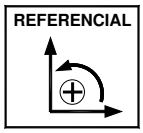
$$\theta_{AB} = 2{,}08 \times 10^{-4} \text{ rad}$$

d) ROTAÇÃO RELATIVA DAS BARRAS $\overline{BC}$ E $\overline{BD}$ (VARIAÇÃO DO ÂNGULO DBC)

- **Sistema virtual**



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_E = 0 \end{cases} \Rightarrow \begin{cases} H_E = 0 \\ V_B = 0 \\ V_E = 0 \end{cases}$$



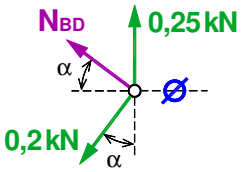
- **Sistema virtual - esforços instalados nas barras**

Nó A $\Rightarrow \begin{cases} N_{AB} = 0 \\ N_{AC} = 0 \end{cases}$

Nó E $\Rightarrow \begin{cases} N_{CE} = 0 \\ N_{DE} = 0 \end{cases}$

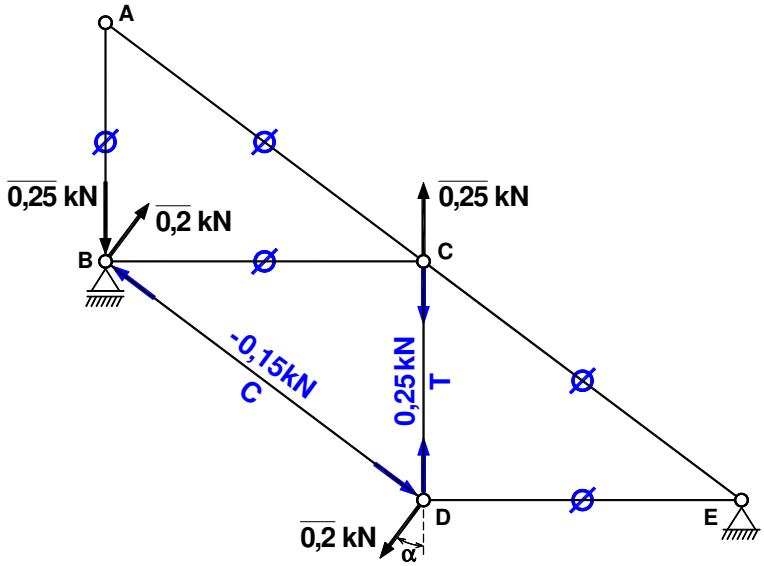
Nó C $\Rightarrow \begin{cases} N_{BC} = 0 \\ N_{CD} = 0{,}25\,\text{kN} \quad (T) \end{cases}$

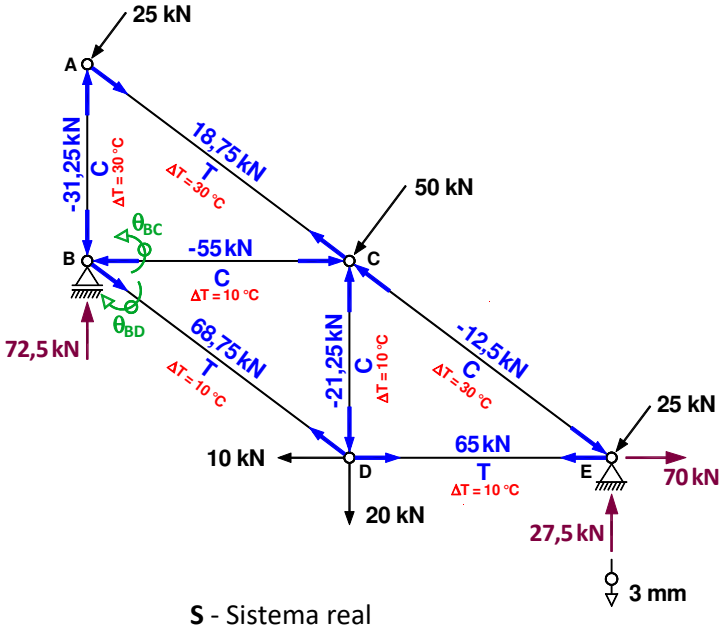
Nó D



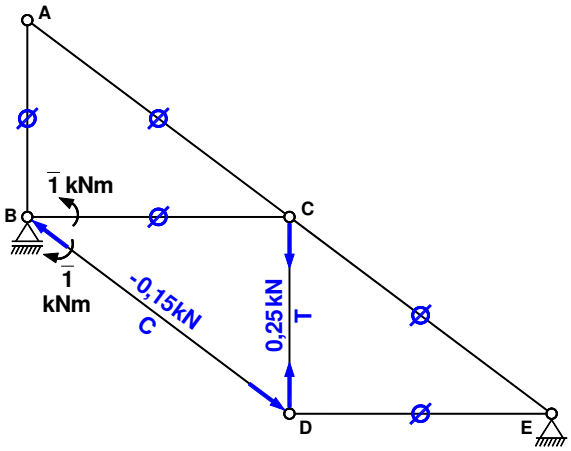
$$\sum F_x = 0 \quad \Rightarrow \quad -0{,}8\,N_{BD} - 0{,}2\times 0{,}6 = 0$$

$$N_{CE} = -0{,}15 \text{ kN} \quad (C)$$





S - Sistema real



$\bar{S}$ - Sistema virtual

$$\bar{1} \times \theta_{BC} + \bar{1} \times \theta_{BD} + \Sigma\, \bar{R} \times \text{assent. apoio} = \Sigma\, \bar{N}\, \frac{N\,L}{E\,A} + \Sigma\, \bar{N} \,.\, \alpha\, \Delta T\, L$$

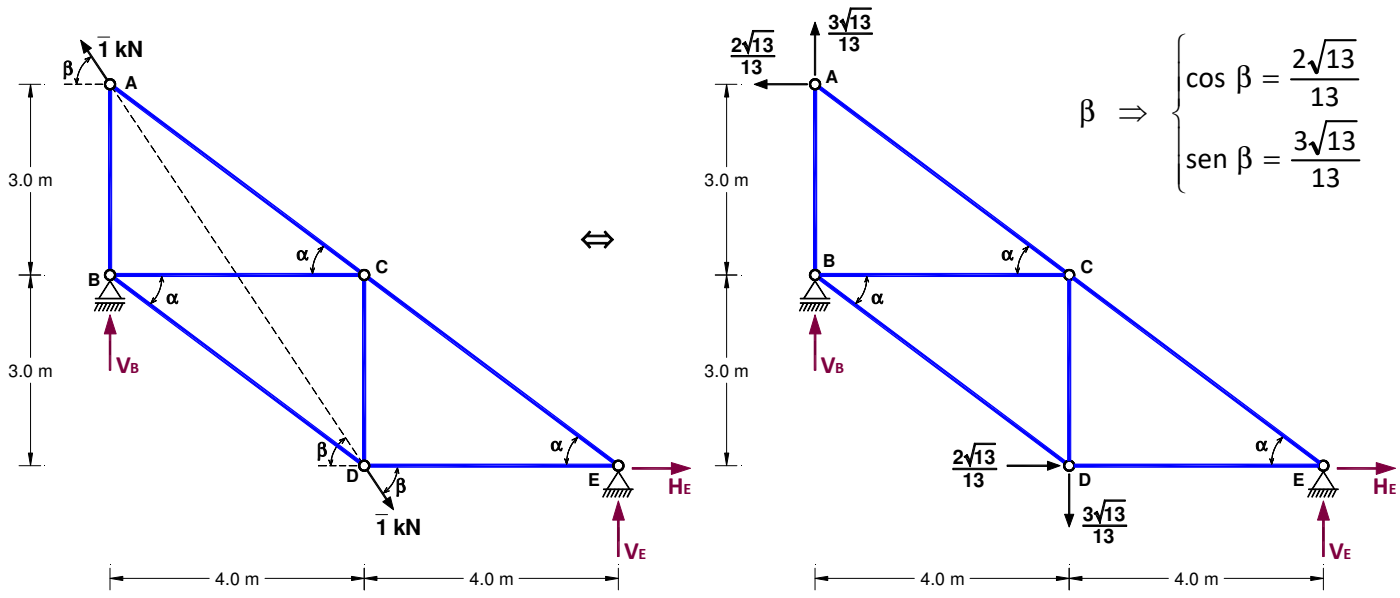
$$\theta_{CB} + \theta_{BD} = -0{,}15 \times \frac{68{,}75 \times 5}{200\text{x}10^{6} \times 24{,}4\text{x}10^{-4}} + 0{,}25 \times \frac{-21{,}25 \times 3}{200\text{x}10^{6} \times 24{,}4\text{x}10^{-4}} +$$

$$- 0{,}15 \times 1{,}2\text{x}10^{-5} \times 10 \times 5 + 0{,}25 \times 1{,}2\text{x}10^{-5} \times 10 \times 3$$

$\theta_{CB} + \theta_{BD}$ = **variação do ângulo DBC** = $-1{,}38 \times 10^{-5}$ rad (o ângulo vai diminuir)

e) VARIAÇÃO DA DISTÂNCIA ENTRE O NÓ A E O NÓ D

- **Sistema virtual**

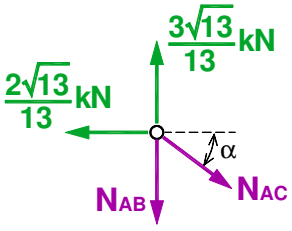


$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_E = 0 \end{cases} \Rightarrow \begin{cases} H_E = 0 \\ V_B = 0 \\ V_E = 0 \end{cases}$$

REFERENCIAL

- **Sistema virtual - esforços instalados nas barras**

Nó A

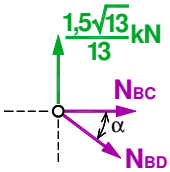


$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} 0{,}8\ N_{AC} - \frac{2\sqrt{13}}{13} = 0 \\ -0{,}6\ N_{AC} - N_{AB} + \frac{3\sqrt{13}}{13} = 0 \end{cases}$$

$$\begin{cases} N_{AC} = \frac{2{,}5\sqrt{13}}{13}\ \text{kN} \quad (T) \\ N_{AB} = \frac{1{,}5\sqrt{13}}{13}\ \text{kN} \quad (T) \end{cases}$$

Nó E $\Rightarrow \begin{cases} N_{CE} = 0 \\ N_{DE} = 0 \end{cases}$

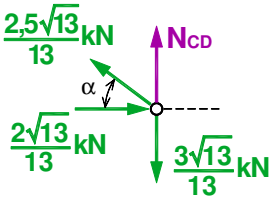
Nó B



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{BC} + 0{,}8\ N_{BD} = 0 \\ -0{,}6\ N_{BD} + \dfrac{1{,}5\sqrt{13}}{13} = 0 \end{cases}$$

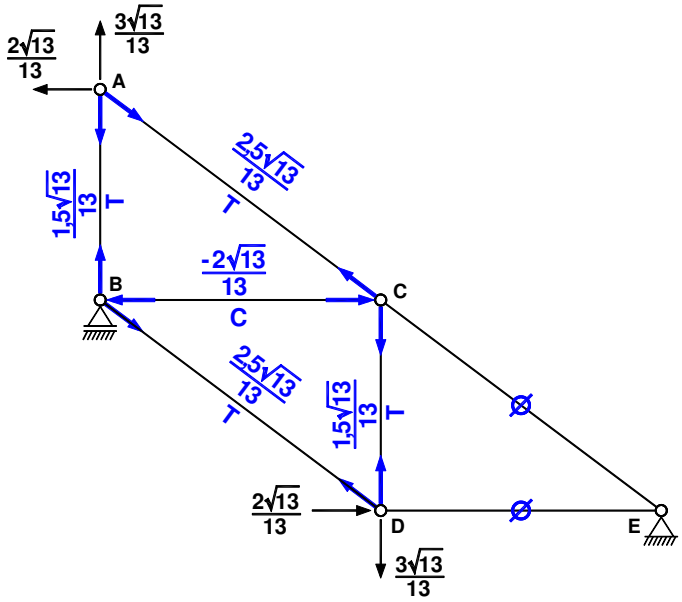
$$\begin{cases} N_{BC} = -\dfrac{2\sqrt{13}}{13}\ \text{kN} \quad (C) \\ N_{BD} = \dfrac{2{,}5\sqrt{13}}{13}\ \text{kN} \quad (T) \end{cases}$$

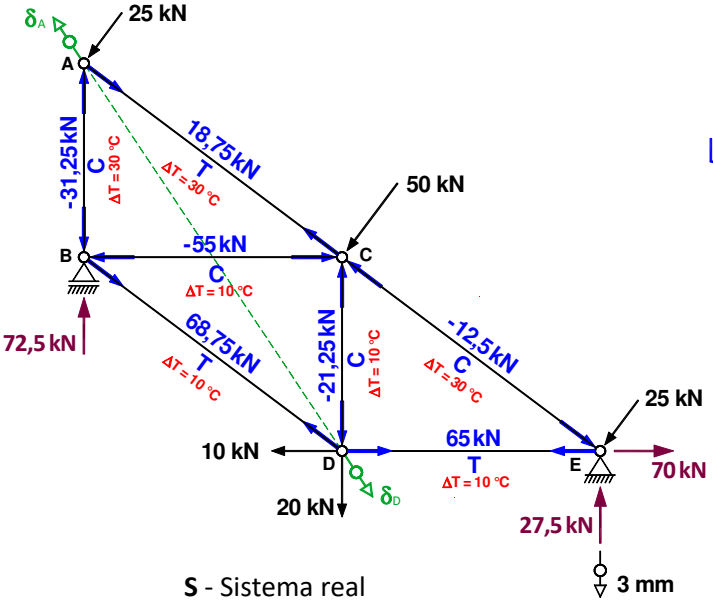
Nó D



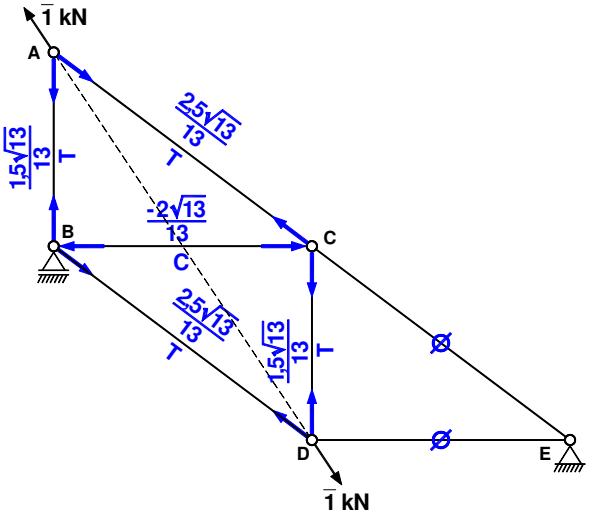
$$\sum F_y = 0 \quad \Rightarrow \quad N_{CD} - \frac{3\sqrt{13}}{13} + 0{,}6 \times \frac{2{,}5\sqrt{13}}{13} = 0$$

$$N_{CD} = \frac{1{,}5\sqrt{13}}{13}\ \text{kN} \quad (T)$$





S - Sistema real



$\overline{\mathbf{S}}$ - Sistema virtual

$$\bar{1} \times \delta_A + \bar{1} \times \delta_D + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{N\,L}{E\,A} + \Sigma \bar{N} \cdot \alpha\, \Delta T\, L$$

BARRAS	**L** (m)	**E** (kPa)	**A** (m^2)	**N** (kN)	$\bar{N}$ (kN)	**ΔT** (°C)	**α.ΔT.L** ($\alpha=1{,}2\times10^{-5}$/°C)	$\bar{N}$ **.α .ΔT. L**	$\bar{N}\frac{NL}{EA}$
AB	3	200×10^6	$17{,}1 \times 10^{-4}$	-31,25	$\frac{1{,}5\sqrt{13}}{13}$	30	108×10^{-5}	$\frac{162\sqrt{13}}{13}\times10^{-5}$	$-11{,}404 \times 10^{-5}$
AC	5	200×10^6	$17{,}1 \times 10^{-4}$	18,75	$\frac{2{,}5\sqrt{13}}{13}$	30	180×10^{-5}	$\frac{450\sqrt{13}}{13}\times10^{-5}$	$19{,}007 \times 10^{-5}$
CE	5	200×10^6	$17{,}1 \times 10^{-4}$	-12,50	0	30	180×10^{-5}	0	0
BC	4	200×10^6	$24{,}4 \times 10^{-4}$	-55,00	$-\frac{2\sqrt{13}}{13}$	10	48×10^{-5}	$-\frac{96\sqrt{13}}{13}\times10^{-5}$	$25{,}007 \times 10^{-5}$
BD	5	200×10^6	$24{,}4 \times 10^{-4}$	68,75	$\frac{2{,}5\sqrt{13}}{13}$	10	60×10^{-5}	$\frac{150\sqrt{13}}{13}\times10^{-5}$	$48{,}841 \times 10^{-5}$
CD	3	200×10^6	$24{,}4 \times 10^{-4}$	-21,25	$\frac{1{,}5\sqrt{13}}{13}$	10	36×10^{-5}	$\frac{54\sqrt{13}}{13}\times10^{-5}$	$-5{,}435 \times 10^{-5}$
DE	4	200×10^6	$24{,}4 \times 10^{-4}$	65,00	0	10	48×10^{-5}	0	0
Σ								$199{,}69 \times 10^{-5}$	$76{,}016 \times 10^{-5}$

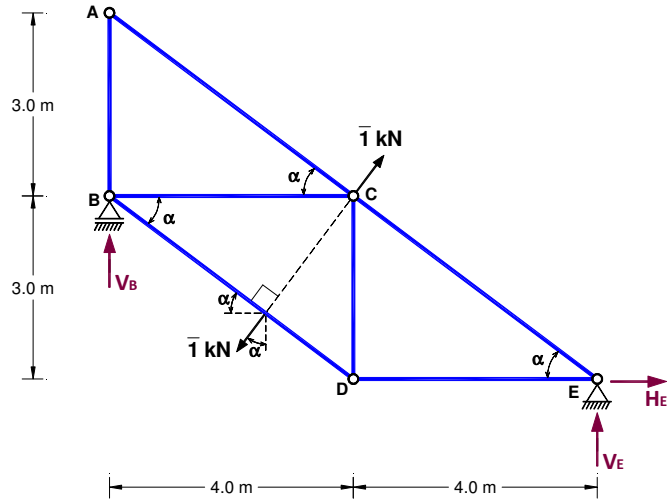
$$\bar{1} \times \delta_A + \bar{1} \times \delta_D + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{N\,L}{E\,A} + \Sigma \bar{N} \cdot \alpha\, \Delta T\, L$$

$\delta_A + \delta_D = 76{,}0616\text{x}10^{-5} + 199{,}69\text{x}10^{-5} = 2{,}757\text{x}10^{-5}\text{m}$

$\delta_A + \delta_D$ = **variação da distância entre o nó A e o nó B** = 2,757 mm (afastamento)

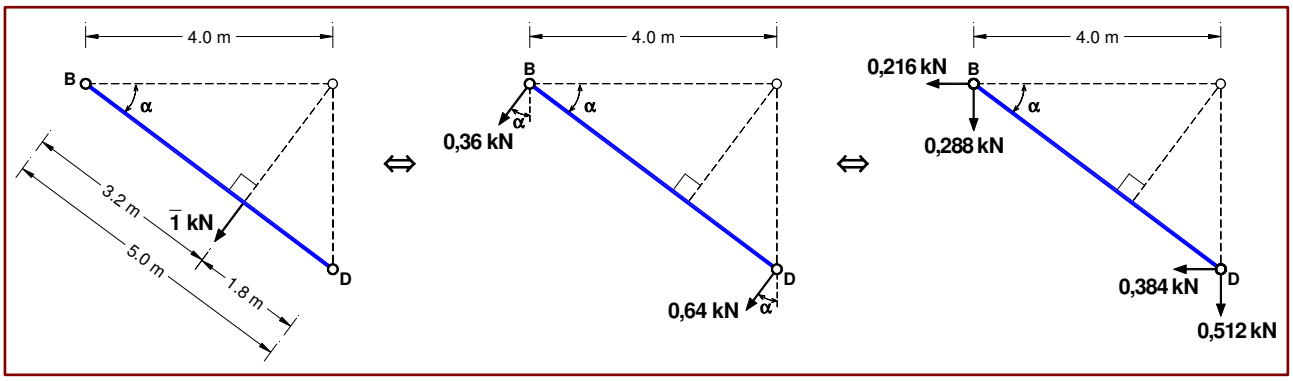
f) VARIAÇÃO DA DISTÂNCIA ENTRE O NÓ C E A BARRA $\overline{BD}$

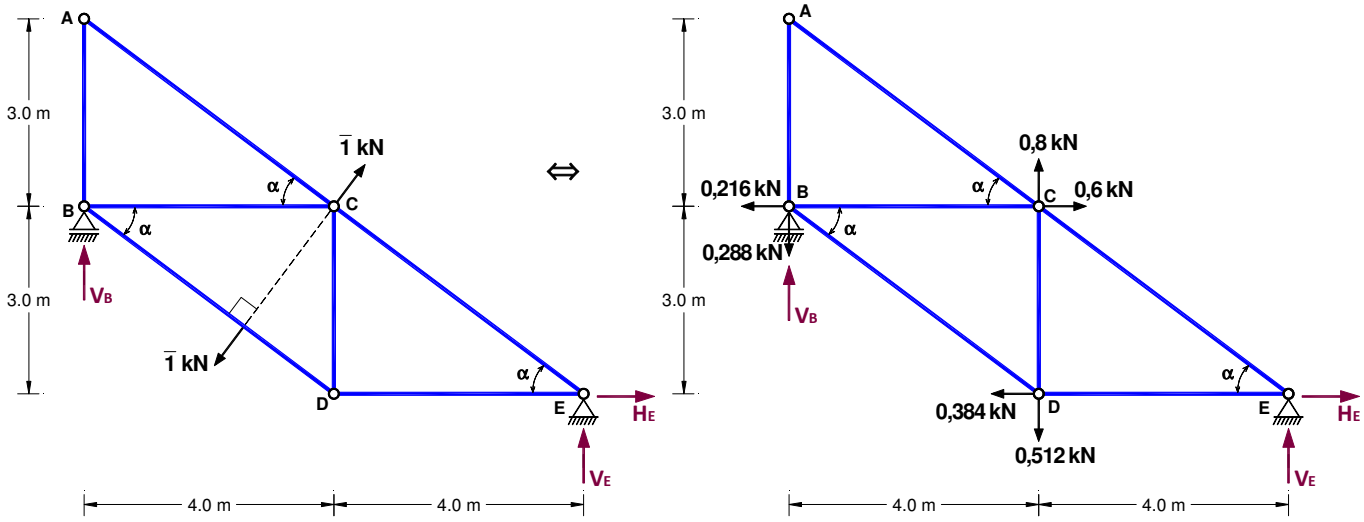
- **Sistema virtual**



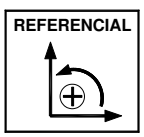
$$\alpha \Rightarrow \begin{cases} \cos\alpha = 0{,}8 \\ \operatorname{sen}\alpha = 0{,}6 \end{cases}$$

$$\begin{cases} F_B = \dfrac{1{,}8}{5} = 0{,}36 \text{ kN} \\ F_D = \dfrac{3{,}2}{5} = 0{,}64 \text{ kN} \end{cases}$$





$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_E = 0 \end{cases} \Rightarrow \begin{cases} H_E = 0 \\ V_B = 0 \\ V_E = 0 \end{cases}$$

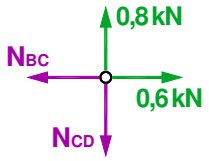


- **Sistema virtual - esforços instalados nas barras**

Nó A $\Rightarrow \begin{cases} N_{AB} = 0 \\ N_{AC} = 0 \end{cases}$

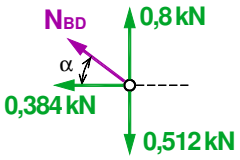
Nó E $\Rightarrow \begin{cases} N_{CE} = 0 \\ N_{DE} = 0 \end{cases}$

Nó C



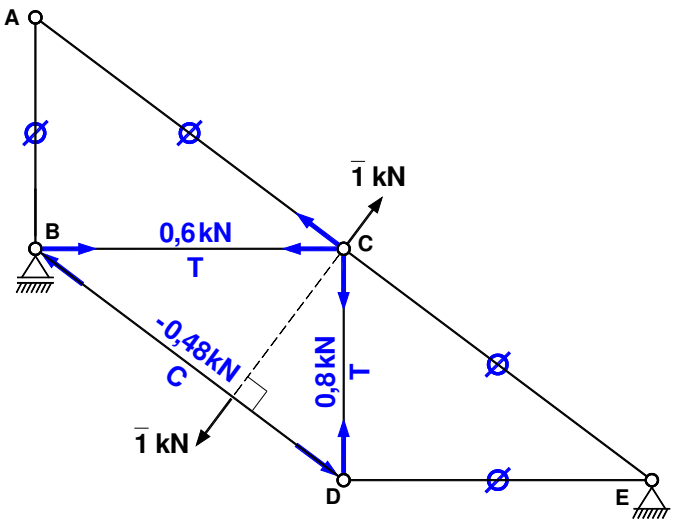
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \Rightarrow \begin{cases} N_{BC} = 0{,}6 \text{ kN} \quad (T) \\ N_{CD} = 0{,}8 \text{ kN} \quad (T) \end{cases}$$

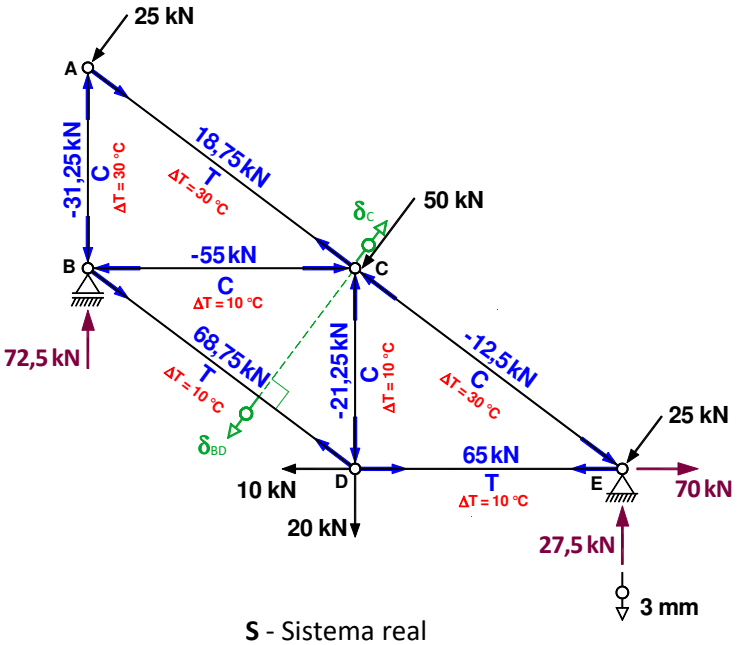
Nó D



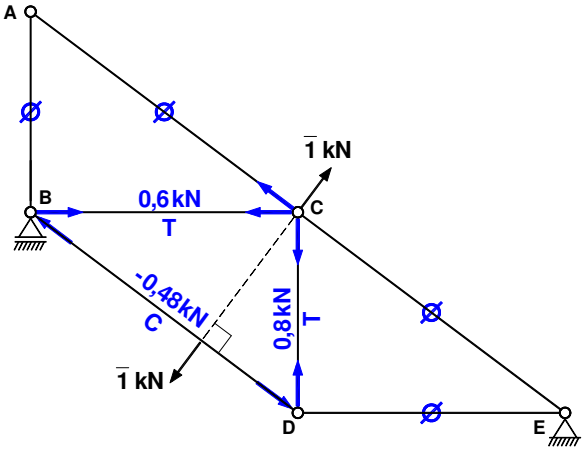
$$\sum F_x = 0 \Rightarrow -0{,}8\,N_{BD} - 0{,}384 = 0$$

$$N_{BD} = -0{,}48 \text{ kN} \quad (C)$$





S - Sistema real



$\bar{S}$ - Sistema virtual

$$\bar{1} \times \delta_{BD} + \bar{1} \times \delta_C + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{N\,L}{E\,A} + \Sigma \bar{N} \cdot \alpha\,\Delta T\,L$$

BARRAS	L (m)	E (kPa)	A (m^2)	N (kN)	$\bar{N}$ (kN)	ΔT (°C)	α.ΔT.L (α=1,2x10^{-5}/°C)	$\bar{N}$.α .ΔT. L	$\bar{N}\frac{N\,L}{E\,A}$
AB	3	200 x 10^6	17,1 x 10^{-4}	-31,25	0	30	108 x 10^{-5}	0	0
AC	5	200 x 10^6	17,1 x 10^{-4}	18,75	0	30	180 x 10^{-5}	0	0
CE	5	200 x 10^6	17,1 x 10^{-4}	-12,50	0	30	180 x 10^{-5}	0	0
BC	4	200 x 10^6	24,4 x 10^{-4}	-55,00	0,6	10	48 x 10^{-5}	28,8 x 10^{-5}	-27,049 x 10^{-5}
BD	5	200 x 10^6	24,4 x 10^{-4}	68,75	- 0,48	10	60 x 10^{-5}	-28,8 x 10^{-5}	-33,811 x 10^{-5}
CD	3	200 x 10^6	24,4 x 10^{-4}	-21,25	0,8	10	36 x 10^{-5}	28,8 x 10^{-5}	-10,451 x 10^{-5}
DE	4	200 x 10^6	24,4 x 10^{-4}	65,00	0	10	48 x 10^{-5}	0	0
Σ								28,8 x 10^{-5}	-71,311 x 10^{-5}

$$\bar{1} \times \delta_{BD} + \bar{1} \times \delta_C + \Sigma \bar{R} \times \text{assent. apoio} = \Sigma \bar{N} \frac{N\,L}{E\,A} + \Sigma \bar{N} \cdot \alpha\,\Delta T\,L$$

$\delta_{BD} + \delta_C = -71{,}311\text{x}10^{-5} + 28{,}8\text{x}10^{-5} = -4{,}251\text{x}10^{-4}\text{m}$

$\delta_{BD} + \delta_C$ = **variação da distância entre o nó C e a barra BD** = -0,425 mm (aproximação)